

Wave propagation in a piezoelectric ceramic plate sandwiched between two semiconductor layers

J.S. Yang* and H.G. Zhou

Department of Engineering Mechanics, University of Nebraska, Lincoln, NE 68588-0526, USA

Abstract. We study propagation of anti-plane (SH) waves in a piezoelectric ceramic plate carrying a thin layer of a semiconducting material on each of its major surfaces. Wave attenuation and dispersion caused by semiconduction as well as wave amplification by a biasing electric field are examined.

1. Introduction

Piezoelectric materials are either dielectrics or semiconductors [1]. An acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss [2]. The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called acoustoelectric effect which is a special case of a more general phenomenon which may be called wave-particle drag [3]. It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by application of a dc electric field [4]. This phenomenon is called acoustoelectric amplification of acoustic waves. Acoustoelectric effect and amplification of acoustic waves can also be achieved in composite structures of piezoelectric dielectrics and nonpiezoelectric semiconductors [5]. In these composites the acoustoelectric effect is due to the combination of the piezoelectric effect and semiconduction in each component phase. Acoustoelectric effect and acoustoelectric amplification of acoustic waves have led to the development of acoustoelectric devices [5–8]. Both surface acoustic waves (SAW) and bulk acoustic waves (BAW) have been used for acoustoelectric devices. The basic behavior of piezoelectric semiconductors and acoustoelectric effect can be described by a linear phenomenological theory [2,4]. More sophisticated nonlinear theories for deformable semiconductors have also been developed [9,10]. Due to multi-field coupling and anisotropy, device modeling by these theories presents complicated mathematical problems. Exact analysis is possible only in rare cases, e.g., the one-dimensional problem of thickness vibrations of plates [11]. Recently two-dimensional equations for a thin piezoelectric semiconductor film were derived in [12] and were used to analyze the propagation of surface acoustic waves over a piezoelectric half-space carrying a thin semiconductor film. The two-dimensional equations for the film simplified the analysis considerably. In this paper

*Corresponding author. Tel.: +1 402 472 0712; Fax: +1 402 472 8292; E-mail: jyang1@unl.edu.

we study the propagation of anti-plane waves in a piezoelectric ceramic plate carrying a thin layer of a nonpiezoelectric semiconductor on each of its two major surfaces. Three-dimensional equations of linear piezoelectric semiconductors are summarized in Section 2. Two-dimensional equations for extensional motions of a thin piezoelectric semiconductor film are given in Section 3 to model the semiconductor layers. Propagation of anti-plane waves under a dc field is analyzed in Section 4. Frequency equations that determine the dispersion relations are obtained. An approximate analytical solution of the frequency equations and numerical results are presented in Section 5. Finally, some conclusions are drawn in Section 6.

2. Three-dimensional equations

Consider a homogeneous, one-carrier piezoelectric semiconductor under a uniform dc electric field \bar{E}_j . The steady state current is $\bar{J}_i = q\bar{n}\mu_{ij}\bar{E}_j$, where q is the carrier charge, \bar{n} is the steady state carrier density which produces electrical neutrality, and μ_{ij} is the carrier mobility. The summation convention for repeated tensor indices is used. When an acoustic wave propagates through the material, perturbations of the electric field, the carrier density and the current are denoted by E_j , n and J_i . The linear theory for small signals [2,11] consists of the equations of motion, Gauss's law, and conservation of charge

$$\begin{aligned} T_{j,i,j} &= \rho\ddot{u}_i, \\ D_{i,i} &= qn, \\ q\dot{n} + J_{i,i} &= 0, \end{aligned} \quad (1)$$

where u_i is the displacement vector, T_{ij} is the stress tensor, ρ is the mass density, and D_i is the electric displacement vector. A comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents differentiation with respect to time. The above equations are accompanied by the following constitutive relations [4]

$$\begin{aligned} T_{ij} &= c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_i &= e_{ijk}S_{jk} + \varepsilon_{ij}E_j, \\ J_i &= q\bar{n}\mu_{ij}E_j + qn\mu_{ij}\bar{E}_j - qd_{ij}n_{,j}, \end{aligned} \quad (2)$$

where the strain tensor S_{ij} and the electric potential ϕ are defined by

$$\begin{aligned} S_{ij} &= (u_{i,j} + u_{j,i})/2, \\ E_i &= -\phi_{,i}. \end{aligned} \quad (3)$$

In Eq. (2), c_{ijkl} , e_{kij} and ε_{ij} are the elastic, piezoelectric and dielectric constants. d_{ij} are the carrier diffusion constants. With successive substitutions from Eqs (2) and (3), Eq. (1) can be written as five equations for \mathbf{u} , ϕ and n

$$\begin{aligned} c_{ijkl}\mathbf{u}_{k,lj} + e_{kij}\phi_{,kj} &= \rho\ddot{u}_i, \\ e_{ikl}\mathbf{u}_{k,li} - \varepsilon_{ij}\phi_{,ij} &= qn, \\ \dot{n} - \bar{n}\mu_{ij}\phi_{,ij} + \mu_{ij}\bar{E}_jn_{,i} - d_{ij}n_{,ij} &= 0. \end{aligned} \quad (4)$$

On the boundary of a finite body with a unit outward normal n_i , the mechanical displacement u_i or the traction vector $T_{ij}n_i$, the electric potential ϕ or the normal component of the electric displacement vector D_in_i , and the carrier density n or the normal current J_in_i may be prescribed [11].

