

A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. What is the expected return to the player, in dollars, for one roll of the dice?

- (A)  $-\frac{1}{8}$    (B)  $-\frac{1}{16}$    (C) 0   (D)  $\frac{1}{16}$    (E)  $\frac{1}{8}$

**2007 AMC 10 B, Problem #22—**

**“Find out the probability of the number appearing 0, 1, and 2 times.”**

**Solution**

**Answer (B):** The probability of the number appearing 0, 1, and 2 times is

$$P(0) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}, \quad P(1) = 2 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{6}{16}, \quad \text{and} \quad P(2) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16},$$

respectively. So the expected return, in dollars, to the player is

$$P(0) \cdot (-1) + P(1) \cdot (1) + P(2) \cdot (2) = \frac{-9 + 6 + 2}{16} = -\frac{1}{16}.$$

**Difficulty:** Hard

**NCTM Standard:** Data Analysis and Probability for Grades 9-12: Understand and apply basic concepts of probability.

**Mathworld.com Classification:** Probability and Statistics > Probability