10 Analyticity, Apriority, Modality

Albert Casullo

In the introduction to his *Critique of Pure Reason*, Immanuel Kant introduces a conceptual framework that involves three distinctions: the epistemic distinction between *a priori* and empirical (or *a posteriori*) knowledge; the metaphysical distinction between necessary and contingent propositions; and the semantic distinction between analytic and synthetic propositions. Kant utilizes his framework to pose four questions: (1) What is *a priori* knowledge? (2) Is there *a priori* knowledge? (3) What is the relationship between the *a priori* and the necessary? (4) Is there synthetic *a priori* knowledge? In response, he (1965: 43) maintains that *a priori* knowledge is "independent of experience", contrasting it with *a posteriori* knowledge, which has its "sources" in experience. He offers two criteria for *a priori* knowledge: necessity and strict universality; and argues that, since mathematical propositions are necessary, mathematical knowledge is *a priori*. Kant's claim that necessity is a criterion of the *a priori* entails

(K1) All knowledge of necessary propositions is *a priori*,

but not

(K2) All propositions known *a priori* are necessary.

Finally, Kant maintains that all propositions of the form "All A are B" are either analytic or synthetic: analytic if the predicate is contained in the subject; synthetic if it is not. Utilizing this distinction, he argues that

(K3) All knowledge of analytic propositions is *a priori*; and

(K4) Some propositions known *a priori* are synthetic.

In support of (K4), Kant claims that the predicate terms of "7 + 5 = 12" and "The straight line between two points is the shortest" are not contained in their respective subjects.

Kant's conceptual framework and the four questions that he poses within it are at the centre of much current philosophical discussion. First, there is disagreement over how to articulate Kant's characterization of *a priori* knowledge and whether that characterization, however articulated, is adequate. Second, the most fundamental division in contemporary philosophy is between those who accept and those who reject the existence of *a priori* knowledge. Kant's supporting argument plays a central role in the debate. Third, Saul Kripke rejects both (K1) and (K2), but his examples of necessary *a posteriori* knowledge and contingent *a posteriori* knowledge remain controversial. Finally, the denial of (K4) by proponents of logical empiricism and W.V. Quine's subsequent rejection of the analytic/synthetic distinction continue to dominate discussions of *a priori* knowledge.

The Concept of *a priori* Knowledge

Kant's characterization of the *a priori* is not fully articulated. He does not spell out the sense in which *a priori* knowledge must be "independent" of experience or the sense in which *a posteriori* knowledge has its "source" in experience. It is generally accepted that, by a source of knowledge, Kant means a source of justification. So the Kantian conception of *a priori* knowledge comes to:

(APK) S knows *a priori* that p if and only if S's belief that p is justified *a priori* and the other conditions on knowledge are satisfied; and

(APJ) S's belief that p is justified *a priori* if and only if S's justification for the belief that p does not depend on experience.

(APJ) has been criticised from two directions. First, some maintain that it is not sufficiently informative; it tells us what *a priori* justification is not, but not what it is. Hence, Laurence BonJour (1985) rejects the Kantian conception of *a priori* justification in favour of the traditional rationalist conception:

(AP1) S's belief that p is justified *a priori* if and only if S intuitively "sees" or apprehends that p is necessarily true.

Alvin Plantinga (1993) and BonJour (1998) offer variants of (AP1). Second, others maintain that the sense of "dependence" relevant to *a priori* justification requires articulation, and have offered two competing accounts. Albert Casullo (2003) endorses

(AP2) S's belief that p is justified *a priori* if and only if S's belief that p

is non-experientially justified (i.e., justified by some non-experiential source).

Hilary Putnam (1983) and Philip Kitcher (1983) favour

(AP3) S's belief that p is justified *a priori* if and only if S's belief that p is non-experientially justified and cannot be defeated by experience.

(AP1) and (AP3) face serious objections.

The term "see" is used metaphorically in (AP1). Let us assume that it shares one basic feature with the literal use of "see": "S sees that p" entails "S believes that p". Hence, (AP1) has the consequence that if S's belief that p is justified *a priori* then S believes that p is necessarily true. This consequence faces two problems. Suppose that Sam is a mathematician who believes some generally accepted theorem T on the basis of a valid proof. Presumably, Sam's belief is justified. But suppose that Sam is also a serious student of philosophy who has come to doubt the cogency of the distinction between necessary and contingent propositions and, as a consequence, refrains from modal beliefs. It is implausible to maintain that Sam's belief that T is not justified *a priori* merely because of his views about a controversial metaphysical thesis. (AP1) is also threatened with a regress. It entails that if S's belief that p is justified *a priori* then S believes that necessarily p. Must S's belief that necessarily p be justified? If not, it is hard to see why it is a necessary condition of having an *a priori* justified belief that p. If so, then presumably it is justified *a priori*. But in order for S's belief that necessarily p to be justified *a priori*, S must believe that necessarily necessarily p, and the same question arises with respect to the latter belief. Must it be justified or not? Hence, (AP1) either faces an infinite regress of justified modal beliefs or is committed to the view that having an unjustified belief that necessarily p is a necessary condition of having a justified belief that p.

