Letters

An Estimate on the Second-Order Normal Acceleration Sensitivity of a Quartz Resonator

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***Abstract*—An estimate is given for a second-order effect in the normal acceleration sensitivity of a perfectly symmetric Y-cut quartz resonator whose first-order normal acceleration sensitivity is zero. The order of the second-order frequency shift is 10^{-11} per g.**

I. Introduction

The first-order perturbation integral in [1] has been widely used to study the effects of various biasing fields in bulk acoustic waves (BAW) and surface acoustic waves (SAW) crystal resonators, including biasing fields due to normal and in-plane accelerations. The first-order perturbation integral is based on the assumption that the biasing fields are infinitesimal. Therefore, only the first-order effect of the biasing fields needs to be considered. Higher-order effects cannot be described by the first-order perturbation integral, and higher-order perturbation is needed to describe these effects. For example, experiments show [2] that frequency shifts in a resonator in fact depend on the acceleration nonlinearly, and the first-order perturbation integral predicts a linear dependence that is valid for small accelerations only. The first-order perturbation also is inadequate when design techniques (e.g., aspect-ratio compensation) are used to eliminate the first-order frequency shift. The second-order frequency shift may not be zero and then must be looked at. Second-order effects are nonlinear in nature and are inherently complicated. A crude estimate was given in [3], and a lengthy second-order perturbation analysis was presented in [4] for second-order effects in acceleration sensitivity. Both [3] and [4] suggest that, for a perfectly symmetric quartz resonator under normal acceleration, although the first-order frequency shift is zero, the second frequency shift is not zero and may provide a lower bound on the normal acceleration sensitivity.

It recently came to our attention that, in fact, there had been earlier studies [5]–[7] on higher-order effects of biasing fields in resonators under in-plane or normal loads. The results in [5]–[7] can be directly used to study second-order effects in acceleration sensitivity when the loads are due to acceleration induced inertial forces. In this short paper we use the results in [6] and [7] to give a simple estimate of a second-order effect in the normal acceleration sensitivity of a quartz resonator whose first-order normal acceleration sensitivity is zero.

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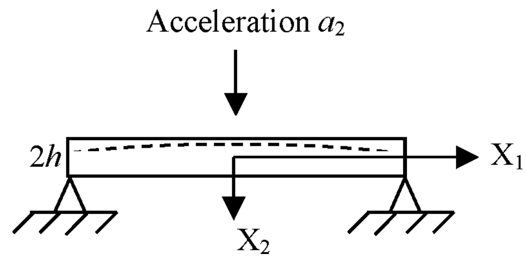

Fig. 1. A simply supported Y-cut quartz plate under normal acceleration.

II. A Second-Order Effect

Consider a Y-cut quartz resonator under normal acceleration (see Fig. 1). The resonator operates with the fundamental thickness-shear mode in the X_1 direction. For our purpose it is sufficient to consider what is called the plane-strain deformation, a widely studied class of resonator problems that has no motion in the X_3 direction and no dependence on X_3. The normal acceleration causes a flexure $w(X_1)$ of the originally flat middle surface as shown by the dotted line in Fig. 1, which is now curved. During flexure the material above the middle surface is in tension and that below the middle surface is in compression. This antisymmetry of the deformation, plus the symmetry of the structure, makes the first-order acceleration sensitivity vanish according to the first-order perturbation integral [1].

It is obvious that the middle surface also becomes a little longer during flexure, which is a second-order effect because the differential arc length of the deformed middle surface is given by $dl = [1 + (w')^2]^{1/2} dX_1 \cong [1+(w')^2/2]dX_1$, which depends on the square of the slope of the deflection and is neglected in all first-order analyses. This stretch of the middle surface causes a contraction of the plate thickness, which is symmetric about the middle surface and may produce a second-order frequency shift. Indeed, an expression for the second-order effect of flexure on the fundamental thickness-shear frequency was given in [6], [7]. For motions independent of X_3, the leading terms in the expression are:

$$\frac{\Delta\omega}{\omega} = E_1^{(0)} + \frac{c_{661}}{2c_{66}} E_1^{(0)} + \frac{c_{662}}{2c_{66}} E_2^{(0)}, \tag{1}$$

where c_{66} is a shear elastic constant, and c_{661} and c_{662} are third-order elastic constants. $E_1^{(0)}$ is the extensional strain of the middle surface near the center of the plate, and $E_2^{(0)}$ is the accompanying thickness strain.

To apply (1) to the problem in Fig. 1 we need to determine the flexure $w(X_1)$ under a_2 and the related $E_1^{(0)}$ and $E_2^{(0)}$. In this case $w(X_1)$ can be written as a trigonometric series [4]. The first term in the series solution represents a good approximation of the deflection surface (neglecting the rest of the terms in the series solution underestimates the center deflection of the plate by an error of 0.39% [4]). Therefore, as an approximation, we take [4]:

$$\begin{aligned} w &= A\cos\alpha X_1, \\ \alpha &= \frac{\pi}{2a},\ A = \frac{12\rho_0 a_2}{h^2\pi R},\ R = -\gamma_{11}\alpha^4, \\ \gamma_{11} &= c_{11} - \frac{c_{12}^2 c_{44} + c_{14}^2 c_{22} - 2c_{12}c_{24}c_{14}}{c_{22}c_{44} - c_{24}^2} \\ &= s_{33}/\left(s_{11}s_{33} - s_{13}^2\right). \end{aligned} \tag{2}$$

The total elongation of the middle surface of the plate is given by:

$$\Delta L = \int_{-a}^{a} \frac{1}{2}(w')^2 dX_1 = \frac{1}{2}A^2\alpha^2 a. \tag{3}$$

We use the average extensional strain for an estimate. Hence:

$$E_1^{(0)} \cong \frac{\Delta L}{2a} = \frac{1}{4}A^2\alpha^2. \tag{4}$$

The corresponding $E_2^{(0)}$ is given by stress relaxation that for a thin plate the normal stress along the thickness is approximately zero. Hence [4]:

$$E_2^{(0)} = -\frac{c_{21}}{c_{22}}E_1^{(0)}. \tag{5}$$

With successive substitutions, we obtain:

$$\begin{aligned} \frac{\Delta\omega}{\omega} &= \left(1 + \frac{c_{661}}{2c_{66}} - \frac{c_{662}}{2c_{66}}\frac{c_{21}}{c_{22}}\right)\frac{1}{4}A^2\alpha^2 \\ &= 0.243\left(1 + \frac{c_{661}}{2c_{66}} - \frac{c_{662}}{2c_{66}}\frac{c_{21}}{c_{22}}\right)\frac{\rho_0^2 a^6 g^2}{\gamma_{11}^2 h^4}\frac{a_2^2}{g^2}. \end{aligned} \tag{6}$$

Note that (6) is quadratic in a_2.

As a numerical example, we consider a typical Y-cut quartz resonator with $a = 7$ mm and $h = 0.097001$ mm. When $a_2 = 1$ g, from (6) and the elastic constants of quartz in [8], [9] we obtain:

$$\frac{\Delta\omega}{\omega} = 0.773 \times 10^{-11}, \tag{7}$$

which is consistent with the results in [3], [4].

III. Conclusions

A second-order effect in the normal acceleration sensitivity of a perfectly symmetric Y-cut quartz resonator is examined. The resonator has no first-order normal acceleration sensitivity. However, results show that its second-order frequency shift is not zero, and it is of the order of 10^{-11} per g.

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