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Analysis of surface acoustic wave pressure sensors

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Abstract

The authors perform sensitivity analysis theoretically on three surface acoustic wave (SAW) pressure sensor structures and two material selections. The analyses take into consideration the effects of mounting structures, ways of transferring pressure to the sensing element, and various physical and geometrical parameters. It is shown that pressure-induced bending produces a larger change of wave speed than pressure-induced extension. A proposed shallow shell structure is demonstrated to increase the sensitivity of SAW pressure sensors, and for this structure, Si SAW exhibits a slightly higher sensitivity than Ge SAW.

Keywords: Wave; Sensor

1. Introduction

Surface acoustic wave (SAW) and bulk acoustic wave (BAW) resonators can be used to make pressure sensors and other acoustic wave sensors [1–7]. These sensors detect pressure by pressure-induced frequency shifts in the case of a stationary wave or change of wave speed when a propagating wave is used. Theoretical modeling of these sensors can be performed by considering small amplitude waves propagating in a medium of finite deformations due to the pressure [8]. Equations governing these waves are obtained by linearizing the nonlinear theory of elasticity (or electroelasticity [9] when a piezoelectric material is used) about initial deformations and fields. Modeling work on pressure sensors remains relatively few, according to a recent review [10], and most of the reported modeling results are concerned with single crystal resonators and sensors. The introduction of MEMs technology into pressure sensor development has largely eliminated the constraint in structure design that has limited the design of single crystal resonators and sensors. Recently, the importance of mounting structures on sensitivity in a BAW pressure sensor was explored in [11]. It was shown that sensitivity can be increased substantially by properly designing the sensor structure. In this paper, we provide a theoretical analysis for SAW pressure sensors. The first-order perturbation integral by Tiersten for calculating effects of pressure-induced initial stress and strain fields on wave frequency or speed is summarized in Section 2. The use of the perturbation integral requires the surface wave modes in an unstrained medium, which are given in Section 3. Then a crystal plate with different mounting structures as SAW pressure sensors is analyzed. These include pressure-induced extension (Section 4), pure bending directly induced by pressure (Section 5) and pure bending indirectly induced by pressure (Section 6). Numerical results for silicon and germanium are presented to illustrate the analysis.

2. Perturbation integral

Consider a homogeneous material body occupying a half-space V with $X_2 > 0$ in a Cartesian coordinate system X_K (see Fig. 1). The body is free from any deformations and fields.

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The mass density is ρ_0. The second- and third-order elastic constants are $c_{K\alpha L\gamma}$ and $c_{K\alpha L\gamma AB}$, respectively. Suppose that the governing equations and boundary conditions allow the propagation of a small-amplitude Rayleigh surface wave with frequency ω, wave number ξ in the X_1 direction, phase speed $V_R = \omega/\xi$ in the X_1 direction, and displacement u_α. Since the region is unbounded, we have an eigenvalue problem with a continuous spectrum. Given a wave number ξ, there always exists a wave with a frequency ω such that $\omega/\xi = V_R$ which depends on material properties only.

When an initial displacement field w with initial stress T^0 and initial strain E^0 is applied, the frequency of the wave of interest is perturbed a little and can be denoted by $\omega + \Delta\omega$. The change of wave frequency due to the initial fields is represented by the following integral from a first-order perturbation analysis [12]:

$$\frac{\Delta\omega}{\omega} \cong \frac{1}{2\omega^2}\frac{\int_V \hat{c}_{L\gamma M\alpha} u_{\gamma,L} u_{\alpha,M}\,\mathrm{d}V}{\int_V \rho_0 u_\alpha u_\alpha\,\mathrm{d}V} \quad (1)$$

where the effective second-order elastic material constants under the initial fields are given by:

$$\hat{c}_{K\alpha L\gamma} = T^0_{KL}\delta_{\alpha\gamma} + c_{K\alpha LN} w_{\gamma,N} + c_{KML\gamma} w_{\alpha,M} + c_{K\alpha L\gamma AB} E^0_{AB} \quad (2)$$

Equivalently, in terms of the wave speed, Eq. (1) can be written as [13]:

$$\Delta V = \frac{1}{2V_R\xi^2}\frac{\int_V \hat{c}_{L\gamma M\alpha} u_{\gamma,L} u_{\alpha,M}\,\mathrm{d}V}{\int_V \rho_0 u_\alpha u_\alpha\,\mathrm{d}V} \quad (3)$$

In order to use the perturbation integral to calculate the change of wave frequency or speed, two things are needed. One is the unperturbed wave in the absence of the initial fields. The other is the initial fields. They will be considered separately in the following sections.