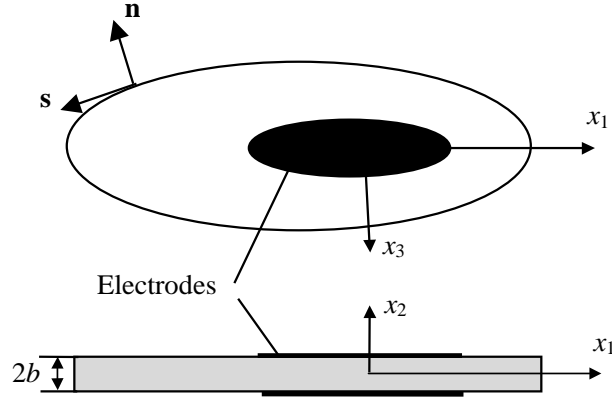


Fig. 1. Plan view and cross section of a piezoelectric semiconductor thin film.

3. Equations for a thin film

The two-dimensional equations for a thin piezoelectric semiconductor film in [12] are summarized below. Consider a piezoelectric semiconductor film of thickness $2b$ as shown in Fig. 1, along with the coordinate system. The film is assumed to be very thin. We need a two-dimensional theory for the film in the manner of the equations for generalized plane stress in elasticity [13]. For a thin film the dominating stress components are T_{11} , T_{33} and T_{13} . We make the usual stress relaxation

$$T_{2j} = 0, \quad j = 1, 2, 3. \quad (5)$$

According to the compact matrix notation [14], with the range of p, q as $1, 2, \dots$ and 6 , Eq. (5) can be written as

$$T_q = 0, \quad q = 2, 4, 6. \quad (6)$$

For convenience we introduce a convention that subscripts u, v, w take the values $2, 4, 6$ while subscripts r, s, t take the remaining values $1, 3, 5$. Then Eq. (2)_{1,2} can be written as

$$\begin{aligned} T_r &= c_{rs}S_s + c_{ru}S_u - e_{kr}E_k, \\ T_v &= c_{vs}S_s + c_{vw}S_w - e_{kv}E_k = 0, \\ D_i &= e_{is}S_s + e_{iu}S_u + \varepsilon_{ij}E_j, \end{aligned} \quad (7)$$

where Eq. (6) has been used. From Eq. (7)₂ we have

$$S_u = -c_{uv}^{-1}c_{vs}S_s + c_{uv}^{-1}e_{kv}E_k. \quad (8)$$

Substitution of Eq. (8) into Eq. (7)_{1,3} gives the mechanical and electric constitutive relations for the membrane

$$\begin{aligned} T_r &= \bar{c}_{rs}S_s - \bar{e}_{kr}E_k, \\ D_i &= \bar{e}_{is}S_s + \bar{\varepsilon}_{ij}E_j, \end{aligned} \quad (9)$$

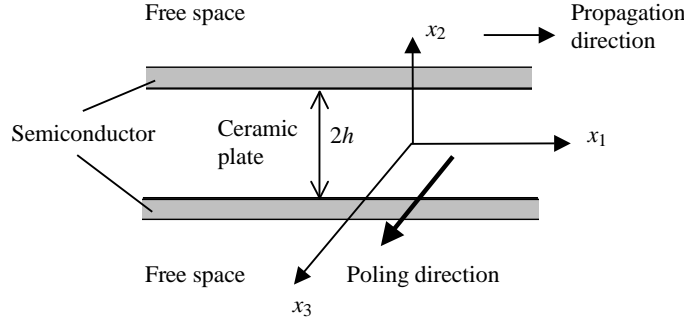


Fig. 2. A ceramic plate with semiconductor films.

where the membrane material constants are

$$\bar{c}_{rs} = c_{rs} - c_{rv}c_{vw}^{-1}c_{ws}, \quad \bar{e}_{ks} = e_{ks} - e_{kw}c_{wv}^{-1}c_{vs}, \quad \bar{\epsilon}_{kj} = \epsilon_{kj} + e_{kv}c_{vw}^{-1}e_{jw}. \quad (10)$$

We now introduce another convention that subscripts a, b, c and d assume 1 and 3 but not 2. Then Eq. (9) can be written as

$$\begin{aligned} T_{ab} &= \bar{c}_{abcd}S_{cd} - \bar{e}_{kab}E_k, \\ D_i &= \bar{e}_{iab}S_{ab} + \bar{\epsilon}_{ij}E_j. \end{aligned} \quad (11)$$

Integrating the equations in Eq. (1)₁ for $i = 1, 3$ and Eq. (1)_{2,3} with respect to x_2 through the film thickness, we obtain the following two-dimensional equations of motion, Gauss's law and conservation of charge

$$\begin{aligned} T_{ab,a} + \frac{1}{2b}[T_{2b}(x_2 = b) - T_{2b}(x_2 = -b)] &= \rho\ddot{u}_b, \\ D_{a,a} + \frac{1}{2b}[D_2(x_2 = b) - D_2(x_2 = -b)] &= qn, \\ q\dot{n} + J_{a,a} + \frac{1}{2b}[J_2(x_2 = b) - J_2(x_2 = -b)] &= 0, \end{aligned} \quad (12)$$

where u_a, T_{ab}, D_a, J_a and n are understood as averages of the corresponding three-dimensional quantities along the plate thickness.

