EPISTEMIC OVERDETERMINATION AND A PRIORI JUSTIFICATION

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Radical empiricism is the view that denies the existence of a priori knowledge. Its most famous proponents are John Stuart Mill and W. V. Quine. Although both reject a priori knowledge, they offer different empiricist accounts of the knowledge alleged by their opponents to be a priori. My concern in this paper is not with the cogency of their positive accounts. My focus is their arguments against a priori knowledge. My goal is to establish that the argument offered by each suffers from a common defect: a failure to appreciate the phenomenon of epistemic overdetermination and its role in the theory of knowledge.

In section 1 of the paper, I consider Mill's position, where the role of epistemic overdetermination in his rejection of the a priori is transparent. In section 2, I go on to elaborate in greater detail the role of epistemic overdetermination in a theory of knowledge. In section 3, I turn to a version of Quine's argument against the a priori, which has been forcefully advanced by both Hilary Putnam and Philip Kitcher. Finally, in section 4, I examine the relationship between epistemic overdetermination and defeasible justification, and articulate its bearing on the existence of a priori knowledge.

1. Mill

Mill's argument against the existence of a priori is presented within the context of offering an empiricist account of our knowledge of geometry and arithmetic. The stage for Mill's account is set by Kant. Kant characterizes a priori knowledge as "independent of experience," contrasting it with a posteriori knowledge, which has its "sources" in experience.[1]

Presumably, when Kant speaks of the "source" of knowledge, he does not mean the source of the belief in question, but the source of its justification. Hence, according to Kant,

(K) S knows a priori that p just in case S's belief that p is justified by some nonexperiential source and the other conditions for knowledge are satisfied.

Kant argued that necessity is a criterion of a priori knowledge, and maintained that "if we have a proposition which in being thought is thought as *necessary*, it is an *a priori* judgment."[2] He goes on to argue that "mathematical propositions, strictly so-called, are always judgments *a priori,* not empirical; because they carry with them necessity, which cannot be derived from experience."[3] Finally, he contends that all mathematical propositions are synthetic, employing an example from arithmetic and an example from geometry to make his case.

Mill begs to differ. He agrees that mathematical propositions are synthetic, but denies that they cannot be known on the basis of experience. Mill's account of mathematical knowledge is a version of inductive empiricism. Inductive empiricism with respect to a domain of knowledge involves two theses. First, some propositions within that domain are epistemically more basic than the others, in the sense that the nonbasic propositions derive their justification from the basic propositions via inference. Second, the basic propositions are known by a process of inductive inference from observed cases. Mill's focus is on the basic propositions of arithmetic and geometry: the axioms and definitions of each domain. His primary goal is to establish that they known by induction from observed cases.

The details of Mill's account are strained. He advances four primary theses regarding the definitions of geometry. First, they are not stipulations regarding the meanings of terms, but involve "an implied assumption that there exists a real thing conformable thereto."[4] Second, no real things–i.e., real points, real lines, real circles, real squares, etc.–conform exactly to the definitions. Third, they are generalizations about the points, lines, circles, and squares of our experience, and sufficiently approximate the truth regarding those things that no significant error occurs if we assume that they are exactly true. Fourth, since the definitions are not true, they are not necessarily true.

Mill advances three primary theses regarding the axioms of geometry. First, they are exactly true of the objects of our experience. Second, they are inductive generalizations based on our experience of those objects. Third, the contention that they are necessary truths is dubious since (a) it is based on the claim that their falsehood is inconceivable, but (b) the inconceivability of their falsehood is explained by the laws of associationist psychology: by the fact that we have experienced many confirming instances of them but no disconfirming instances.

Mill's account, taken in the crude form in which he presents it, is untenable, and my goal here is not to attempt to rehabilitate it in some way.[5] Instead, I propose to concede that he has accomplished his goal: that he has offered a plausible inductive empiricist account of mathematical knowledge, at least in the case of arithmetic and geometry. How does this concession bear on the existence of a priori knowledge? It *does* show that Kant was wrong in claiming that mathematical knowledge *cannot* be derived from experience. It does *not*, however, show that Kant was wrong in maintaining that mathematical knowledge *is* a priori. The fact that mathematical knowledge is or can be derived from experience does not immediately entail that it is not or cannot be derived from some nonexperiential source. Mill recognizes that more needs to be said at this juncture. In particular, he recognizes that his opponents can concede that the axioms are originally *suggested* by experience, but deny that experience is necessary to *prove* them.

