A finite sequence of three-digit integers has the property that the tens and units digits of each term are, respectively, the hundreds and tens digits of the next term, and the tens and units digits of the last term are, respectively, the hundreds and tens digits of the first term. For example, such a sequence might begin with terms 247, 475, and 756 and end with the term 824. Let S be the sum of all the terms in the sequence. What is the largest prime number that always divides S?

(A) 3 **(B)** 7 **(C)** 13 **(D)** 37 **(E)** 43

2007 AMC 10 A, Problem #22—

"A given digit appears as the hundreds digit, the tens digit, and the units digit of a term the same number of times."

Solution

Answer (D): A given digit appears as the hundreds digit, the tens digit, and the units digit of a term the same number of times. Let k be the sum of the units digits in all the terms. Then $S = 111k = 3 \cdot 37k$, so S must be divisible by 37. To see that S need not be divisible by any larger prime, note that the sequence 123, 231, 312 gives $S = 666 = 2 \cdot 3^2 \cdot 37$.

Difficulty: Hard

NCTM Standard: Number and Operations Standard: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

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