4. Propagation of plate waves

Consider a piezoelectric plate of polarized ceramics which carries a semiconductor layer of silicon on each of its major surfaces (Fig. 2). Silicon is a cubic crystal with m3m symmetry which does not have piezoelectric coupling. The films are assumed to be very thin as compared to the thickness of the plate and the wavelength we are interested in, and can be modeled by the film equations in the previous section. In this case the equations of the thin films will appear as interface continuity conditions between the ceramic plate and the free space at $x_2 = \pm h$. This type of approximation has been used to model

surface waves guided by thin films in elasticity [15]. Consider the ceramic plate first. The ceramic is poled in the x_3 direction with [14]

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{31} & c_{31} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \begin{pmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^T, \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \tag{13}$$

where $c_{66} = (c_{11} - c_{12})/2$. We are interested in anti-plane motions with

$$u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2, t), \quad \phi = \phi(x_1, x_2, t). \tag{14}$$

The non-vanishing strain and electric field components are

$$\begin{Bmatrix} 2S_{13} \\ 2S_{23} \end{Bmatrix} = \nabla u_3, \quad \begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = -\nabla \phi, \tag{15}$$

where $\nabla = \mathbf{i}_1 \partial_1 + \mathbf{i}_2 \partial_2$ is the two-dimensional gradient operator. The nontrivial components of T_{ij} and D_i are

$$\begin{Bmatrix} T_{13} \\ T_{23} \end{Bmatrix} = c_{44} \nabla u_3 + e_{15} \nabla \phi, \quad \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = e_{15} \nabla u_3 - \varepsilon_{11} \nabla \phi, \tag{16}$$

The nontrivial ones of the equations of motion and charge take the following form:

$$\begin{aligned} c_{44} \nabla^2 u_3 + e_{15} \nabla^2 \phi &= \rho \ddot{u}_3, \\ e_{15} \nabla^2 u_3 - \varepsilon_{11} \nabla^2 \phi &= 0, \end{aligned} \tag{17}$$

where ∇^2 is the two-dimensional Laplacian $\nabla^2 = \partial_1^2 + \partial_2^2$. Introducing [16]

$$\psi = \phi - \frac{e_{15}}{\varepsilon_{11}} u_3, \tag{18}$$

then

$$\begin{aligned} T_{23} &= \bar{c}_{44} u_{3,2} + e_{15} \psi_{,2}, \\ T_{31} &= \bar{c}_{44} u_{3,1} + e_{15} \psi_{,1}, \\ D_1 &= -\varepsilon_{11} \psi_{,1}, \\ D_2 &= -\varepsilon_{11} \psi_{,2}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} \bar{c}_{44} \nabla^2 u_3 &= \rho \ddot{u}_3, \\ \nabla^2 \psi &= 0, \end{aligned} \tag{20}$$

where

$$\bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}} = c_{44}(1 + k_{15}^2), \quad k_{15}^2 = \frac{e_{15}^2}{\varepsilon_{11} c_{44}}. \tag{21}$$

4.1. Symmetric waves

Consider the possibility of solutions in the following form:

$$\begin{aligned} u_3 &= A \cos \xi_2 x_2 e^{i(\xi_1 x_1 - \omega t)}, \\ \psi &= B \cosh \xi_1 x_2 e^{i(\xi_1 x_1 - \omega t)}, \end{aligned} \quad (22)$$

where A and B are undetermined constants. Eq. (22)₂ already satisfies Eq. (20)₂. For Eq. (22)₁ to satisfy Eq. (20)₁ we must have

$$\bar{c}_{44}(\xi_1^2 + \xi_2^2) = \rho\omega^2, \quad (23)$$

which leads to the following expression for ξ_2

$$\xi_2^2 = \frac{\rho\omega^2}{\bar{c}_{44}} - \xi_1^2 = \xi_1^2 \left(\frac{v^2}{v_T^2} - 1 \right), \quad (24)$$

where

$$v^2 = \frac{\omega^2}{\xi_1^2}, \quad v_T^2 = \frac{\bar{c}_{44}}{\rho}. \quad (25)$$

The following will be needed for boundary and continuity conditions:

$$\begin{aligned} \phi &= \left(\frac{e_{15}}{\varepsilon_{11}} A \cos \xi_2 x_2 + B \cosh \xi_1 x_2 \right) e^{i(\xi_1 x_1 - \omega t)}, \\ T_{23} &= (-A \bar{c}_{44} \xi_2 \sin \xi_2 x_2 + e_{15} B \xi_1 \sinh \xi_1 x_2) e^{i(\xi_1 x_1 - \omega t)}, \\ D_2 &= -\varepsilon_{11} B \xi_1 \sinh \xi_1 x_2 e^{i(\xi_1 x_1 - \omega t)}. \end{aligned} \quad (26)$$