Mill attempts to close the gap in his argument with the following observations:

> They cannot, however, but allow that the truth of the axiom, "Two straight lines cannot inclose a space," even if evident independently of experience, is also evident from experience. Whether the axiom needs confirmation or not, it receives confirmation in almost every instant of our lives, since we cannot look at any two straight lines which intersect one another without seeing that from that point they continue to diverge more and more. . . . Where, then, is the necessity for assuming that our recognition of these truths has a different origin from the rest of our knowledge when its existence is perfectly accounted for by supposing its origin to be the same? . . . The burden of proof lies on the advocates of the contrary opinion; it is for them to point out some fact inconsistent with the supposition that this part of our knowledge of nature is derived from the same sources as every other part.[6]

Mill thinks that he can move from the premise that inductive empiricism provides an account of knowledge of mathematical axioms to the stronger conclusion that knowledge of such axioms is not a priori. The key premise in his argument involves an appeal to a version of the *Explanatory Simplicity Principle* (ES): If a putative source of knowledge is not necessary to explain knowledge of the propositions within some domain, then it is not a source of knowledge of the propositions within that domain. Since Mill has offered an account of mathematical knowledge based on inductive generalization from observed cases, the explanatory simplicity principle yields the conclusion that mathematical knowledge is not a priori. There is no need to introduce a nonexperiential source in order to explain mathematical knowledge. Mill's argument can be articulated as follows:

(M1) Inductive empiricism provides an account of mathematical knowledge based on inductive generalization from observed cases.

(ES) ϕ is a source of knowledge for some domain D of knowledge only if ϕ is necessary to explain knowledge of some propositions within D.

(M2) Therefore, mathematical knowledge is not a priori.

Clearly, the burden of the argument is carried by (ES): the Explanatory Simplicity Principle. We now turn to understanding the principle and its consequences.

2. Epistemic Overdetermination

(ES) is ambiguous. In order to bring out the ambiguity, consider the *Single Source Principle* (SS):

(SS) For each domain D of knowledge, there is only a single source of justification for the propositions within that domain.

(ES) does not entail (SS) since it leaves open the possibility that a domain of knowledge is *epistemically segregated:* i.e., some propositions within D are justified only by source A, and some other propositions within D are justified only by a different source B. In such a situation, since both A and B are necessary to justify some propositions within D, (ES) allows that both are sources of justification for the propositions within D.

(ES), however, does entail the following weaker version of (SS):

(SS1) For *some* propositions within D, there is only a single source of justification.

Consider again our epistemically segregated domain of knowledge. If source A justifies every proposition justified by source B, then source B would be unnecessary to explain knowledge of domain D. Similarly, if source B justifies every proposition justified by source A, then source A would be unnecessary to explain knowledge of domain D. So, in order for A and B both to be necessary to explain knowledge of domain D, there must be at least one proposition within D justified by A but not B, and at least one proposition within D justified by B but not A.

An interesting question arises at this juncture. Does (ES) also entail

(SS2) For *each* proposition within D, there is only a single source of justification?

If we again consider our epistemically segregated domain of knowledge, (SS2) entails that for every proposition p within domain D, p is justified by either source A or source B but not both. In order to simplify matters, let us assume that domain D consists solely of four propositions: P_1, P_2, P_3, and P_4. Let us also assume that source A justifies P_1, P_2, P_3, but not P_4. Hence, source B is necessary to explain only the justification of P_4. Does (ES) leave open the possibility that source B justifies P_1, P_2, or P_3 as well?

There are two readings of (ES):

(ES1) ϕ is a source of knowledge for $P_1, \ldots, P_n$ of domain D only if ϕ is necessary to explain the justification of *some* propositions within D.

(ES2) ϕ is a source of knowledge for $P_1, \ldots, P_n$ of domain D only if ϕ is necessary to explain the justification of $P_1, \ldots, P_n$.

The second, more stringent, reading of (ES) does entail (SS2). According to (ES2), source A justifies only those propositions within D that are not justified by some other source. The first, more liberal, reading of (ES) does not entail (SS2). According to (ES1), once source A is introduced to explain the justification of P_4, it can also justify other propositions within D, such as P_1. Although (ES1) does not entail (SS2), it does entail

(SS3) If some source ϕ explains the justification of all propositions within domain D, then for *each* proposition within D, there is only a single source of justification.

(ES2) strikes me as more the more plausible reading of (ES). Consider again our example. Let P_1 be Euclid's first postulate: A straight line can be drawn from any point to any other point. Let P_4 be Euclid's second postulate: Any straight line can be extended continuously in a straight line. Assume that reason is necessary to explain only the justification of P_4. Moreover, assume that reason also justifies P_1. If P_1 is indeed justified by reason, then it is in virtue of some relationship between the evidence generated by reason and the content of P_1. One familiar account is that reason produces an intuition that P_1, and the intuition that P_1 justifies the belief that P_1. Presumably, neither the fact that reason produces the intuition that P_1 nor the fact that the intuition justifies P_1 depends on whether reason also justifies P_4, let alone on whether it is necessary to justify P_4. (ES1), however, entails the following puzzling counterfactual:

(C) If reason were not necessary to justify P_4, then reason would not justify P_1.