(AP3) is also open to serious objection. Kripke (1980) and Kitcher (1983) maintain that an adequate conception of *a priori* knowledge should allow for the possibility that a person knows empirically some proposition that he or she can know *a priori*. (AP3) precludes this possibility. Assume that

(A) S knows empirically that p and S can know *a priori* that p.

From the left conjunct of (A), it follows that

(1) S's belief that p is justified$_k$ empirically,

where "justified$_k$" abbreviates "justified to the degree minimally sufficient for knowledge". Consider now the empirical sources that have been alleged

to justify mathematical propositions: counting objects, reading a textbook, consulting a mathematician, and computer results. Each of these sources is fallible in an important respect. The justification each confers on a belief that p is defeasible by an empirically justified overriding defeater; that is, by an empirically justified belief that not-p. If S's belief that p is justified by counting a collection of objects and arriving at a particular result, then it is possible that S recounts the collection and arrives at a different result. If S's belief that p is justified by a textbook (mathematician, computer result) that states that p, then it is possible that S encounters a different textbook (mathematician, computer result) that states that not-p. In each case, the latter result is an empirically justified overriding defeater for S's original justification. Hence, given the fallible character of empirical justification, it follows that

(2) S's empirical justification for the belief that p is defeasible by an empirically justified belief that not-p.

(2), however, entails that

(3) S's belief that not-p is justifiable$_d$ empirically,

where "justifiable$_d$" abbreviates "justifiable to the degree minimally sufficient to defeat S's justified$_k$ belief that p". Furthermore, the conjunction of (AP3) and the right conjunct of (A) entails

(4) It is not the case that S's non-experiential justification$_k$ for the belief that p is defeasible by S's empirically justified belief that not-p.

(4), however, entails that

(5) It is not the case that S's belief that not-p is justifiable$_d$ empirically.

The conjunction of (3) and (5) is a contradiction. Hence, (AP3) is incompatible with (A). (AP2), however, is compatible with (A) since the conjunction of (AP2) and the right conjunct of (A) does not entail (4). Since both the traditional rationalist conception and the Putnam–Kitcher articulation of the Kantian conception of *a priori* justification are open to serious objections which (AP2) avoids, (AP2) provides the superior articulation of the concept of *a priori* justification.

The Existence of *a priori* Knowledge

Traditional apriorism: Kant

Kant offers the most influential traditional argument for the existence of *a priori* knowledge. Kant (1965: 43) holds that necessity is a criterion of the *a priori*: "if we have a proposition which in being thought is thought as *necessary*, it is an *a priori* judgment; ... " He (1965: 52) then goes on to argue that "mathematical propositions, strictly so called, are always judgments *a priori*, not empirical; because they carry with them necessity, which cannot be derived from experience." Kant's argument, the *Argument from Necessity*, can be presented as follows:

(N1) Mathematical propositions are necessary.

(N2) One cannot know a necessary proposition on the basis of experience.

(N3) Therefore, one cannot know mathematical propositions on the basis of experience.

The phrase "know a necessary proposition" in (N2) masks some important distinctions:

(A) S knows the *truth-value* of p just in case S knows that p is true or S knows that p is false.

(B) S knows the *general modal status* of p just in case S knows that p is a necessary proposition (i.e., necessarily true or necessarily false) or S knows that p is a contingent proposition (i.e., contingently true or contingently false).

(C) S knows the *specific modal status* of p just in case S knows that p is necessarily true or S knows that p is necessarily false or S knows that p is contingently true or S knows that p is contingently false.

(A) and (B) are logically independent: one can know one but not the other. One can know that the Goldbach Conjecture is either necessarily true or necessarily false but not know whether it is true or false. Similarly, one can know that the Pythagorean Theorem is true, but not know whether it is necessarily true or contingently true. The specific modal status of a proposition is just the conjunction of its *truth-value* and its general modal status. Therefore, one cannot know the specific modal status of a proposition unless one knows both its *truth-value* and its general modal status.

We can now distinguish two readings of (N2):

(N2A) One cannot know the *truth-value* of a necessary proposition on the basis of experience; and

(N2B) One cannot know the *general modal status* of a necessary proposition on the basis of experience.