Electric fields can also exist in the free space of $x_2 > h$, which is governed by

$$\begin{aligned} \nabla^2 \phi &= 0, \quad x_2 > h, \\ \phi &\rightarrow 0, \quad x_2 \rightarrow +\infty. \end{aligned} \quad (27)$$

The relevant solution to Eq. (27) is

$$\begin{aligned} \phi &= C e^{\xi_1(h-x_2)} e^{i(\xi_1 x_1 - \omega t)}, \\ D_2 &= -\varepsilon_0 \phi_{,2} = \varepsilon_0 \xi_1 C e^{\xi_1(h-x_2)} e^{i(\xi_1 x_1 - \omega t)}. \end{aligned} \quad (28)$$

where C is an undetermined constant.

For long, anti-plane waves, fields in the semiconductor film at $x_2 = h$ depends approximately on x_1 and time only. Denote the fields in the film by

$$u_3 = A \cos \xi_2 h e^{i(\xi_1 x_1 - \omega t)}, \quad \phi = C e^{i(\xi_1 x_1 - \omega t)}, \quad n = N e^{i(\xi_1 x_1 - \omega t)}, \quad (29)$$

where N is an undetermined constant. At $x_2 = h$, Eq. (29) already satisfies the continuity of mechanical displacement between the film and the ceramic plate, and the continuity of electric potential between the

film and the free space. We use a prime to indicate the elastic and dielectric constants as well as the mass density of the film. The elastic and dielectric constants are given by

$$\begin{pmatrix} c'_{11} & c'_{12} & c'_{12} & 0 & 0 & 0 \\ c'_{12} & c'_{11} & c'_{12} & 0 & 0 & 0 \\ c'_{12} & c'_{12} & c'_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{44} \end{pmatrix}, \begin{pmatrix} \varepsilon'_{11} & 0 & 0 \\ 0 & \varepsilon'_{11} & 0 \\ 0 & 0 & \varepsilon'_{11} \end{pmatrix}. \tag{30}$$

Consider the case when the dc biasing electric field is in the x_1 direction. From Eq. (11) and Eq. (2)₃ we obtain

$$\begin{aligned} T_{13} &= \bar{c}'_{55} S_{13} = \bar{c}'_{44} u_{3,1} = \bar{c}'_{44} i \xi_1 A \cos \xi_2 h e^{i(\xi_1 x_1 - \omega t)}, \\ D_1 &= \bar{\varepsilon}'_{11} E_1 = -\bar{\varepsilon}'_{11} \phi_{,1} = -\bar{\varepsilon}'_{11} i \xi_1 C e^{i(\xi_1 x_1 - \omega t)}, \\ J_1 &= -q \bar{n} \mu_{11} \phi_{,1} + q n \mu_{11} \bar{E}_1 - q d_{11} n_{,1} \\ &= (-q \bar{n} \mu_{11} i \xi_1 C + q N \mu_{11} \bar{E}_1 - q d_{11} i \xi_1 N) e^{i(\xi_1 x_1 - \omega t)}. \end{aligned} \tag{31}$$

Substitution of Eqs (26), (28), (29) and (31) into the continuity condition of the electric potential between the ceramic plate and the film, Eq. (12)₁ for $b = 3$, and Eq. (12)_{2,3} yields

$$\begin{aligned} \frac{e_{15}}{\varepsilon_{11}} A \cos \xi_2 h + B \cosh \xi_1 h &= C, \\ -\bar{c}'_{44} \xi_1^2 A \cos \xi_2 h - \frac{1}{2b} (-A \bar{c}_{44} \xi_2 \sin \xi_2 h + e_{15} B \xi_1 \sinh \xi_1 h) &= -\rho' \omega^2 A \cos \xi_2 h, \\ \bar{\varepsilon}'_{11} \xi_1^2 C + \frac{1}{2b} (\varepsilon_0 \xi_1 C + \varepsilon_{11} B \xi_1 \sinh \xi_1 h) &= q N, \\ -q i \omega N + i \xi_1 (-q \bar{n} \mu_{11} i \xi_1 C + q N \mu_{11} \bar{E}_1 - q d_{11} i \xi_1 N) &= 0, \end{aligned} \tag{32}$$

which is a system of linear homogeneous equations for A , B , C and N . For nontrivial solutions the determinant of the coefficient matrix has to vanish

$$\begin{vmatrix} \frac{e_{15}}{\varepsilon_{11}} \cos \xi_2 h & \cosh \xi_1 h & -1 & 0 \\ (\rho' \omega^2 - \bar{c}'_{44} \xi_1^2) \cos \xi_2 h & & & \\ + \frac{\bar{c}_{44} \xi_2}{2b} \sin \xi_2 h & -\frac{e_{15} \xi_1}{2b} \sinh \xi_1 h & 0 & 0 \\ 0 & \frac{\varepsilon_{11} \xi_1}{2b} \sinh \xi_1 h & \bar{\varepsilon}'_{11} \xi_1^2 + \frac{\varepsilon_0 \xi_1}{2b} & -q \\ 0 & 0 & q \bar{n} \mu_{11} \xi_1^2 & -q i \omega + i \xi_1 q \mu_{11} \bar{E}_1 \\ & & & + q d_{11} \xi_1^2 \end{vmatrix} = 0, \tag{33}$$