Since P_1 does not depend for its justification on the justification of P_4, it is unclear why P_1's being justified by reason depends on P_4's being justified by reason. Since this argument against (ES1) is not conclusive, I will address both versions of (ES) in the subsequent discussion.

Both (ES1) and (ES2) should be rejected. (ES2) entails (SS2), and (ES1) entails (SS3). But (SS2) and (SS3) should be rejected for two reasons. The first is methodological: both (SS2) and (SS3) settle by fiat substantive epistemological issues. Epistemology is concerned with the sources and extent of human knowledge. Addressing these concerns involves two different tasks.

The first is identifying the most general domains of human knowledge and the putative sources of knowledge for each domain. The second is establishing that the propositions in the target domain are indeed justified by the sources in question.

A source of knowledge is a cognitive capacity that, when exercised properly, produces knowledge. A cognitive capacity produces knowledge by justifying the beliefs produced by its proper exercise. Two observations about cognitive capacities and their role in a theory of knowledge are important for our purposes. First, the question of how many cognitive capacities humans possess that have the requisite properties for justifying beliefs, and the question of the range of beliefs produced by each capacity, are broadly empirical. They are answered by empirical investigation. In some cases, say visual perception, the empirical evidence that we possess such a capacity and that it is a source of beliefs about a certain range of properties of physical objects may be so readily available that no further empirical investigation is necessary. But epistemologists have also proposed more controversial, putative sources of knowledge. Roderick Chisholm offers two examples. Some maintain that, in order to explain our knowledge of other minds, "there must be another source–possibly the *Verstehen*, or 'intuitive understanding', of German philosophy and psychology."[7] The second example pertains to knowledge of religious truths: "Hugh of St. Victor held, in the twelfth century, that in addition to the *oculis canis*, by means of which we know the physical world, and the *oculis rationis*, by means of which we know our own states of mind, there is an *oculis contemplationis*, by means of which we know the truths of religion."[8] Each of these claims entails that human beings have a certain cognitive capacity that is a source of beliefs falling into a particular domain. The questions of whether such capacities exist and, if so, what beliefs they produce can be settled only on the basis of empirical evidence.

Second, the goal of a theory of knowledge should be to provide a complete inventory of the basic cognitive capacities that humans have for acquiring knowledge. In the absence of compelling independent evidence, there is no basis for assuming that if humans possess some cognitive capacity that justifies all propositions within some particular domain of knowledge, they have no other cognitive capacity that justifies any beliefs within that domain. Both (SS2) and (SS3) make such an assumption in the absence of any independent evidence. For example, let us suppose that we have an adequate account of the justification of a certain domain of propositions about other minds on the basis of the observation of the behavior of others and some legitimate process of inductive inference. It follows from both (SS2) and (SS3) either that humans lack intuitive understanding or that it is not a source of justification of beliefs about other minds. Hence, (ES1) and (ES2) settle by fiat a substantive epistemological question. The questions of whether intuitive understanding exists as a cognitive capacity and, if it does, whether it produces beliefs about other minds in a manner that confers justification on those beliefs should not be settled by a methodological assumption that has no independent support. Similarly, if we suppose that rational theology justifies all the propositions within some domain of religious knowledge, (SS2) and (SS3) entail either that either humans lack an *oculis contemplationis* or that it is not a source of justification of religious beliefs. Once again, such questions should not be settled by a methodological assumption that has no independent support.

There is a second reason for rejecting (SS2) and (SS3). They conflict with an undisputable feature of our epistemic lives: epistemic overdetermination. It is a familiar fact of our epistemic lives that the justification of some of our beliefs is overdetermined–i.e., for some of our beliefs, we have more than one justification, and each of the justifications, in the absence of the others, is sufficient to justify the belief in question. For example, you attended a party last night and someone asks you if Jill also attended. You didn't interact with her at the party, so you have to stop and think about it. You suddenly recall that you saw her talking to Jack, and that recollection triggers a host of additional recollections of Jill's being at the party. Presumably, your original recollection justifies your belief that Jill was at the party. But each of the other recollections that came in its wake also justifies your belief. So you have many different recollections, each of which is sufficient to justify your belief that Jill was at the party. Your justification for that belief is overdetermined.