3. Unperturbed surface wave

Consider a half-space of silicon as shown in Fig. 1. Silicon is of cubic symmetry ($m3m$) and allows plane-strain motions with $u_3 = 0$ and $\partial_3 = 0$. Rayleigh type surface wave solutions in a half-space of $m3m$ crystals with a traction-free boundary surface is given in [14]:

$$u_1 = \exp\left(-2\pi\beta\frac{X_2}{\lambda_R}\right)\cos\left(2\pi g\frac{X_2}{\lambda_R} + \alpha\right) \times \exp i\left[\omega\left(t - \frac{X_1}{V_R}\right) - \alpha\right],$$
$$u_2 = ir\exp\left(-2\pi\beta\frac{X_2}{\lambda_R}\right)\cos\left(2\pi g\frac{X_2}{\lambda_R} - \alpha\right) \times \exp i\left[\omega\left(t - \frac{X_1}{V_R}\right) - \alpha\right] \quad (4)$$

where β, λ_R, g, α, V_R and r depend on material parameters.

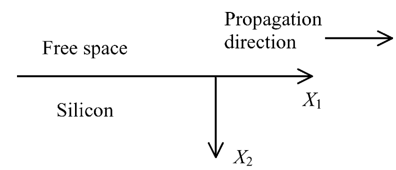

Fig. 1. An elastic half-space and coordinate system for surface waves.

4. Initial extension due to pressure

For pressure-induced initial deformations we examine several cases. In real SAW devices surface waves in fact propagate on one side of a plate. First consider a crystal plate sealed in a shallow shell [11] (see Fig. 2). When the structure in Fig. 2 is subject to a surrounding pressure p, the following extensional force develops in the crystal plate [11]:

$$N = \frac{pl^2}{d} \quad (5)$$

The strain fields in the plate are [11]:

$$E^0_1 = w_{1,1} = -\frac{N}{2h\bar{c}_{11}},$$
$$E^0_2 = w_{2,2} = -\frac{c_{21}}{c_{22}}w_{1,1} = \frac{c_{21}}{c_{22}}\frac{N}{2h\bar{c}_{11}} \quad (6)$$

where

$$\bar{c}_{11} = c_{11} - \frac{c_{12}^2}{c_{22}} \quad (7)$$

all other displacement gradients vanish.

With the unperturbed modes in Eq. (4) and the initial fields in Eq. (6) we are ready to calculate the perturbation integral. To illustrate the analysis, we consider two materials, silicon and germanium, in the following analysis. For silicon $\rho_0 = 2332\,\mathrm{kg/m^3}$ and [15]:

$$c_{11} = 16.57, \qquad c_{44} = 7.956,$$
$$c_{12} = 6.39 \times 10^{10}\,\mathrm{N/m^2} \quad (8)$$

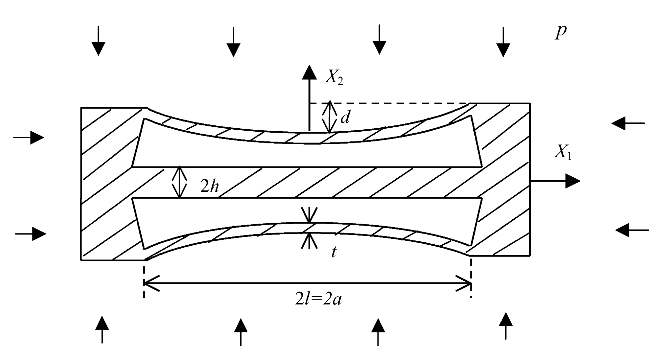

Fig. 2. A plate in extension in a shallow shell under pressure.

Then the parameters in the surface wave solution Eq. (4) can be determined as:

$$V_R = 4917\,\mathrm{m/s}, \quad g = 0.4808, \quad \beta = 0.4556,$$
$$r = 1.226, \quad \alpha = 58^\circ 1' \tag{9}$$

For the third-order elastic constants there are 20 nonzero ones among which six are independent [15]:

$$c_{111} = -825, \quad c_{112} = -451, \quad c_{123} = -64,$$
$$c_{144} = 12, \quad c_{155} = -310, \quad c_{456} = -64\,\mathrm{GPa} \tag{10}$$