which determines the dispersion relation, a relation between ω and ξ_1 , of the waves. Equation (33) can be further written as

$$\bar{k}_{15}^2 \tanh \xi_1 h + \frac{\xi_2}{\xi_1} \tan \xi_2 h + \left(\frac{\omega^2}{\xi_1^2} \frac{\rho'}{\bar{c}'_{44}} - 1 \right) \frac{\bar{c}'_{44}}{\bar{c}_{44}} \xi_1 2b \tag{34}$$

$$= \frac{\bar{k}_{15}^2 \tanh^2 \xi_1 h}{\tanh \xi_1 h + \frac{\varepsilon_0}{\varepsilon_{11}} + \frac{\bar{\varepsilon}'_{11}}{\varepsilon_{11}} \xi_1 2b - \frac{2bq\bar{n}\mu_{11}}{\varepsilon_{11} \left[i \left(\frac{\omega}{\xi_1} - \mu_{11} \bar{E}_1 \right) - d_{11} \xi_1 \right]}}$$

where $\bar{k}_{15}^2 = e^2 / (\varepsilon_{11} \bar{c}_{44})$. The symmetric fields for $x_2 < 0$ and the continuity conditions at $x_2 = -h$ yield the same dispersion relation. For symmetric fields, all fields in the lower film are the same as those in the upper film.

4.2. Anti-symmetric waves

Consider the following fields in the plate

$$\begin{aligned} u_3 &= A \sin \xi_2 x_2 e^{i(\xi_1 x_1 - \omega t)}, \\ \psi &= B \sinh \xi_1 x_2 e^{i(\xi_1 x_1 - \omega t)}. \end{aligned} \quad (35)$$

Equation (35)₂ satisfies Eq. (20)₂. For Eq. (35)₁ to satisfy Eq. (20)₁ we again have Eqs (23)–(25). The fields needed for boundary and continuity conditions are

$$\begin{aligned} \phi &= \left(\frac{e_{15}}{\varepsilon_{11}} A \sin \xi_2 x_2 + B \sinh \xi_1 x_2 \right) e^{i(\xi_1 x_1 - \omega t)}, \\ T_{23} &= (A \bar{c}_{44} \xi_2 \cos \xi_2 x_2 + e_{15} B \xi_1 \cosh \xi_1 x_2) e^{i(\xi_1 x_1 - \omega t)}, \\ D_2 &= -\varepsilon_{11} B \xi_1 \cosh \xi_1 x_2 e^{i(\xi_1 x_1 - \omega t)}. \end{aligned} \quad (36)$$

The fields in the free space of for $x_2 > 0$ are still given by Eq. (28). For fields in the film at $x_2 = h$, Eq. (29)₁ should be replaced by

$$u_3 = A \sin \xi_2 h e^{i(\xi_1 x_1 - \omega t)}, \quad (37)$$

and Eq. (31)₁ should be replaced by

$$T_{13} = \bar{c}'_{44} i \xi_1 A \sin \xi_2 h e^{i(\xi_1 x_1 - \omega t)}. \quad (38)$$

Substitution of the relevant equations in Eqs (36), (28), (37), (29)_{2,3}, (38) and (31)_{2,3} into the continuity condition of the electric potential between the ceramic plate and the film, Eq. (12)₁ for $b = 3$, and Eq. (12)_{2,3} yields

$$\begin{aligned} \frac{e_{15}}{\varepsilon_{11}} A \sin \xi_2 h + B \sinh \xi_1 h &= C, \\ -\bar{c}'_{44} \xi_1^2 A \sin \xi_2 h - \frac{1}{2b} (A \bar{c}_{44} \xi_2 \cos \xi_2 h + e_{15} B \xi_1 \cosh \xi_1 h) &= -\rho' \omega^2 A \sin \xi_2 h, \\ \bar{\varepsilon}'_{11} \xi_1^2 C + \frac{1}{2b} (\varepsilon_0 \xi_1 C + \varepsilon_{11} B \xi_1 \cosh \xi_1 h) &= qN, \\ -qi\omega N + i\xi_1 (-q\bar{n}\mu_{11} i \xi_1 C + qN\mu_{11} \bar{E}_1 - qd_{11} i \xi_1 N) &= 0. \end{aligned} \quad (39)$$