Kant (1965: 52) supports (N2) with the observation that "Experience teaches us that a thing is so and so, but not that it cannot be otherwise." This observation supports (N2B) but not (N2A), since Kant allows that experience can provide evidence that something is the case, but denies that it can provide evidence that something must be the case. The conclusion of the argument, however, is that knowledge of the *truth-value* of mathematical propositions, such as that 7 + 5 = 12, is *a priori*.

Kant's argument, the *Kantian Argument*, can now be articulated as follows:

(N1) Mathematical propositions are necessary.

(N2B) One cannot know the *general modal status* of a necessary proposition on the basis of experience.

(N3A) Therefore, one cannot know the *truth-value* of mathematical propositions on the basis of experience.

The Kantian Argument turns on this principle:

(KP) If the general modal status of p is knowable only *a priori*, then the *truth-value* of p is knowable only *a priori*.

(KP), however, is false. If one can know only *a priori* that a proposition is necessary, then one can know only *a priori* that a proposition is contingent. The evidence relevant to determining the latter is the same as that relevant to determining the former. For example, if one determines that "2 + 2 = 4" is necessary by trying to conceive of its falsehood and failing, one determines that "Kant is a philosopher" is contingent by trying to conceive of its falsehood and succeeding. But from the fact that one can know only *a priori* that the proposition "Kant is a philosopher" is contingent, it does not follow that one can know only *a priori* that the proposition "Kant is a philosopher" is true. Clearly, it is knowable *a posteriori*.

Roderick Chisholm (1977) suggests the following reformulation of the Argument from Necessity, the *Modal Argument*:

(N1) Mathematical propositions are necessary.

(N2B) One cannot know the *general modal status* of a necessary proposition on the basis of experience.

(N3B) Therefore, one cannot know the *general modal status* of mathematical propositions on the basis of experience.

The Modal Argument faces a different problem. Why accept (N2B)? Kant maintains that experience can teach us only what is the case. But a good deal of our ordinary practical knowledge and the bulk of our scientific knowledge provide clear counterexamples to the claim. My knowledge that my pen will fall if I drop it does not provide information about what is the case for the antecedent is contrary-to-fact. Scientific laws are not mere descriptions of the actual world. They support counterfactual conditionals and, hence, provide information beyond what is true of the actual world. In the absence of further support, (N2B) should be rejected.

Moderate Apriorism: Logical Empiricism

A second strategy for defending the existence of *a priori* knowledge is offered by proponents of logical empiricism, such as A. J. Ayer (1952) and Carl Hempel (1972), who reject John Stuart Mill's contention that knowledge of basic mathematical propositions, such as that 2 x 5 = 10, is based on induction from observed cases. Both draw attention to the fact that, if one is justified in believing that some general proposition is true on the basis of experience, then contrary experiences should justify one in believing that the proposition is false. But no experiences would justify one in believing that a mathematical proposition, such as that 2 x 5 = 10, is false. Suppose, for example, that one were to count what appear to be five pairs of shoes and arrive at the result that there were only nine shoes. Ayer (1952: 75–6) contends that

> [o]ne would say that I was wrong in supposing that there were five pairs of objects to start with, or that one of the objects had been taken away while I was counting, or that two of them had coalesced, or that I had counted wrongly. One would adopt as an explanation whatever empirical hypothesis fitted in best with the accredited facts. The one explanation which would in no circumstances be adopted is that ten is not always the product of two and five.

Since Ayer maintains that we would not regard any experiences as evidence

that a mathematical proposition is false, he concludes that no experiences provide evidence that they are true.

Ayer's argument, the *Irrefutability Argument*, can be stated as follows:

(A1) No experiences provide evidence that mathematical propositions are false.

(A2) If no experiences provide evidence that mathematical propositions are false, then no experiences provide evidence that they are true.

(A3) Therefore, no experiences provide evidence that mathematical propositions are true.

Ayer's example provides very weak support for (A1) because (a) it does not take into account the number of experiences that confirm the proposition in question, (b) it involves only a single experience that disconfirms the proposition, and (c) the hypotheses which are invoked to explain away the disconfirming experience as apparent are not subjected to independent empirical test. In a situation where there is a strong background of supporting experiential evidence for an inductive generalization and an isolated disconfirming experience, it is reasonable to discount the disconfirming experience as apparent and to explain it away on whatever empirical grounds are most plausible. But it does not follow that the generalization in question cannot be disconfirmed by experience.

In order to provide stronger support for (A1), Ayer's example must be revised as follows: increase the number of experiences that disconfirm the proposition so that it is large relative to the number of experiences that confirm it; and subject the hypotheses invoked to explain away the disconfirming experiences as apparent to independent tests that fail to support them. Let us now suppose that one has a very large number of experiences that disconfirm the proposition that 2 x 5 = 10 and, furthermore, that empirical investigations of the hypotheses invoked to explain away these disconfirming experiences as apparent produce very little, if any, support for the hypotheses. Given these revisions, Ayer can continue to endorse premise (A1) only at the expense of holding empirical beliefs that are at odds with the available evidence.