There are two different types of epistemic overdetermination: epistemic overdetermination by the same source, and epistemic overdetermination by different sources. The first type of epistemic overdetermination occurs when we have more than one justification for a particular belief, and each of them comes from the same source. Our example in the previous paragraph is of the first type. The second type of epistemic overdetermination occurs when we have more than one justification for a particular belief, and at least some of them come from different sources. Here's an example. I've misplaced my wallet again. I wonder where I might have left it. I suddenly recall having left it on the kitchen counter when I came in from the garage last night. Presumably, my recollection justifies me in believing that my wallet is on the kitchen counter. But, just to be sure, I walk out to the kitchen to check. To my relief, I see my wallet on the counter. Presumably, my seeing my wallet on the counter also justifies me in believing that my wallet is on the counter. So here my justification is overdetermined by different sources. Both my recollection and my visual experience justify my belief about my wallet, and each is sufficient to justify that belief in the absence of the other.

Both (SS2) and (SS3) have significant consequences regarding the possibility of epistemic overdetermination. Assume that

(EO) S's belief that p is epistemically overdetermined by two different sources.

It follows that

(EO1) S's belief that p is justified by two different sources.

But (EO1) is incompatible with (SS2). The example in the previous paragraph shows that (EO1) is true. Therefore, (SS2) and, *a fortiori*, (ES2) are false.

The relationship between (SS3) and epistemic overdetermination is more complicated. Although (SS3) is compatible with (EO1), it is incompatible with

(EO2) S's belief that p is justified by some source A, where p is a member of domain D and A justifies all propositions within domain D, and S's belief that p is justified by source B.

Providing an uncontroversial example that shows that (EO2) is true is difficult since Mill provides little information about how to individuate domains and sources of knowledge. Hence, we must rely on the intuitive principles of individuation that epistemologists typically employ. Consider the traditional problem of perception and knowledge of the external world. The traditional view is that perception is the source of justification for all propositions about the present existence of medium-sized physical objects in our immediate vicinity. Let D be the domain of all such propositions. Moreover, we assume with the tradition that all propositions within D are justified by perception. Now consider the following example. I'm at my desk working on a paper. I jot down some ideas with my pencil and stop to think. A new idea occurs to me, which I want to jot down quickly before I lose it. But I can't find my pencil. I suddenly recall having seen it roll off the desk a few minutes ago. I look down and see it. My belief that my pencil is on the floor is justified both by my seeing it there and by my recollection of having seen it roll off the desk. In this case, perception justifies my belief that the pencil is on the floor. Moreover, the proposition that the pencil is on the floor is a member of a domain all of whose members are justified by perception. Hence, (SS3) entails that my belief is not also justified by memory. Since it is also justified by memory, (SS3) and, *a fortiori*, (ES1) are false.

3. Quine

Although Quine and Mill both reject a priori knowledge, they offer very different accounts of mathematical knowledge. Quine rejects inductive empiricism: he rejects the idea that there are basic mathematical propositions which, taken in isolation, are directly justified by observation and inductive generalization. Quine's account of mathematical knowledge is a version of holistic empiricism. Mathematical propositions are components of scientific theories. Their epistemic properties are analogous to those of the more theoretical propositions of such theories. They are not tested directly against observation, but only indirectly via their observational consequences. Moreover, they don't have observational consequences in isolation, but only in conjunction with the other propositions of the theory. Hence, according to holistic empiricism, mathematical propositions are highly theoretical components of scientific theories, and entire scientific theories, including their mathematical components, are indirectly confirmed or disconfirmed by experience via their observational consequences.

Our main concern, however, is not with Quine's positive account of mathematical knowledge, but with his case against a priori knowledge. Here we find a second contrast with Mill. Mill is straightforward in presenting his case against a priori knowledge; Quine's case is much more elusive. Some take Quine's program of naturalized epistemology to be the primary challenge to the existence of a priori knowledge.[9] Others contend that the case had already been made in his classic paper, "Two Dogmas of Empiricism."[10] But even if we focus exclusively on the latter, as I propose to do, the argument remains difficult to articulate.

The primary target of Quine's attack in "Two Dogmas" is a conception of analyticity inspired by Frege: a statement is analytic if it can be turned into a logical truth by replacing synonyms with synonyms. The primary points of the attack can be summarized as follows:

(1) Definition presupposes synonymy rather than explaining it.

(2) Interchangeability *salva veritate* is a sufficient condition of cognitive synonymy only in relation to a language containing an intensional adverb 'necessarily'.

(3) Semantic rules do not explain 'Statement S is analytic for language L', with variable 'S' and 'L', even if 'L' is limited to artificial languages.

(4) The verification theory of meaning provides an account of statement synonymy that presupposes reductionism. Reductionism fails but survives as the view that individual statements admit of confirmation or infirmation.

(5) Any statement can be held to be true come what may. No statement is immune to revision.