The other 14 are determined from the following relations [16]:

$$c_{113} = c_{112}, \quad c_{122} = c_{112}, \quad c_{133} = c_{112},$$
$$c_{166} = c_{155}, \quad c_{222} = c_{111}, \quad c_{223} = c_{112},$$
$$c_{233} = c_{112}, \quad c_{244} = c_{155}, \quad c_{255} = c_{144},$$
$$c_{266} = c_{155}, \quad c_{333} = c_{111}, \quad c_{344} = c_{155},$$
$$c_{355} = c_{155}, \quad c_{366} = c_{144} \tag{11}$$

For germanium we have [15] $\rho = 5332\,\mathrm{kg/m^3}$ and

$$c_{11} = 12.9, \quad c_{44} = 6.68,$$
$$c_{12} = 4.9 \times 10^{10}\,\mathrm{N/m^2} \tag{12}$$

Then the following can be determined:

$$V_R = 2924\,\mathrm{m/s}, \quad g = 0.5122, \quad \beta = 0.4378,$$
$$r = 1.1938, \quad \alpha = 55^\circ 53' \tag{13}$$

The third-order material constants are:

$$c_{111} = -710, \quad c_{112} = -389, \quad c_{123} = -18,$$
$$c_{144} = -23, \quad c_{155} = -292, \quad c_{456} = -53\,\mathrm{GPa} \tag{14}$$

We are interested in SAW at $f = 1$ GHz. The corresponding wave length is $\lambda = 4.92 \times 10^{-3}$ mm for Si and 2.92×10^{-3} mm for Ge. For geometric parameters we choose $a = 20h$, $b = 20h$ and $2h = 20\lambda$. The plate thickness $2h$ is sufficiently large as compared to the wave length so that the plate is effectively a half-space for the SAW we are considering.

Pressure-induced change of wave speed is shown in Fig. 3. The relations are linear for small pressure. This is ideal for a pressure sensor. For large pressure the response will be nonlinear. In order to calculate the nonlinear response and determine the range for a linear response the fourth-order elastic constants are needed, which are unavailable in the literature, unfortunately. The slopes of the lines in the figure are related to sensitivity, and they confirm that a plate in a shallower shell has higher sensitivity. The sensitivity of Si is slightly larger than that of Ge.

5. Initial flexure due to direct action of pressure

Next we consider the plate shown in Fig. 4. To find the initial deformations we use the classical plate theory of flexure given in [17]. The plate is in pure bending with a bending moment of $M = pb^2/2$. For plane-strain motions with $w_3 = 0$ and $\partial_3 = 0$, the deflection $w_2^{(0)}$ is found to be:

$$\frac{2}{3}h^3\bar{c}_{11}w_2^{(0)} = \frac{M}{2}(X_1^2 - a^2) = \frac{pb^2}{4}(X_1^2 - a^2) \tag{15}$$

Then the displacement gradients needed in the perturbation integral can be found from the procedure in [17] as:

$$w_{1,1} = -\frac{3pb^2}{4\gamma_{11}h^3}X_2, \quad w_{1,2} = -\frac{3pb^2}{4\gamma_{11}h^3}X_2,$$
$$w_{1,3} = 0, \quad w_{2,1} = \frac{3pb^2}{4\gamma_{11}h^3}X_2,$$
$$w_{2,2} = \frac{c_{21}}{c_{11}}\frac{3pb^2}{4\gamma_{11}h^3}X_2, \quad w_{2,3} = 0,$$
$$w_{3,1} = 0, \quad w_{3,2} = 0, \quad w_{3,3} = 0 \tag{16}$$

Response of the SAW to the applied pressure is shown in Fig. 5. Note that the sign of the change of wave speed is opposite to that in Fig. 3 because Fig. 3 is for extension and In Fig. 4 the SAW propagates on the side of the beam in compression. Comparisons of Figs. 3 and 5 show that under the same applied pressure bending is more sensitive than extension because in bending the pressure-induced strain is mostly near the plate surface where the SAW is propagating.

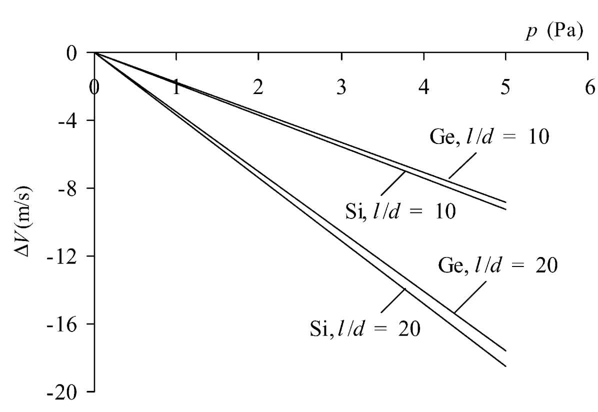

Fig. 3. Change of wave speed due to extension.