For nontrivial solutions

$$\begin{vmatrix} \frac{\epsilon_{15}}{\epsilon_{11}} \sin \xi_2 h & \sinh \xi_1 h & -1 & 0 \\ (\rho' \omega^2 - \bar{c}'_{44} \xi_1^2) \sin \xi_2 h & & & \\ -\frac{\bar{c}_{44} \xi_2}{2b} \cos \xi_2 h & -\frac{\epsilon_{15} \xi_1}{2b} \cosh \xi_1 h & 0 & 0 \\ 0 & \frac{\epsilon_{11} \xi_1}{2b} \cosh \xi_1 h & \bar{\epsilon}'_{11} \xi_1^2 + \frac{\epsilon_0 \xi_1}{2b} & -q \\ 0 & 0 & q \bar{n} \mu_{11} \xi_1^2 & -qi\omega + i\xi_1 q \mu_{11} \bar{E}_1 \\ & & & +qd_{11} \xi_1^2 \end{vmatrix} = 0. \tag{40}$$

Equation (40) can be further written as

$$\begin{aligned} & \bar{k}_{15}^2 \tan \xi_2 h - \frac{\xi_2}{\xi_1} \tanh \xi_1 h + \left(\frac{\omega^2 \rho'}{\xi_1^2 \bar{c}'_{44}} - 1 \right) \frac{\bar{c}'_{44}}{\bar{c}_{44}} \xi_1 2b \tan \xi_2 h \tanh \xi_1 h \\ & = \frac{\bar{k}_{15}^2 \tan \xi_2 h}{1 + \left(\frac{\epsilon_0}{\epsilon_{11}} + \frac{\bar{\epsilon}'_{11}}{\epsilon_{11}} \xi_1 2b \right) \tanh \xi_1 h - \frac{2bq \bar{n} \mu_{11} \tanh \xi_1 h}{\epsilon_{11} \left[i \left(\frac{\omega}{\xi_1} - \mu_{11} \bar{E}_1 \right) - d_{11} \xi_1 \right]}}. \end{aligned} \tag{41}$$

The anti-symmetric fields for $x_2 < 0$ and the continuity conditions at $x_2 = -h$ yield the same dispersion relation. Note that for anti-symmetric fields the carrier density perturbation n of the lower film has an opposite sign to that of the upper film, but q, \bar{n} and \bar{E}_1 are the same as those of the upper film.

Equations (34) and (41) show that the waves are dispersive. When $b = 0$, i.e., the semiconductor films do not exist, Eqs (34) and (41) reduce to the frequency equations in [17] for piezoelectric waves in a ceramic plate.

5. Discussion and numerical results

As an example we consider the lowest branch of the dispersion relations of the symmetric waves in Eq. (34). This particular branch determines what is called face-shear waves which are often used for devices applications. For long face-shear waves with $\xi_1 h \ll 1$, we have

$$\tan \xi_1 h \cong \xi_1 h, \quad \tanh \xi_1 h \cong \xi_1 h. \tag{42}$$

In this case, Eq. (34) reduces to

$$\begin{aligned} & \bar{k}_{15}^2 + \left[\left(\frac{v}{v_T} \right)^2 - 1 \right] + \left[\left(\frac{v}{v'_T} \right)^2 - 1 \right] \frac{\bar{c}'_{44}}{\bar{c}_{44}} \frac{2b}{h} \\ & = \frac{\bar{k}_{15}^2}{1 + \frac{\epsilon_0}{\epsilon_{11} \xi_1 h} + \frac{\bar{\epsilon}'_{11}}{\epsilon_{11}} \frac{2b}{h} - \frac{2bq \bar{n} \mu_{11}}{\epsilon_{11} \xi_1 h [i(v - \bar{E}_1 \mu_{11}) - d_{11} \xi_1]}} \end{aligned} \tag{43}$$

where

$$v_T'^2 = \frac{\bar{c}'_{44}}{\rho'}. \tag{44}$$

The denominator of the right hand side of Eq. (44) indicates that complex wave speed may be expected and the imaginary part of the complex wave speed may change its sign (wave amplification) when $\mu_{11}\bar{E}_1\xi_1 - \omega$ changes its sign or

$$v = \frac{\omega}{\xi_1} = \mu_{11}\bar{E}_1, \quad (45)$$

i.e., the acoustic wave speed is equal to the carrier drift speed. When semiconduction is small, Eq. (43) can be solved by an iteration or perturbation procedure. As the lowest (zero) order of approximation, we neglect the semiconduction and denote the zero-order solution by $v_{(0)}$. Then, from Eq. (43)

$$\bar{k}_{15}^2 + \left[\left(\frac{v_{(0)}}{v_T} \right)^2 - 1 \right] + \left[\left(\frac{v_{(0)}}{v'_T} \right)^2 - 1 \right] \frac{\bar{c}'_{44} 2b}{\bar{c}_{44} h} = \frac{\bar{k}_{15}^2}{1 + \frac{\varepsilon_0}{\varepsilon_{11}\xi_1 h} + \frac{\bar{\varepsilon}'_{11} 2b}{\varepsilon_{11} h}}. \quad (46)$$

For the next order we substitute $v_{(0)}$ into the right-hand side of Eq. (43) and obtain the following equation for $v_{(1)}$

$$\begin{aligned} \bar{k}_{15}^2 + \left[\left(\frac{v_{(1)}}{v_T} \right)^2 - 1 \right] + \left[\left(\frac{v_{(1)}}{v'_T} \right)^2 - 1 \right] \frac{\bar{c}'_{44} 2b}{\bar{c}_{44} h} \\ = \frac{\bar{k}_{15}^2}{1 + \frac{\varepsilon_0}{\varepsilon_{11}\xi_1 h} + \frac{\bar{\varepsilon}'_{11} 2b}{\varepsilon_{11} h} - \frac{2bq\bar{n}\mu_{11}}{\varepsilon_{11}\xi_1 h [i(v_{(0)} - \bar{E}_1\mu_{11}) - d_{11}\xi_1]}}, \end{aligned} \quad (47)$$

which suggests a wave that is both dispersive and dissipative.