There are two striking aspects of the attack. First, it is directed at two different targets: points (1), (2) and (3) target the notion of synonymy; points (4) and (5) target the doctrine of reductionism. Second, none of the points is explicitly directed at a priori knowledge. Hence, if "Two Dogmas" does indeed present a challenge to the existence of a priori knowledge, as many of its champions claim, then some additional premise is necessary that connects one of the two targets to the a priori. What is the additional premise?

One common response is that Quine is attacking the logical empiricist conception of a priori knowledge, whose central claim is

(LE) All a priori knowledge is of analytic truths,

and that his argument purports to show that the concept of an analytic truth is incoherent. Let us grant both claims. Does this concession impugn the existence of a priori knowledge? Clearly, it does not. (LE) is a thesis about the nature of the propositions alleged to be known a priori: it claims that they are analytically equivalent to logical truths. If Quine is right, then (LE) itself is incoherent. But from the fact that a thesis about the nature of propositions known a priori is incoherent, it does not follow that there is no a priori knowledge.

An alternative response is to take (LE) as a conceptual claim–i.e., to take it as claiming that the concept of a priori knowledge involves, either implicitly or explicitly, the concept of analytic truth. On this reading, the incoherence of the concept of analytic truth entails the incoherence of the concept of a priori knowledge. This response, however, rests on a false conceptual claim. Since I have argued this point in detail elsewhere, I will be brief here.[11] First, the concept of analytic truth is not explicitly part of the concept of a priori knowledge. As we saw earlier, the traditional concept of a priori knowledge is the concept of knowledge whose justification is nonexperiential. That concept, taken alone, neither states nor immediately entails anything about the nature of the propositions so justified. Second, the most promising route to maintaining that the concept of analytic truth is implicitly part of the concept of a priori knowledge is to endorse two theses: (a) the view that all a priori knowledge is of necessary truths; and (b) some version of the so-called "linguistic theory" of necessary truth, which maintains that the concept of necessary truth is analyzable in terms of the concept of analytic truth. There are, however, two problems with this route. First, the concept of a priori knowledge does not involve, either explicitly or implicitly, the concept of necessary truth. Second, no one has ever offered an even remotely plausible analysis of the concept of necessary truth in terms of the concept of analytic truth.

Some recent champions of "Two Dogmas" have proposed an alternative reading of its main argument that explicitly articulates the additional premise necessary to establish the conclusion that there is no a priori knowledge. Hilary Putnam rejects the orthodox reading of Quine's argument: namely, that he was offering an attack on the analytic/synthetic distinction that purported to establish that all attempts to define 'analyticity' are ultimately circular.[12] Putnam views this reading as too simplistic because Quine's arguments are not all directed toward the same target. In particular, point (5) attacks the concept of a statement that is confirmed no matter what. But, according to Putnam, this concept, unlike the concept of analyticity, is epistemic and not semantic:

> But why should this concept, the concept of a statement which is confirmed no matter what, be considered a concept of *analyticity*? Confirmation, in the positivist sense, has something to do with rational belief. A statement which is highly confirmed is a statement which it is rational to believe, or rational to believe to a high degree. If there are indeed statements which have the maximum degree of confirmation in all

> circumstances, then these are simply truths which it is *always rational to believe*, nay, more, truths which it is never rational to even begin to doubt. . . . On the face of it, then, the concept of a truth which is confirmed no matter what is not a concept of *analyticity* but a concept of *apriority.*[13]

Moreover, Quine's argument against the concept of a truth that is confirmed no matter what is not based on some alleged circularity. Instead, according to Putnam, it is "an argument from what is clearly a normative description of the history of modern science," which Putnam locates in the following celebrated passage:

> Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. Even a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. Conversely, by the same token, no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?[14]

Putnam goes on to endorse Quine's argument and to maintain that the importance of "Two Dogmas" lies not in its rejection of the analytic/synthetic distinction but in its rejection of a priori knowledge.

Philip Kitcher agrees that the importance of "Two Dogmas" lies in its rejection of a priori knowledge:

> Defenders of analyticity have often construed the main thrust of Quine's most famous attack, "Two Dogmas of Empiricism," as arguing that the concept of analyticity is undefinable in notions Quine takes to be unproblematic. . . . I locate Quine's central point elsewhere. The importance of the article stems from its final section, a section which challenges not the existence of analytic truths but the claim that analytic truths are knowable a priori.[15]

Kitcher goes on to maintain that Quine's central argument is located in the very same passage that Putnam quotes, and reconstructs it the following manner:

> Quine connects analyticity to apriority *via* the notion of unrevisability. If we can know a priori that *p* then no experience could deprive us of our warrant to believe that *p*. . . . But "no statement is immune from revision." It follows that analytic statements, hailed by Quine's empiricist predecessors and contemporaries as a priori, cannot be a priori; . . .[16]

Hence, Putnam and Kitcher agree on three points. First, the most important argument in "Two Dogmas" targets the existence of a priori knowledge. Second, the argument is located in the passage in which Quine claims that no statement is immune to revision. Finally, and most importantly, the argument involves an implicit premise that connects the concept of the a priori with the concept of rational unrevisability.

