Reduction of crystal resonator second-order normal acceleration sensitivity by overhang plates

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We propose to use overhang plates for reducing the second-order normal acceleration sensitivity of crystal resonators. A theoretical analysis is performed. Results show that the second-order normal acceleration sensitivity of a thickness-shear resonator can be reduced by two orders of magnitude. [DOI: 10.1063/1.2202726]

Crystal resonators are key components for oscillators and filters for time keeping, telecommunication, and control. These devices are often mounted on objects in motion, e.g., missiles and satellites. Resonators usually have shapes of plates. They operate with bulk acoustic waves (BAW, e.g., thickness-shear) or surface acoustic waves (SAW). When a plate resonator is under a normal acceleration (see Fig. 1), the inertial force due to the acceleration causes initial or biasing fields in the resonator, which in turn cause frequency shifts in the resonator. The first-order perturbation integral by Tiersten[1] has been widely used to calculate frequency shifts induced by normal accelerations in both BAW and SAW resonators. In particular, for a perfectly symmetric thickness-shear BAW resonator, the first-order frequency shift is found to be zero. Recently, it has been shown that when the first-order frequency shift vanishes, the second-order frequency shift may be nonzero and may thus provide a nonzero lower bound on the frequency shift.[2,3] Estimates show that the order of magnitude of the second-order normal acceleration sensitivity of BAW resonators is about 10^{-12}–10^{-11}/g.[2,3] Further reduction of this second-order effect is crucial to the ongoing research aimed at advancing the present 10^{-10}/g technology to the 10^{-12}/g technology in the future.

We propose to use overhang plates for reducing the second-order normal acceleration sensitivity. This technique had been used to reduce the first-order acceleration sensitivity.[4,5] Consider a crystal plate as shown in Fig. 2. The plate has two overhang portions in $a<|X_1|<a+b$. The active portion of the resonator is the central portion of $|X_1|<a$. For example, for a thickness-shear BAW resonator, vibrations can be confined to the central portion by energy trapping due to the electrode mass (see Fig. 2 where the thick lines represent electrodes when the plate represents a BAW resonator). When such a plate is under a normal acceleration, the inertial force in the overhang portions tends to reduce the deflection in the central portion due to the acceleration. This provides a possibility for design and optimization. By properly choosing the overhang length b, we can try to reduce or even remove the normal acceleration sensitivity.

First examine the effect of the overhangs on the deflection in the central portion of the plate under a normal acceleration. Due to symmetry, consider half of the plate in $0<X_1<a+b$, with a unit thickness in the X_3 direction. The effect of the overhang portion $a<X_1<a+b$ on the right half of the central portion $0<X_1<a$ can be represented by a shear force and a bending moment. Therefore we only need to analyze the portion in Fig. 3. In the figure, the acceleration a_2 is replaced by its inertial force. For this portion, the governing equation and boundary conditions for the deflection $w(X_1)$ are[6]

$$EIw'''' = -\rho a_2 2h, \quad 0<X_1<a,$$

$$w(a)=0, \quad EIw''(a) = -\rho a_2 h b^2, \quad w'(0)=0, \quad EIw'''(0) = 0, \tag{1}$$

where a prime is a derivative with respect to X_1 and, for a planar deformation independent of X_3 with $w_3=0$, we have the following effective Young's modulus E and the moment of inertia I:

$$E = c_{11} - c_{12}^2/c_{22}, \quad I = \frac{2h^3}{3}. \tag{2}$$

The solution to Eq. (1) is

$$EIw = \tfrac{1}{12}\rho a_2 h (a^2 - X_1^2)(X_1^2 + 6b^2 - 5a^2). \tag{3}$$

With Eq. (3), we can choose b to adjust the state of flexure in the central portion of the plate and the acceleration sensitivity. For example, with $b^2=5a^2/6$ we can make $w_2(0)=0$.

To show the technique specifically, consider a Y-cut quartz plate. The plate vibrates with the fundamental thickness-shear mode in the X_1 direction. First consider the case when the plate has no overhang ($b=0$, see Fig. 1). The normal acceleration causes a flexure $w(X_1)$ of the originally flat middle plane, as shown by the dotted line in Fig. 1, which is now curved. During flexure the material above the middle plane is in compression, and that below the middle

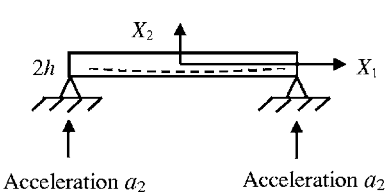

FIG. 1. A simply supported plate under normal acceleration.