For the ceramic plate consider PZT-5H with $\rho_0 = 7500 \text{ kg/m}^3$ and [1]

$$\begin{aligned} c_{11} &= 12.6, & c_{33} &= 11.7, & c_{44} &= 2.30, \\ c_{12} &= 7.95, & c_{13} &= 8.41 \times 10^{10} \text{ N/m}^2, & c_{66} &= (c_{11} - c_{12})/2, \\ e_{15} &= 17.0, & e_{31} &= -6.5, & e_{33} &= 23.3 \text{ C/m}^2, \\ \varepsilon_{11} &= 1700\varepsilon_0, & \varepsilon_{33} &= 1470\varepsilon_0, & \varepsilon_0 &= 8.854 \times 10^{-12} \text{ farads/m}. \end{aligned} \quad (48)$$

For silicon with $\rho_0 = 2332 \text{ kg/m}^3$, we have [18,19]

$$\begin{aligned} c'_{11} &= 16.57, & c'_{44} &= 7.956, & c'_{12} &= 6.39 \times 10^{10} \text{ N/m}^2, \\ \varepsilon'_{11} &= 11.8\varepsilon_0. \end{aligned} \quad (49)$$

The mobility of electrons and holes of silicon at 300°K are [20]

$$\mu_n = 1500, \quad \mu_p = 480 \text{ cm}^2/\text{V} - \text{sec}. \quad (50)$$

The diffusion constants can be determined from the Einstein relation [20]

$$D = \frac{kT}{q_e} \mu, \quad (51)$$

where T is the absolute temperature, and k the Boltzmann constant. At room temperature $kT/q_e = 0.026 \text{ V}$ [20] where $q_e = 1.602 \times 10^{-9} \text{ coulomb}$ is the electronic charge.

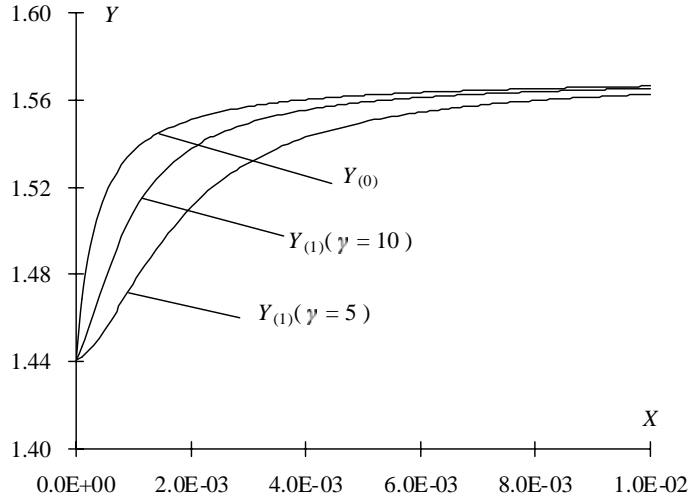


Fig. 3. Dispersion relations.

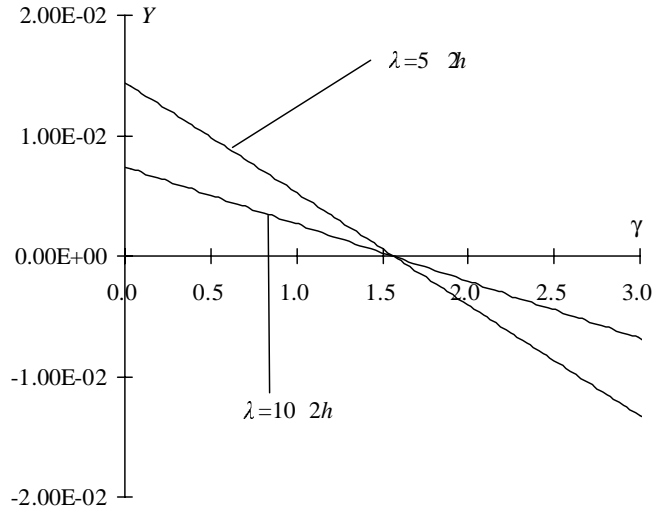


Fig. 4. Dissipation as a function of the dc bias. $Y = \text{Im}\{v_{(1)}\}/v_T$, $\lambda = 2\pi/\xi_1$.