Both Putnam and Kitcher maintain that rational unrevisability is a necessary condition for a priori knowledge. But the conditions that they propose are different. Putnam maintains that the concept of a statement that is confirmed no matter what is a concept of apriority. A statement that is confirmed no matter what is, according to Putnam, one that it is always rational to believe, one that it is never rational to doubt. But a statement that is *always* rational to believe is one that is rational to believe in the face of *any* evidence to the contrary. Hence, Putnam endorses a *strong* unrevisability condition:

(SC) If S's belief that p is justified a priori, then S's belief that p is not rationally revisable in light of *any* evidence.

Kitcher's proposal is more modest. He maintains that if S knows that p a priori then "no experience could deprive us of our warrant to believe that p." Hence, Kitcher endorses a *weak* unrevisability condition:

(WC) If S's belief that p is justified a priori, then S's belief that p is not rationally revisable in light of any *experiential* evidence.

We will focus on (WC) for two reasons. First, it is more plausible than (SC). (SC) entails that if S's justified belief that p is rationally revisable, then S's belief that p is justified a posteriori. But consider a competent mathematician who, working within her field of research expertise, carefully constructs a proof that A entails B and believes, on that basis, that A entails B. Presumably, such a belief is justified. Suppose, however, upon later reviewing the proof, she discovers a subtle flaw and, as a consequence, withholds the belief that A entails B. (SC) has the implausible consequence that the mathematician's belief that A entails B is justified a posteriori even if her original justification for that belief is exclusively nonexperiential, and her subsequent justification for the belief that her proof is flawed is also exclusively nonexperiential. Second, even if (SC) is defensible, it is not necessary to secure the validity of Quine's argument; (WC) is sufficient. Hence, the Putnam-Kitcher version of Quine's argument can be stated as follows:

(Q1) No statement is immune to revision in light of recalcitrant experience.

(WC) If S's belief that p is justified a priori, then S's belief that p is not rationally revisable in light of any *experiential* evidence.

(Q2) Therefore, no knowledge is a priori.

Although (Q1) is open to dispute, I propose to grant it in order to assess its bearing on the existence of a priori knowledge. My focus will be on (WC).

4. Defeasible Justification

We rejected Mill's argument against the existence of a priori knowledge on the grounds that it rules out by fiat the possibility of epistemic overdetermination. Quine's argument, as reconstructed by Putnam and Kitcher, does not appear to make any commitments with respect to the possibility of epistemic overdetermination. The appearances, however, are misleading. To show why they are misleading, we need to explore more carefully the relationship between defeasible justification and epistemic overdetermination.

We begin by introducing some additional conceptual resources. Experiential sources of justification, such as visual perception, are fallible. Some beliefs justified by visual perception are false. For example, if I carefully visually examine a sheet of paper that is on the table before me in ordinary lighting conditions and, on that basis, conclude that it is square, but fail to notice that two sides of the sheet are slightly longer that than the other two sides, I have a false belief justified by visual perception. Fortunately, however, experiential sources also have the capacity to correct errors. Returning to our example, if I were to visually inspect the sheet of paper a second time and notice that two of its sides are slightly longer than the other two, I would have a belief, justified by visual perception, that the sheet is not square but rectangular. So, in such a situation, a single source of justification justifies both the belief that p and the belief that not-p. It appears that all experiential sources of justification have this feature: if they can justify the belief that p, they can also justify the belief that not-p and vice-versa.[17] Let us say that a self-revising source of justification is one that satisfies the following condition:

(SR) Source ϕ can justify S's belief that p just in case ϕ can justify S's belief that not-p.

We now return to the concept of defeasibility and distinguish two types of defeaters for a justified belief. Let us assume that S's belief that p is justified by source A. (Call a belief justified by source A an 'A-justified belief'.) There are two types of defeaters for S's A-justified belief that p. An overriding defeater for S's A-justified belief that p is

(O) S's justified belief that not-p.

An undermining defeater for S's A-justified belief that p is

(U) S's justified belief that S's A-justification for the belief that p is inadequate or defective.

Finally, recall our distinction between epistemic overdetermination by a single source and epistemic overdetermination by different sources. My goal is to show that, in the case of self-revising sources of justification, there is an important connection between defeasibility by overriding defeaters and epistemic overdetermination by different sources.