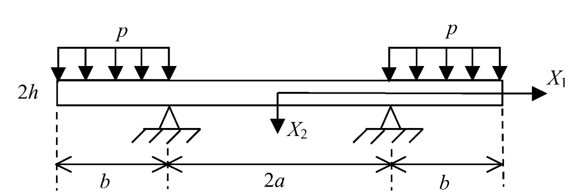

Fig. 4. A plate in pure bending directly under pressure.

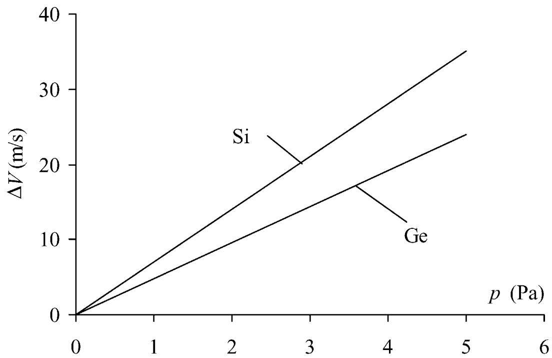

Fig. 5. Change of wave speed due to pure bending under direct pressure.

6. Initial flexure due to indirect action of pressure through a shallow shell

We now consider pure bending of a crystal plate in a shallow shell (see Fig. 6). To isolate and exhibit the effect of the shallow shell we imagine that the crystal plates are sealed such that the pressure is transmitted from the shallow shell to the crystal plates only. In this case the bending moment is approximately given by $M = Nb/2$, where N is given in Eq. (5). Then the deflection is:

$$\frac{2}{3}h^3\bar{c}_{11}w_2^{(0)} = \frac{M}{2}(X_1^2 - a^2) = \frac{Nb}{4}(X_1^2 - a^2) = \frac{pl^2}{d}\frac{b}{4}(X_1^2 - a^2) \quad (17)$$

In real sensor manufacturing the seals are not necessary. Then, the pressure acting on the crystal plates directly may be used to further increase the sensitivity. The displacement gradient can be found in a way similar to Eq. (16), with $pb^2/4$ in Eq. (15) replaced by $pl^2b/(4d)$.

Change of wave speed versus pressure is shown in Fig. 7 for different shallow shells. Again shallower shells imply higher sensitivity. We note that for the same pressure SAW in a shallow shell has a much higher sensitivity than the two previous cases.

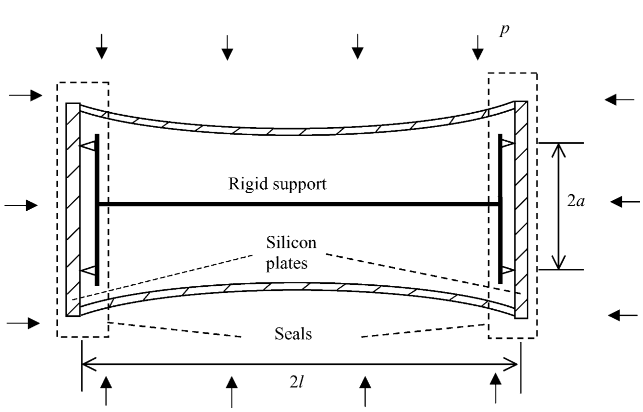

Fig. 6. A plate in pure bending in a shallow shell under pressure.

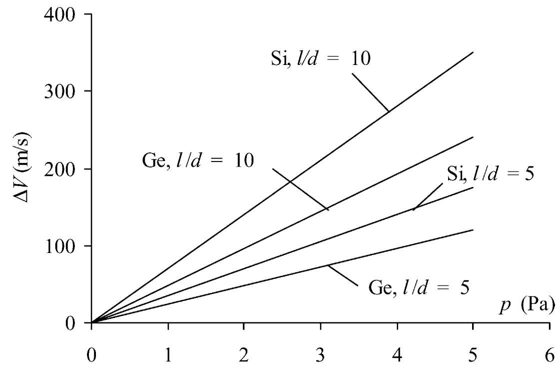

Fig. 7. Change of wave speed due to pure bending through a shallow shell.

7. Conclusion

Pressure-induced bending produces a larger change of wave speed than pressure-induced extension. A proposed shallow shell structure is demonstrated to increase the sensitivity of SAW pressure sensors, and for this structure, Si SAW exhibits a slightly higher sensitivity than Ge SAW.

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Biographies

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