For each positive integer $n$, let $S(n)$ denote the sum of the digits of $n$. For how many values of $n$ is $n + S(n) + S(S(n)) = 2007$?

**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** $5$

## 2007 AMC 10 A, Problem #25— 2007 AMC 12 A, Problem #22— "$n \geq 2007 - 28 - 10 = 1969$ and $n, S(n),$ and $S(S(n))$ must all be multiples of 3."

### Solution

**Answer (D):** If $n \leq 2007$, then $S(n) \leq S(1999) = 28$. If $n \leq 28$, then $S(n) \leq S(28) = 10$. Therefore if $n$ satisfies the required condition it must also satisfy

$$n \geq 2007 - 28 - 10 = 1969.$$

In addition, $n, S(n),$ and $S(S(n))$ all leave the same remainder when divided by 9. Because 2007 is a multiple of 9, it follows that $n, S(n),$ and $S(S(n))$ must all be multiples of 3. The required condition is satisfied by 4 multiples of 3 between 1969 and 2007, namely 1977, 1980, 1983, and 2001.

Note: There appear to be many cases to check, that is, all the multiples of 3 between 1969 and 2007. However, for $1987 \leq n \leq 1999$, we have $n + S(n) \geq 1990 + 19 = 2009$, so these numbers are eliminated. Thus we need only check 1971, 1974, 1977, 1980, 1983, 1986, 2001, and 2004.

**Difficulty:** Hard

**NCTM Standard:** Algebra Standard: analyze change in various contexts.

**Mathworld.com Classification:** Algebra > Sums