Let us begin by assuming that S's belief that p is epistemically overdetermined by two self-revising sources of justification:

(1) S's belief that p is justified by self-revising source A and can be justified by self-revising source B.

(1) entails

(2) Source B can justify S's belief that p.

Since B is a self-revising source of justified beliefs, it follows that

(3) Source B can justify S's belief that not-p.

Since S's justified belief that not-p is an overriding defeater for S's justified belief that p, it follows that

(4) S's A-justified belief that p is defeasible by an overriding defeater justified by source B.

The converse entailment also holds. If we assume (4), it follows that

(3) Source B can justify S's belief that not-p.

Since A and B are self-revising sources, it follows that

(2) Source B can justify S's belief that p,

and that

(1) S's belief that p is justified by self-revising source A and can be justified by self-revising source B.

So we have established that there is a connection between epistemic overdetermination by two different self-revising sources and defeasibility by overriding defeaters.

We now turn to the implications of this connection for the Putnam-Kitcher version of Quine's argument. Here we are faced with an immediate problem: Quine's argument focuses exclusively on the conditions for rational belief revision, but it does not explicitly address the conditions under which beliefs are justified. Since the case of Euclidean geometry provides the most striking example of putative experiential disconfirmation of a mathematical proposition, and since it is alleged to exemplify the holistic empiricist's account of how such propositions are disconfirmed by experience, we will focus on it. Moreover, we will assume that the very same empirical evidence that disconfirms the principles of Euclidean geometry also confirms the principles of the alternative non-Euclidean geometry. In short, we will assume that the Quinean story about the rational revision of the principles of Euclidean geometry is also the Quinean story about rational adoption of the principles of non-Euclidean geometry. This assumption is controversial since, when generalized, it leads to a potentially embarrassing consequence for Quine's account. We will return to that issue later since it will not impact the argument that we offer against the account.

The Quinean story about the rational revision of the principles of Euclidean geometry is familiar. The principles of Euclidean geometry are part of an overall scientific theory describing the structure of physical space, which includes, in addition to the geometrical theory, a physical theory. Scientific theories are accepted or rejected on the basis of standard criteria such as conformity to observational data, explanatory power, conservatism, and simplicity. The principles of Euclidean geometry were rejected in favor of the principles of non-Euclidean geometry because the conjunction of Euclidean geometry with physical theory yielded an overall theory inferior, when measured by the standard criteria, to the overall theory yielded by the conjunction of non-Euclidean geometry with physical theory

It is important to note here that Quine's claim that no statement is immune to revision by recalcitrant experience applies to the newly adopted principles of non-Euclidean geometry. Hence, more generally, the claim that no statement is immune to revision in light of recalcitrant experience entails that the empirical evidence relevant to the justification of mathematical beliefs is self-revising in the sense defined earlier. If there is some set of experiences that confirm the principles of some geometry, there is also some alternative set of experiences such that were they to occur, they would justify the denial of those principles, and vice versa.

How does this bear on (WC)? Let us assume the basic thesis of radical empiricism:

(RE) If p is a mathematical statement and S's belief that p is justified, then S's belief that p is justified by experiential evidence.

We argued in the previous paragraph that

(SR*) The experiential evidence alleged by Quine to justify mathematical statements is self-revising.

Therefore, the conjunction of (RE) and (SR*) entails

(5) If S's belief that p, where p is some mathematical statement, is justified by experiential evidence, then S's belief that not-p can be justified by experiential evidence.

Since a justified belief that not-p is a defeater for S's justified belief that p, (5) entails

(6) If S's belief that p, where p is some mathematical statement, is justified by experiential evidence, then S's belief that p is defeasible by experiential evidence.

The conjunction of (6) and

(WC) If S's belief that p is justified a priori, then S's belief that p is not revisable in light of any experiential evidence

entails

(7) If S's belief that p, where p is some mathematical statement, is justified by experiential evidence, then S's belief that p is not justified a priori.

(7) is incompatible with the following thesis of epistemic overdetermination:

(8) If p is a mathematical statement, then S's belief that p is justified both a priori and by experiential evidence.

Hence, (WC) settles by fiat a substantive epistemological question.

Let me briefly summarize. Quine's remarks in "Two Dogmas" can be parlayed into an argument against a priori knowledge only by introducing a substantive necessary condition on a priori justification: (WC). (WC), however, has the consequence of ruling out the possibility that mathematical statements are justifiable both a priori and by experiential evidence. But since this is a substantive epistemological question, any conception of the a priori that settles it by fiat should be rejected. Hence, Quine's argument, like Mill's, ultimately fails because it fails to take into account the possibility of epistemic overdetermination.

