2004 AMC 12A-#21

(Series, Trigonometric Functions)

If $\sum_{n=0}^{\infty} \cos^{2n}\theta = 5$, what is the value of $\cos 2\theta$?

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{\sqrt{5}}{5}$

(D) $\frac{3}{5}$

(E) $\frac{4}{5}$

1998 AIME - #15

(Eulerian Circuit)

Define a *domino* to be an ordered pair of *distinct* positive integers. A *proper sequence* of dominos is a list of distinct dominos in which the immediately preceding pair, and in which (i,j) and (j,i) do not *both* appear for any i and j. Let D_{40} be the set of all dominos whose coordinates are no larger than 40. Find the length of the longest proper sequence of dominos that can be formed using the dominos of D_{40}.

2006 AMC 12A-#18

(Function notation)

The function f has the property that for each real number x in its domain, $1/x$ is also in its domain and

$$f(x) + f\left(\frac{1}{x}\right) = x.$$

What is the largest set of real numbers that can be in the domain of f?

(A) $\{x \mid x \neq 0\}$

(B) $\{x \mid x < 0\}$

(C) $\{x \mid x > 0\}$

(D) $\{x \mid x \neq -1 \text{ and } x \neq 0 \text{ and } x \neq 1\}$

(E) $\{-1, 1\}$

2005 AMC 12B-#22

(Complex numbers, Sequences)

A sequence of complex numbers $z_0, z_1, z_2, \ldots$ is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}},$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many values are there for z_0?

(A) 1

(B) 2

(C) 4

(D) 2005

(E) 2^{2005}

2005 USAMO - #2

(Fermat's Theorem, Diophantine Equations)

Prove that the system

$$x^6 + x^3 + x^3y + y = 147^{157}$$

$$x^3 + x^3y + y^2 + y + z^9 = 157^{147}$$

has no solutions in integers x, y, and z.

2005 AMC 12A-#23

(Logarithms)

Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots, 2^{25}\}$. What is the probability that $\log_a b$ is an integer?

(A) $\frac{2}{25}$

(B) $\frac{31}{300}$

(C) $\frac{13}{100}$

(D) $\frac{7}{50}$

(E) $\frac{1}{2}$

These past problems from the upper levels of the American Mathematics Competitions are based on Eulerian Concepts · give them a try . . .

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Euler

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