Inductive Radical Empiricism: Mill

Radical empiricism is the view that denies the existence of *a priori* knowledge. One strategy for denying the existence of *a priori* knowledge is to offer radical empiricist accounts of those domains of knowledge that proponents of the *a*

priori allege to be knowable only *a priori*. Since mathematical knowledge has received the most attention, we will focus on it. Radical empiricist accounts of mathematical knowledge fall into two broad categories: inductive and holistic.

John Stuart Mill (1973) offers an inductive empiricist account of mathematical knowledge. Inductive empiricism with respect to a domain of knowledge involves two theses: (1) some propositions within that domain are epistemically more basic than the others, in the sense that the non-basic propositions derive their justification from the basic propositions via inference; and (2) the basic propositions are known by inductive inference from observed cases. Mill's primary thesis is that the basic propositions, the axioms and definitions, of arithmetic and geometry are known by induction from observed cases.

Mill's account faces serious objections, such as those offered by Gottlob Frege (1974). Let us assume, however, that these objections can be deflected and that Mill provides a defensible inductive empiricist account of mathematical knowledge. Does this show that mathematical knowledge is not *a priori*? If Mill's account is defensible, then it follows that Kant's claim that one cannot know mathematical propositions on the basis of experience is false. It does not follow, however, that the weaker claim that there is *a priori* knowledge of mathematical propositions is false. From the fact that one knows (or can know) mathematical propositions on the basis of experience, it does not immediately follow that one does not (or cannot) know mathematical propositions *a priori*.

Mill (1973: 231–2) addresses the gap in his argument with the following considerations:

> They cannot, however, but allow that the truth of the axiom, Two straight lines cannot inclose a space, even if evident independently of experience, is also evident from experience. ... Where then is the necessity for assuming that our recognition of these truths has a different origin from the rest of our knowledge, when its existence is perfectly accounted for by supposing its origin to be the same? ... The burden of proof lies on the advocates of the contrary opinion: it is for them to point out some fact, inconsistent with the supposition that this part of our knowledge of nature is derived from the same sources as every other part.

He attempts to close the gap by appealing to a version of the Explanatory Simplicity Principle: If a putative source of knowledge is not necessary to explain knowledge of the propositions within some domain, then it is not a source of knowledge of the propositions within that domain.

Mill's argument, the *Explanatory Simplicity Argument*, can be articulated as follows:

(M1) Inductive empiricism provides an account of mathematical knowledge based on inductive generalization from observed cases.

(M2) α is a source of knowledge for some domain D only if α is necessary to explain knowledge of some propositions within D.

(M3) Therefore, mathematical knowledge is not *a priori*.

The burden of the argument is carried by (M2), the Explanatory Simplicity Principle.

Casullo (2005) argues that the Explanatory Simplicity Principle is false because it rules out the possibility of a familiar form of epistemic overdetermination. The justification of some of our beliefs is overdetermined by different sources. There are some beliefs for which we have more than one justification, each of those justifications derives from a different source, and each, in the absence of the others, is sufficient to justify the belief in question. For example, I've misplaced my wallet and wonder where I might have left it. I suddenly recall having left it on the kitchen table last night. My recollection justifies my belief that my wallet is on the kitchen table. But, just to be sure, I walk out to the kitchen to check. To my relief, I see my wallet on the table. My seeing my wallet on the table also justifies my belief that my wallet is on the table. So here my justification is overdetermined by different sources. If the justification of my belief is overdetermined by two different sources, it follows that my belief is justified by two different sources. Hence, in the absence of an argument against the possibility of epistemic overdetermination by different sources, Mill's appeal to the Explanatory Simplicity Principle simply begs the question.

Holistic Radical Empiricism: Quine

Quine rejects inductive empiricism. He rejects the idea that there are basic mathematical propositions which, taken in isolation, are directly justified by observation and inductive generalization. Quine's account of mathematical knowledge is a version of holistic empiricism. Mathematical propositions are components of scientific theories. They are not tested directly against observation, but only indirectly via their observational consequences. Moreover, they don't have observational consequences in isolation, but only in conjunction with the other propositions of the theory. Hence, according to holistic empiricism, entire scientific theories, including their mathematical components, are indirectly confirmed or disconfirmed by experience via their observational consequences.