My argument against the Quinean account is based on the assumption that mathematical propositions are confirmed by being embedded in an overall scientific theory that is confirmed by experience. That assumption, however, leads to the consequence that, in the absence of a good deal of scientific knowledge, one cannot be justified in believing mathematical propositions. But it is implausible to deny that skilled craftsmen, who build cabinets or musical instruments but know little physics, and educated adults, who have studied geometry and calculus but not physics, are not justified in believing any propositions of geometry. Hence, some theorists who are sympathetic to both Quine's radical empiricism and his account of rational belief revision offer alternative accounts of the original justification of such beliefs. Philip Kitcher, for example, stresses the authority of textbooks and teachers in accounting for one's rudimentary mathematical knowledge.[18] An alternative strategy employed by neo-Quineans is to endorse a version of epistemic conservatism: a belief is prima facie justified to some degree merely in virtue of being held. William Lycan, for example, endorses the Principle of Credulity: “Accept at the outset each of those things that seems to be true.”[19] Gilbert Harman endorses General Foundationalism: “A general foundations theory holds that all of one’s beliefs and inferential procedures at a given time are foundational at that time.”[20] A belief or inferential procedure is foundational for a person at a time just in case it is non-inferentially prima facie justified for that person at that time.

Adopting either of these alternative accounts of the source of our initial justification for mathematical beliefs does not escape the argument that we offered against (WC), because both of these alternative sources of justification, like the one that we originally considered, are self-revising. If I can be justified in believing that p on the basis of reading that p in a textbook or hearing a teacher assert that p, then I can also be justified in believing that not-p on the basis of reading that not-p in a textbook or hearing a teacher assert that not-p, and vice-versa. If I can be justified in believing some statement that p on the basis of believing that p, then I can also be justified in believing that not-p on the basis of believing that not-p, and vice-versa. Hence, the alternative accounts, when conjoined with (WC), also rule out the possibility of epistemic overdetermination by fiat.

There is a second problem with (WC) that is worth noting. Suppose that we concede that the Putnam-Kitcher version of Quine’s argument is sound. The theoretical motivation of proponents of the argument is to support radical empiricism. The conjunction of (Q2) and

(9) There is mathematical knowledge

does not entail (RE). It entails only the weaker

(RE*) If p is a mathematical statement and S's belief that p is justified, then *either* S's belief that p is justified by experiential evidence *or* S's belief that p is justified by nonexperiential evidence that is defeasible by experiential evidence.

(RE*), however, is compatible with Kant's claim that mathematical statements are justified nonexperientially. Hence, the Putnam-Kitcher version of Quine's argument fails to establish that mathematical statements are justified by experience.[21]

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ENDNOTES

1. Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith (New York: St Martin's Press, 1965), 43.

2. Ibid.

3. Ibid., 52.

4. John Stuart Mill, *A System of Logic*, ed. J. M. Robson (Toronto: University of Toronto Press, 1973), 224.

5. Philip Kitcher offers a sympathetic defense of Mill's position in "Arithmetic for the Millian," *Philosophical Studies* 37 (1980): 215-236. Kitcher's defense is challenged in chapter 5 of Albert Casullo, *A Priori Justification* (New York: Oxford University Press, 2003).

6. Mill, 231-232.

7. R. M. Chisholm, *Theory of Knowledge*, 1st ed (Englewood Cliffs: Prentice-Hall, Inc., 1966), 65.

8. Ibid., 67.

9. W. V. Quine, "Epistemology Naturalized," in *Ontological Relativity and Other Essays* (New York: Columbia University Press, 1969).

10. W. V. Quine, "Two Dogmas of Empiricism," in *From a Logical Point of View*, 2nd ed revised (New York: Harper and Row, 1963).

11. See Casullo, chapters 1 and 8.

12. Hilary Putnam, "'Two Dogmas' Revisited," in *Realism and Reason: Philosophical Papers, Vol. 3* (Cambridge: Cambridge University Press, 1983).

13. Ibid., 90.

14. Quine, "Two Dogmas of Empiricism," 43; quoted by Putnam, ibid.

15. Philip Kitcher, *The Nature of Mathematical Knowledge* (New York: Oxford University Press, 1983), 80.

16. Ibid.

17. Some might deny that introspection has this feature. Since introspection plays no role in my subsequent arguments, this contention can be conceded.

18. Kitcher, 91-95.

19. William Lycan, "Bealer on the Possibility of Philosophical Knowledge," *Philosophical Studies* 81(1996), 145.

20. Gilbert Harman, "General Foundations versus Rational Insight," *Philosophy and Phenomenological Research* 63 (2001), 657.

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