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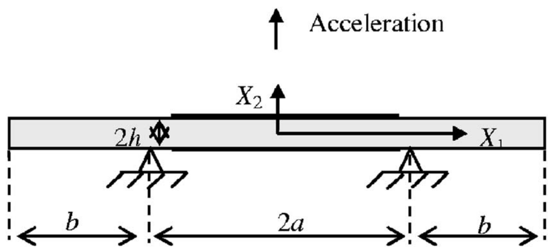

FIG. 2. A plate with overhangs under normal acceleration.

plane is in tension. This antisymmetry of the deformation, together with the symmetry of the structure in the X_1 direction, makes the first-order acceleration sensitivity vanish according to the first-order perturbation integral.[1] It is obvious that during flexure the middle plane of the plate becomes a little longer, which is a second-order effect because the differential arc length of the deformed middle plane is given by $dl=[1+(w')^2]^{1/2}\ dX_1 \cong [1+(w')^2/2]dX_1$, which depends on the square of the slope of the deflection. This stretch of the middle plane causes a contraction of the plate thickness which is symmetric about the middle plane and produces a second-order frequency shift.[2,3]

We attempt to use the plate with overhang portions in Fig. 2 to minimize this second-order acceleration sensitivity by adjusting b. An expression for the frequency shift of the fundamental thickness-shear mode due to the stretch of the middle plane is known.[7,8] The leading terms in the expression are

$$\frac{\Delta\omega}{\omega} = E_1^{(0)} + \frac{c_{661}}{2c_{66}}E_1^{(0)} + \frac{c_{662}}{2c_{66}}E_2^{(0)}, \tag{4}$$

where c_{66} is a shear elastic constant, and c_{661} and c_{662} are third-order elastic constants. $E_1^{(0)}$ is the extensional strain of the middle plane near the center of the plate, and $E_2^{(0)}$ is the accompanying thickness contraction due to Poisson's effect. In order to use (4) we need to calculate the following from Eq. (3). The total elongation of the middle plane of the plate is given by

$$\Delta L = \int_0^a \frac{1}{2}(w')^2 dX_1 = \frac{17\rho^2 a_2^2 a^7}{70E^2h^4}\left(1 - \frac{42}{17}\frac{b^2}{a^2} + \frac{105}{68}\frac{b^4}{a^4}\right). \tag{5}$$

We use the average extensional strain for an estimate. Hence $E_1^{(0)} \cong \Delta L/a$. The corresponding $E_2^{(0)}$ is given by a stress relaxation that for a thin plate the normal stress along the thickness is approximately zero. Hence

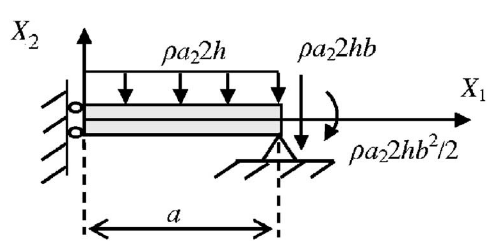

FIG. 3. Part of the plate in Fig. 2.

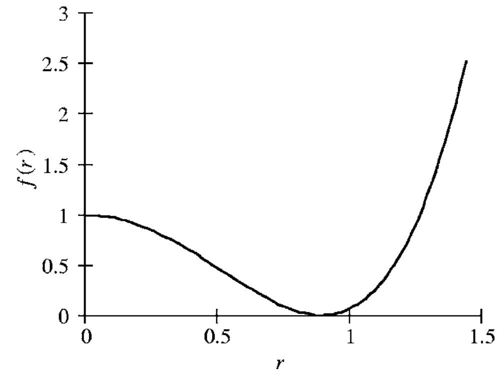

FIG. 4. Effect of overhang on normal acceleration sensitivity.

$$E_2^{(0)} = -\frac{c_{21}}{c_{22}}E_1^{(0)}. \tag{6}$$

With successive substitutions, we obtain

$$\frac{\Delta\omega}{\omega} = 0.243\left(1 + \frac{c_{661}}{2c_{66}} - \frac{c_{662}}{2c_{66}}\frac{c_{21}}{c_{22}}\right)\frac{\rho^2 a^6 g^2}{E^2h^4}\frac{a_2^2}{g^2}f(r),$$
$$f(r) = 1 - \tfrac{42}{17}r^2 + \tfrac{105}{68}r^4, \quad r = \frac{b}{a}. \tag{7}$$

Equation (7) is quadratic in a_2. For a Y-cut quartz resonator with $a=7$ mm, $b=0$ (no overhang), $h=0.097\,001$ mm, and the elastic constants of quartz,[9,10] when $a_2=1$ g, Eq. (7) yields a value of 0.773×10^{-11}, which is a lower bound of the present technology. To further reduce this acceleration sensitivity, we can adjust b in Eq. (7). We plot $f(r)$ in Fig. 4. The figure shows that when r is approximately equal to 0.9, $f(r)$ is approximately 0.01. Therefore by properly choosing the length of the overhang the normal acceleration sensitivity can be reduced by two orders of magnitude, which is the present goal for military needs.

In summary we have shown that plates with overhangs can be used for the reduction of the second-order normal acceleration sensitivity. The thickness-shear BAW resonator analyzed as an example shows a reduction of two orders of magnitude.

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