AMC 12 Student Practice Questions

- At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2005 AMC 12 B, Problem #3

2005 AMC 10 B, Problem #6— "How many quizzes does she need to earn an A on?"

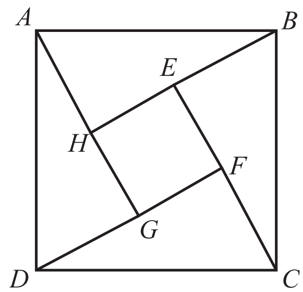
- **Solution (B)** To earn an A on at least 80% of her quizzes, Lisa needs to receive an A on at least $(0.8)(50) = 40$ quizzes. Thus she must earn an A on at least $40 - 22 = 18$ of the remaining 20. So she can earn a grade lower than an A on at most 2 of the remaining quizzes.

Difficulty: Easy

NCTM Standard: Problem Solving Standard: Solve problems that arise in mathematics and in other contexts

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Percent

- Square $EFGH$ is inside square $ABCD$ so that each side of $EFGH$ can be extended to pass through a vertex of $ABCD$. Square $ABCD$ has side length $\sqrt{50}$, and $BE = 1$. What is the area of the inner square $EFGH$?



(A) 25 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 42

2005 AMC 12 A, Problem #7— "What can one say about triangle BEC?"

- **Solution (C)** The symmetry of the figure implies that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles. So

$$BH = CE = \sqrt{BC^2 - BE^2} = \sqrt{50 - 1} = 7,$$

and $EH = BH - BE = 7 - 1 = 6$. Hence the square $EFGH$ has area $6^2 = 36$.

OR

As in the first solution, $BH = 7$. Now note that $\triangle ABH$, $\triangle BCE$, $\triangle CDF$, and $\triangle DAG$ are congruent right triangles, so

$$\text{Area}(EFGH) = \text{Area}(ABCD) - 4\text{Area}(\triangle BCE) = 50 - 4\left(\tfrac{1}{2} \cdot 1 \cdot 7\right) = 36.$$

Difficulty: Medium-easy
NCTM Standard: Geometry Standard: Use visualization, spatial reasoning, and geometric modeling to solve problems.
Mathworld.com Classification:
Geometry > Plane Geometry > Squares > Square

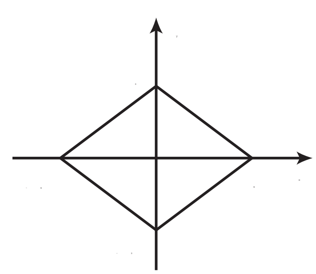
- What is the area enclosed by the graph of $|3x| + |4y| = 12$?

(A) 6 **(B)** 12 **(C)** 16 **(D)** 24 **(E)** 25

2005 AMC 12 B, Problem #7— "What is the shape of the region?"

- **Solution (D)** The graph is symmetric with respect to both coordinate axes, and in the first quadrant it coincides with the graph of the line $3x + 4y = 12$. Therefore the region is a rhombus, and the area is

$$\text{Area} = 4\left(\frac{1}{2}(4 \cdot 3)\right) = 24.$$



Difficulty: Medium-hard

NCTM Standard: Geometry Standard: Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Mathworld.com Classification:
Geometry > Plane Geometry > Quadrilaterals > Rhombus

- On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

(A) $\frac{5}{11}$ **(B)** $\frac{10}{21}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{11}{21}$ **(E)** $\frac{6}{11}$

2005 AMC 12 A, Problem #14— "What are the effects on each face?"

- **Solution (D)** A standard die has a total of 21 dots. For $1 \le n \le 6$, a dot is removed from the face with n dots with probability $n/21$. That face is left with an odd number of dots with probability $n/21$ if n is even and $1 - n/21$ if n is odd. Each face is the top face with probability $1/6$. Therefore the top face has an odd number of dots with probability

$$\begin{aligned}\frac{1}{6}\left(\left(1-\frac{1}{21}\right)+\frac{2}{21}+\left(1-\frac{3}{21}\right)+\frac{4}{21}+\left(1-\frac{5}{21}\right)+\frac{6}{21}\right) &= \frac{1}{6}\left(3+\frac{3}{21}\right)\\ &= \frac{1}{6}\cdot\frac{66}{21}=\frac{11}{21}.\end{aligned}$$

OR

The probability that the top face is odd is $1/3$ if a dot is removed from an odd face, and the probability that the top face is odd is $2/3$ if a dot is removed from an even face. Because each dot has the probability $1/21$ of being removed, the top face is odd with probability

$$\left(\frac{1}{3}\right)\left(\frac{1+3+5}{21}\right)+\left(\frac{2}{3}\right)\left(\frac{2+4+6}{21}\right)=\frac{33}{63}=\frac{11}{21}.$$

Difficulty: Hard
NCTM Standard: Data Analysis and Probability Standard: Understand and apply basic concepts of probability
Mathworld.com Classification:
Probability and Statistics > Probability > Probability