Our main concern is whether Quine's account of mathematical knowledge

provides an argument against the existence of *a priori* knowledge. The argument of Quine's (1963) classic paper, "Two Dogmas of Empiricism", remains controversial. His attack is directed at a variant of Frege's conception of analyticity: a statement is analytic if it can be turned into a logical truth by replacing synonyms with synonyms. His primary target is the notion of *synonymy* and his leading contentions can be summarized as follows. First, synonymy cannot be explained in terms of definition, interchangeability *salve veritate*, or semantic rules. Second, the verification theory of meaning does provide an account of statement synonymy; but the theory presupposes radical reductionism, which is a failed programme. A vestige of that programme survives in the view that individual statements admit of confirmation or disconfirmation. Quine objects to this vestige since it lends credence to the idea that there are statements confirmed no matter what, which he (1963: 43) rejects on the grounds that "no statement is immune to revision".

There are two strands to Quine's argument. The first challenges the cogency of semantic concepts such as synonymy. The second challenges the remaining vestige of reductionism. Neither contention, however, is explicitly directed at *a priori* knowledge. Hence, if Quine's argument does present a challenge to the existence of *a priori* knowledge, then some additional premise is necessary that connects one of its explicit targets to the *a priori*.

One standard reading of Quine's argument is that his goal is to undermine the central tenet of logical empiricism,

(LE) All *a priori* knowledge is of analytic truths,

by showing that the analytic/synthetic distinction is not cogent. Suppose we grant that (LE) is indeed Quine's target and that his arguments establish that the analytic/synthetic distinction is not cogent. It does not follow that either the claim of proponents of (LE) that there is *a priori* knowledge or their supporting argument for that claim is not cogent. Logical empiricists, such as Ayer, do not take (LE) to be constitutive of the concept of *a priori* knowledge. Moreover, they do not base their case for the existence of *a priori* knowledge on a premise, such as (LE), that involves the concept of analytic truth. They endorse the Kantian conception of *a priori* knowledge and base their case for *a priori* knowledge on the Irrefutability Argument. They then go on to offer independent arguments to show that propositions known *a priori* are analytic. Hence, Quine's argument establishes only that their thesis about the nature of the propositions known *a priori* is not cogent. But from this it does not follow that either their claim that there is *a priori* knowledge or their supporting argument for that claim is not cogent.

One might attempt to bolster Quine's argument by maintaining that (LE) is constitutive of the concept of *a priori* knowledge. If the concept of *a priori* knowledge involves the concept of analytic truth and the latter concept is incoherent, then the former is also incoherent. There are two ways in which the concept of *a priori* knowledge might involve the semantic concept of analytic truth: explicitly or implicitly. As we saw in section 1, neither Kant's conception of *a priori* knowledge, (APK), nor his conception of *a priori* justification, (APJ), explicitly involves the concept of analytic truth. The only plausible case for maintaining that the concept of *a priori* knowledge implicitly involves that concept is based on two premises: (1) the concept of *a priori* knowledge involves the concept of necessary truth; and (2) the concept of necessary truth is analysable in terms of the concept of analytic truth. Both premises are problematic since (APK) does not involve the concept of necessary truth, and there is no available analysis of the concept of necessary truth in terms of the concept of analytic truth.

Putnam (1983) proposes an alternative connection between Quine's contentions and the rejection of the *a priori*. He maintains that Quine's contentions are directed towards two different targets. His initial contentions are directed towards the semantic concept of synonymy. His later contentions, however, are directed towards the concept of a statement that is confirmed no matter what, which is not a semantic concept. It is an epistemic concept; it is a concept of apriority. Kitcher (1983: 80) endorses Putnam's reading of Quine's argument: "If we can know *a priori* that *p* then no experience could deprive us of our warrant to believe that *p*." Hence, the Putnam–Kitcher version of Quine's argument, the *Unrevisability Argument*, can be stated as follows:

(Q1) No statement is immune to revision in light of recalcitrant experience.

(Q2) If S's belief that p is justified *a priori*, then S's belief that p is not rationally revisable in light of any experiential evidence.

(Q3) Therefore, no knowledge is *a priori*.

The argument fails. Premise (Q2) is open to the objection presented against (AP3) on pages 231–2.

The Relationship Between *a priori* Knowledge and Necessary Truth

Current interest in the relationship between *a priori* knowledge and necessary truth is due to Kripke (1971, 1980), who makes two striking epistemological claims:

(E1) There are necessary *a posteriori* truths; and

(E2) There are contingent *a priori* truths.

Kripke maintains that (E1) is a consequence of one of his primary metaphysical theses:

(MT) Identity statements involving proper names are necessarily true if true,

and that (E2) is a consequence of one of his primary semantic theses:

(ST) A definite description that is employed to introduce a name fixes the reference of that name rather than providing its sense.

He also acknowledges that it is a widely held view, one that he associates with Kant, that

(K) All knowledge of necessary truths is *a priori* and all *a priori* knowledge is of necessary truths.

