Letters

Thickness-Twist Edge Modes in a Semi-Infinite Piezoelectric Plate of Crystals with 6mm Symmetry

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***Abstract*—It is shown that simple and exact thickness-twist edge modes exist in a semi-infinite piezoelectric plate of crystals with 6mm symmetry. This suggests the possibility of new thickness-twist plate edge mode resonators and acoustic wave sensors with certain advantages.**

I. Introduction

Edge modes in plates and wedges often are used in acoustic wave resonators [1]–[4]. Many bulk acoustic wave resonators are based on infinite plate thickness-shear and thickness-stretch modes. However, real devices are always with finite sizes. Finite resonators operating with infinite plate modes suffer from edge effects. Edge modes, however, are exact at the edges and, therefore, always are desirable. Known edge modes are mainly plate flexural modes [1], [2] and thickness-stretch modes [3]. Edge thickness-twist modes do not seem to have been studied ever. The reason may be that, as to be shown in this paper, the existence of edge thickness-twist modes relies on piezoelectric coupling, and these modes do not have an elastic counterpart. In the analysis of edge modes, approximate two-dimensional plate equations often are used [2]. Our analysis below will be exact and is based on the three-dimensional equations of piezoelectricity. We are interested in thickness-twist vibration modes of crystal plates because they often are used as the operating modes for resonators [5], [6]. In addition to quartz and ceramic plates, which have been used for a long time, recently thin AlN and ZnO plates are of growing interest because of the development of thin-film resonators [7], [8]. They are crystals of 6mm symmetry. Plates with both normal and in-plane six-fold axes are being developed. Polarized ceramics like PZT are transversely isotropic. The material matrices for their linear behavior have the same structures as those of 6mm crystals. Therefore, our analysis also is valid for polarized ceramics.

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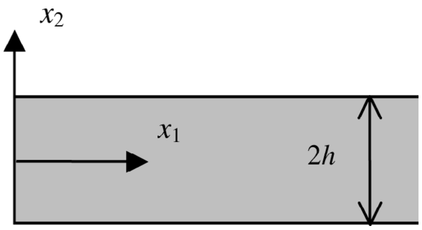

Fig. 1. A semi-infinite piezoelectric plate of 6mm crystals.

II. Governing Equations

Consider the semi-infinite plate in Fig. 1. The six-fold axis (or the poling direction of ceramics) is along x_3. All surfaces are traction-free. The two major surfaces at $x_2 = \pm h$ are unelectroded. Thickness-twist motions are governed by:

$$\begin{aligned} &u_1 = u_2 = 0, \\ &u_3 = u(x_1, x_2, t), \quad \phi = \phi(x_1, x_2, t), \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the displacement vector, and ϕ is the electric potential. A function ψ can be introduced through [9], [10]:

$$\phi = \psi + \frac{e}{\varepsilon} u, \tag{2}$$

where $e = e_{15}$ and $\varepsilon = \varepsilon_{11}$ are the relevant piezoelectric and dielectric constants. The governing equations for u and ψ are [9], [10]:

$$\begin{aligned} \overline{c}\nabla^2 u &= \rho \ddot{u}, \\ \nabla^2 \psi &= 0, \end{aligned} \tag{3}$$

where ∇^2 is the two-dimensional Laplacian, ρ is the mass density, $\overline{c} = c + e^2/\varepsilon$ and $c = c_{44}$ are the relevant shear elastic constants. The nonzero stress $\mathbf{T}$ and electric displacement $\mathbf{D}$ components are [9], [10]:

$$\begin{aligned} &T_{23} = \overline{c}u_{,2} + e\psi_{,2}, \quad T_{13} = \overline{c}u_{,1} + e\psi_{,1}, \\ &D_1 = -\varepsilon\psi_{,1}, \qquad\qquad D_2 = -\varepsilon\psi_{,2}, \end{aligned} \tag{4}$$

where an index after a comma denotes partial differentiation with respect to the coordinate associated with the index. At the plate major surfaces we consider traction-free boundary conditions with:

$$\begin{aligned} T_{23} = 0, \quad x_2 = \pm h, \\ D_2 = 0, \quad x_2 = \pm h, \end{aligned} \tag{5}$$

or, equivalently, in terms of u and ψ:

$$u_{,2} = 0, \quad \psi_{,2} = 0, \quad x_2 = \pm h. \tag{6}$$

III. Edge Modes

It can be verified by direct substitution that the following fields satisfy (3) and (6), and they can be classified into waves symmetric or antisymmetric in x_2. For symmetric fields:

$$\begin{aligned} u &= \cos\xi_2 x_2 A \exp(-\xi_1 x_1)\exp(i\omega t), \\ \psi &= \cos\xi_2 x_2 B \exp(-\xi_2 x_1)\exp(i\omega t), \\ \xi_2 &= \frac{m\pi}{2h}, \quad m = 0, 2, 4, \ldots, \end{aligned} \tag{7}$$

and for antisymmetric fields:

$$\begin{aligned} u &= \sin\xi_2 x_2 A \exp(-\xi_1 x_1)\exp(i\omega t), \\ \psi &= \sin\xi_2 x_2 B \exp(-\xi_2 x_1)\exp(i\omega t), \\ \xi_2 &= \frac{m\pi}{2h}, \quad m = 1, 3, 5, \ldots, \end{aligned} \tag{8}$$

where ω is time frequency, A and B are undetermined constants, and:

$$\begin{aligned} &\xi_1 = \sqrt{\xi_2^2 - \frac{\rho\omega^2}{\overline{c}}} = \sqrt{\frac{\rho}{\overline{c}}}\sqrt{\left(\frac{m\pi}{2h}\right)^2 - \omega^2\frac{\overline{c}}{\rho}} = \frac{1}{v_T}\sqrt{\omega_m^2 - \omega^2}, \\ &v_T = \sqrt{\frac{\overline{c}}{\rho}}, \quad \omega_m^2 = \left(\frac{m\pi}{2h}\right)^2\frac{\overline{c}}{\rho}, \end{aligned} \tag{9}$$

v_T is the plane shear wave speed with piezoelectric coupling or stiffening. ω_m is the cutoff frequency of thickness-twist waves in an unbounded plate. In particular, $m = 0$ is called a face-shear mode. We will not consider this mode because from (9) this mode cannot be an edge mode. The modes corresponding to $m > 0$ decay exponentially from the free edge at $x_1 = 0$ and, therefore, are called edge modes.

Eq. (7) or (8) still need to satisfy the boundary conditions on the traction-free minor surface at $x_1 = 0$. For the electrical boundary conditions, we consider an electroded surface with the electrode grounded:

$$T_{13} = 0, \quad \phi = 0, \quad x_1 = 0. \tag{10}$$

Then, from the fields in (7) and (8), for symmetric or antisymmetric fields, we have the same equations for A and B as below:

$$\begin{aligned} \overline{c}(-\xi_1)A + e(-\xi_2)B &= 0, \\ \frac{e}{\varepsilon}A + B &= 0. \end{aligned} \tag{11}$$

For nontrivial solutions, we must require that:

$$\begin{vmatrix} \overline{c}\xi_1 & e\xi_2 \\ e/\varepsilon & 1 \end{vmatrix} = \overline{c}\xi_1 - \frac{e^2}{\varepsilon}\xi_2 = 0. \tag{12}$$

which can be written as:

$$\frac{1}{v_T}\sqrt{\omega_m^2 - \omega^2} = \overline{k}_{15}^2\xi_2, \tag{13}$$

or:

$$\omega_m^2 - \omega^2 = v_T^2\overline{k}_{15}^4\xi_2^2, \tag{14}$$

or:

$$\begin{aligned} &\omega^2 = \omega_m^2 - v_T^2\overline{k}_{15}^4\xi_2^2 = (1 - \overline{k}_{15}^4)\frac{\overline{c}}{\rho}\left(\frac{m\pi}{2h}\right)^2, \\ &m = 1, 2, 3, \ldots, \end{aligned} \tag{15}$$

where:

$$\overline{k}_{15}^2 = \frac{e^2}{\overline{c}\varepsilon}. \tag{16}$$

Eq. (15) determines the frequencies of the edge modes. If the minor face at $x_1 = 0$ is unelectroded, or if it is mechanically fixed, no edge modes can be found.

IV. Summary

Exact thickness-twist edge modes have been obtained for a plate with traction-free faces all around, unelectroded major faces, and an electroded end face. The results can be used to make new edge mode resonators.

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