We plot the real parts of $v_{(0)}$ and $v_{(1)}$ versus ξ_1 in Fig. 3. The dimensionless wave number X and the dimensionless wave speed Y of different orders are defined by

$$X = \xi_1 / \frac{\pi}{2h}, \quad Y_{(0)} = v_{(0)}/v_T, \quad Y_{(1)} = \text{Re}\{v_{(1)}\}/v_T. \tag{52}$$

γ is a dimensionless number given by

$$\gamma = \mu_{11} \bar{E}_1 / v_T, \tag{53}$$

which may be considered as a normalized electric field. It represents the ratio of the electron drift velocity and the speed of plane shear waves in the ceramic. Because of the use of the thin film equations

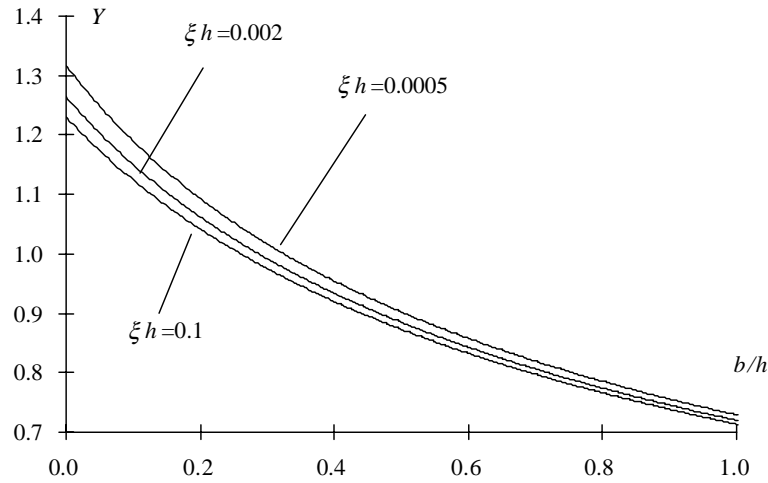


Fig. 5. The critical electric field $Y = \mu_{11} \bar{E}_1 / v^{(0)}$ for a plate of PZT-5H.

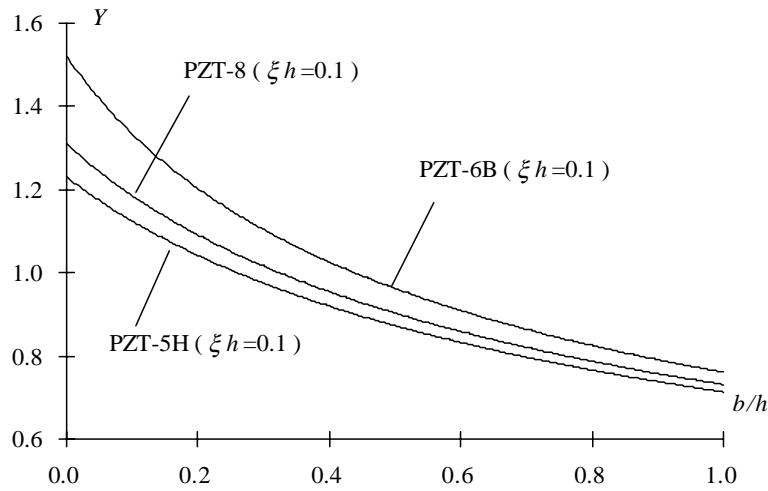


Fig. 6. The critical electric field $Y = \mu_{11} \bar{E}_1 / v^{(0)}$ for plates of different ceramics.

for the semiconductor layers and the approximations in Eq. (42), our solutions are valid only when the wavelength is much larger than the film thickness and the plate thickness. A sufficient condition for this is $X \ll 1$. It can be seen that semiconduction causes additional dispersion. This conduction induced dispersion varies according to the dc biasing electric field.

Figure 4 shows the imaginary part of $v_{(1)}$ versus γ . The dimensionless number describing the decaying behavior of the waves is defined by

$$Y = \text{Im}\{v_{(1)}\} / v_T. \tag{54}$$

The most important thing in design for acoustoelectric amplifiers is the critical electric field for wave amplification. Figure 4 shows that when the dc bias is sufficiently large, the decay constant becomes negative indicating wave amplification. The transition from damped waves to growing waves indeed

occurs when Eq. (45) is true for $v_{(0)}$. These agree qualitatively with the behavior of the 1-D plane waves studied in [4].

Since wave amplification occurs when Eq. (45) is true for $v_{(0)}$, we plot the normalized critical electric field at the transition, $Y = \mu_{11}\bar{E}_1/v^{(0)}$, versus the main design parameter b/h in Fig. 5 for waves with different wavelengths, where $v^{(0)}$ is determined from Eq. (46). These are all long waves. The figure shows a relatively simple behavior that the critical electric field decreases for increasing film thickness. This is as expected because the only origin of the energy source for wave amplification is the biasing electric field in the film. The critical electric field versus b/h for different ceramics is given in Fig. 6.

6. Conclusion

Dispersion relations for SH waves in a piezoelectric ceramic plate carrying a semiconductor film on each of its major surfaces are obtained. It is shown that semi-conduction causes dispersion and acoustic loss in piezoelectric waves. Acoustoelectric amplification occurs when the electron drift speed under a dc bias is larger than the acoustic wave speed. The results are useful for the design of plate acoustoelectric amplifiers. The equations for thin films simplify the analysis.

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