Therefore, he argues against (K) in order to defuse a potential objection to (MT) and (ST).

In order to assess how Kripke's claims bear on Kant's account of the relationship between *a priori* knowledge and necessary truth, four preliminary observations are in order. First, (K) is the conjunction of two principles:

(K1) All knowledge of necessary truths is *a priori*; and

(K2) All *a priori* knowledge is of necessary truths.

Second, Kant's conception of *a priori* knowledge (KAP),

(KAP) S knows *a priori* that p if and only if S's justification for the belief that p is independent of all experience and the other conditions on knowledge are satisfied,

does not underwrite either (K1) or (K2) since necessity is not constitutive of (KAP). Third, Kant's contention that necessity is a criterion of *a priori* knowledge, where a criterion is a sufficient condition that is not constitutive of the concept of *a priori* knowledge, underwrites (K1). Fourth, neither Kant's conception of *a priori* knowledge nor his contention that necessity is a criterion of the *a priori* underwrites (K2). (K2) plays no role in the framework for discussing the *a priori* that Kant articulates in his introduction to the *Critique*.

Casullo (2010) maintains that (K2) draws its support from a different source, the traditional rationalist conception of *a priori* knowledge:

(RAP) S knows *a priori* that p just in case S intuitively "sees" (or apprehends) that p is necessarily true and the other conditions on knowledge are satisfied.

Since "'seeing' that p is necessarily true" entails "p is necessarily true", it follows from (RAP) that *a priori* knowledge is restricted to necessary truths. Therefore, only (E1) bears on Kant's account of the relationship between *a priori* knowledge and necessary truth.

Kripke initially provides two different examples in support of (E1): (a) statements in which an essential property is attributed to a physical object; and (b) identity statements involving different co-referential proper names. He later extends his discussion of identity statements to include theoretical identity statements. We will focus on (a) and (b). Let "a" be the name of a particular lectern and "F" be the property of being made of wood. Suppose that someone knows that Fa – i.e., that this lectern is made of wood. Such knowledge is *a posteriori* since one knows that something is made from wood as opposed to, say, water frozen from the river Thames on the basis of how it looks and feels. Yet, if Fa is true, it is necessarily true since F is an essential property of a. In any possible world in which a exists, a is F. Hence, one who knows that Fa has *a posteriori* knowledge of a necessary truth.

To assess the implications of Kripke's example, we must keep in mind that the expression "*a posteriori* knowledge of a necessary truth" is ambiguous since it does not distinguish between (A) *a posteriori* knowledge of the *truth-value* of a necessary proposition, (B) *a posteriori* knowledge of the general modal status of a necessary proposition, and (C) *a posteriori* knowledge of the specific modal status of a necessary proposition. Kripke's case is an example of *a posteriori* knowledge of the *truth-value* of Fa since one discovers via experience that the lectern is made of wood. What about knowledge of its general modal status? Here Kripke (1971: 153) is explicit in maintaining that we know by "*a priori* philosophical analysis" that if Fa is true, then it is necessarily true. Hence, Kripke's case is not an example of *a posteriori* knowledge of the general modal status of a necessary proposition. Kripke maintains that such knowledge is *a priori*. Finally, one who knows (*a posteriori*) that Fa and (*a priori*) that if Fa, then necessarily Fa can infer, and thereby know, that necessarily Fa. Knowledge that necessarily Fa is knowledge of the specific modal status of Fa. Since knowledge of the specific modal status of Fa is based (in part) on *a posteriori* knowledge of its truth, it is also *a posteriori*.

The same observations apply to Kripke's example of identity statements involving proper names. Since, according to Kripke, ordinary proper names,

such as "Hesperus" and "Phosphorus", are rigid designators, each picks out the same object in all possible worlds in which it picks out any object. Therefore, if both pick out the same object in the actual world, both pick out the same object in all possible worlds in which they pick out any object. Hence, if "Hesperus is Phosphorus" is true, it is necessarily true. On the other hand, it was an astronomical discovery that Hesperus is Phosphorus. So, once again, Kripke has provided an example of *a posteriori* knowledge of the *truth-value* of a necessary proposition. Moreover, he (1980: 109) maintains that we know "by a priori philosophical analysis" that such identity statements are necessarily true if true. Hence, Kripke's case is not an example of *a posteriori* knowledge of the general modal status of a necessary proposition. Finally, one who knows (*a posteriori*) that Hesperus is Phosphorus and (*a priori*) that if Hesperus is Phosphorus, then necessarily Hesperus is Phosphorus can infer, and thereby know (*a posteriori*), that necessarily Hesperus is Phosphorus.

How does (E1) bear on Kant's account of the relationship between *a priori* knowledge and necessary truth? The question cannot be answered straightforwardly because neither Kant nor Kripke makes the appropriate distinctions. (K1) is ambiguous. There are two ways of reading it:

(K1A) All knowledge of the *truth-value* of necessary propositions is *a priori*; and

(K1B) All knowledge of the *general modal status* of necessary propositions is *a priori*.

Although Kant endorses both (K1A) and (K1B), the argument he offers in support of (K1) supports only (K1B). Kripke's examples of necessary *a posteriori* truths are examples of *a posteriori* knowledge of the *truth-value* of a necessary proposition. He, however, denies that they are examples of *a posteriori* knowledge of the general modal status of a necessary proposition. Hence, Kripke's claims challenge (K1A) but not (K1B).

Both Kant and Kripke contend that knowledge of the general modal status of propositions is possible. Yet Kripke's claim that there are necessary *a posteriori* truths presents a significant challenge to that contention. Prior to his arguments to the contrary, most held the false belief that necessary *a posteriori* truths, such as that Hesperus is Phosphorus, are contingent truths. Moreover, there remains a strong intuition that appears to support that false belief. This suggests that modal intuitions are systematically unreliable and that they result in widespread error regarding the general modal status of propositions. Such widespread error, in turn, threatens modal knowledge.

Kripke recognizes the challenge to his position and responds to it in a manner that is hospitable to modal knowledge. Kripke maintains that, given that Hesperus is Phosphorus, there is no possible world in which Hesperus

is not Phosphorus. So it is false that it might turn out that Hesperus is not Phosphorus. Yet he (1980: 103) acknowledges that "this seems very strange because in advance, we are inclined to say, the answer to the question whether Hesperus is Phosphorus might have turned out either way." Kripke (1980: 103–4) attempts to resolve this tension as follows:

> The evidence I have before I know that Hesperus is Phosphorus is that I see a certain star or a certain heavenly body in the evening and call it "Hesperus", and in the morning and call it "Phosphorus". I know these things. There certainly is a possible world in which a man should have seen a certain star at a certain position in the evening and called it "Hesperus" and a certain star in the morning and called it "Phosphorus"; and should have concluded – should have found out by empirical investigation – that he names two different stars, or two different heavenly bodies. ... And so it's true that given the evidence that someone has antecedent to his empirical investigation, he can be placed in a sense in exactly the same situation, that is a qualitatively identical epistemic situation, and call two heavenly bodies "Hesperus" and "Phosphorus", without their being identical. So in that sense we can say that it might have turned out either way. Not that it might have turned out either way as to Hesperus's being Phosphorus. Though for all we knew in advance, Hesperus wasn't Phosphorus, that couldn't have turned out any other way, in a sense.

Kripke (1980: 142) generalizes his answer to the puzzle as follows:

> Any necessary truth, whether a priori or a posteriori, could not have turned out otherwise. In the case of some necessary a posteriori truths, however, we can say that under appropriate qualitatively identical evidential situations, an appropriate corresponding qualitative statement might have been false.

So there is a sense in which a necessary *a posteriori* truth might have turned out to be false, but that sense does not entail that it is not a necessary truth.

There are two different senses in which p might turn out to be false or, alternatively, two senses in which it is possible that p is false. The first is metaphysical since it pertains to whether there is a possible world in which p is false. The second is epistemic since it pertains to whether the falsehood of p (or, more precisely, p*, where p* is the appropriate qualitative analogue to p) is compatible with one's qualitative evidence. According to Kripke, where p is a necessary *a posteriori* truth and one has an intuition that p might turn out to be false, one does not have an intuition that the falsehood of p is metaphysically possible. Instead, such an intuition, when properly understood and accurately reported, is an intuition that the falsehood of p is epistemically possible. In

other words, where E is one's original qualitative evidence for p, one has the intuition that the falsehood of p* is compatible with E*, where p* is a qualitative statement that appropriately corresponds to p and E* is an evidential situation that is appropriately qualitatively identical to E. Therefore, the intuition does not call into question the necessary truth of p.

Kripke's account is hospitable to modal knowledge for two reasons. First, when modal intuitions are properly understood and accurately reported, the modal beliefs that they support are true. Modal error arises when one confuses epistemic possibility with metaphysical possibility. Second, modal error is tractable in that (a) it is systematic and widespread only in the case of *a posteriori* necessities, but (b) Kripke's account identifies the source of the error, which enables us to avoid it or, at least, to correct it. These features of modal intuition align it favourably with other fallible sources of knowledge, such as perception.

Two-dimensional semantics occupies a prominent place in the contemporary discussion of modal knowledge. One of its virtues, according to its proponents, is that it provides a perspicuous account of the two types of possibility distinguished by Kripke. David Chalmers (2006: 59) lucidly summarizes the two-dimensional approach as follows:

> The core idea of two-dimensional semantics is that there are two different ways in which the extension of an expression depends on possible states of the world. First, the actual extension of an expression depends on the character of the actual world in which an expression is uttered. Second, the counterfactual extension of an expression depends on the character of the counterfactual world in which the expression is evaluated. Corresponding to these two sorts of dependence, expressions correspondingly have two sorts of intensions, associating possible states of the world with extensions in different ways.

The two sorts of intensions, according to Chalmers (2006: 59), yield two different ways of thinking about possibilities:

> In the first case, one thinks of a possibility as representing a way the actual world might turn out to be: or as it is sometimes put, *one considers a possibility as actual*. In the second case, one acknowledges that the actual world is fixed, and thinks of a possibility as a way the world might have been but is not: or as it is sometimes put, *one considers a possibility as counterfactual*.

The two different ways of thinking about possibilities can result in different extensions being assigned to an expression relative to a possible world. Consider again the possible world described by Kripke, in which someone sees

a certain star in a certain position in the evening sky and calls it "Hesperus", and also sees a certain star in a certain position in the morning sky and calls it "Phosphorus", but the two stars are not identical. If we think of this possibility as counterfactual, then "Hesperus" and "Phosphorus" both pick out Venus given that both pick out Venus in the actual world. Hence, in that world considered as counterfactual, Hesperus is Phosphorus. Moreover, in any world considered as counterfactual, "Hesperus" and "Phosphorus" both pick out Venus given that both pick out Venus in the actual world. So there is no world, considered as counterfactual, in which Hesperus is not Phosphorus. On the other hand, if we think of Kripke's possibility as actual, then "Hesperus" and "Phosphorus" pick out different objects. Hence, in that world considered as actual, Hesperus is not Phosphorus. So, thinking of possibilities as counterfactual, captures the sense in which it could not have turned out that Hesperus is not Phosphorus; but thinking of them as actual captures the sense in which, for all we knew in advance, it might have turned out that Hesperus is not Phosphorus.

Synthetic *a priori* Knowledge

Kant's most enduring contribution to the controversy surrounding *a priori* knowledge is his defence of

(K4) Some propositions known *a priori* are synthetic.

The literature on the *a priori* over the past 150 years is dominated by this issue. In addressing that literature, one question immediately arises: Why is the existence of synthetic *a priori* knowledge *epistemologically* significant? Kant regards it as significant because it sets the stage for his primary theoretical undertaking, which is to answer the question: How is synthetic *a priori* knowledge possible? Kant's question, however, is puzzling in one respect. Having established that there is *a priori* knowledge, he is in a position to pose the question: How is *a priori* knowledge possible? The fact that he deems it necessary to draw the analytic/synthetic distinction and to defend (K4) indicates that Kant does not think that *a priori* knowledge *in general* is problematic. In particular, he views analytic *a priori* knowledge as unproblematic.

If synthetic *a priori* knowledge is epistemologically problematic but analytic *a priori* knowledge is not, then they must differ in some way. What, according to Kant, is the difference? Kant maintains that knowledge of analytic propositions requires only possession of the relevant concepts and the principle of contradiction. Synthetic *a priori* knowledge, however, requires more. For example, in order to know that 7 + 5 = 12, Kant (1965: 53) maintains: "We have to go outside

these concepts, and call in the aid of the intuition which corresponds to one of them." Synthetic *a priori* knowledge raises special epistemological problems because of its alleged source in intuition.

The significance of (K4) is rooted in the assumption that the source of synthetic *a priori* knowledge is different from the source of analytic *a priori* knowledge. Kant, however, does not defend this assumption. Although he maintains that knowledge of analytic propositions requires only knowledge of the principle of contradiction and the content of concepts, he does not explicitly address the source of such knowledge. Since he does not explicitly address the source of analytic *a priori* knowledge, Kant has no basis for claiming that the source of such knowledge is different from the source of synthetic *a priori* knowledge, let alone that the latter is epistemologically more problematic than the former. Consequently, the epistemological significance of (K4) is presupposed rather than established.

Reactions to (K4) fall into three broad categories. Those in the first endorse (K4) but take issue with some of Kant's examples. Frege, for example, agrees that the truths of geometry are synthetic *a priori* but maintains that the truths of arithmetic are analytic. Those in the second reject (K4). Logical empiricists, such as Ayer, argue that alleged examples of synthetic *a priori* truths are either analytic or *a posteriori*. The reactions in the third category, which draw their inspiration from Quine, deny the cogency of the analytic/synthetic distinction and, *a fortiori*, the cogency of (K4). The epistemological import of these reactions is minimal.

Frege endorses (K4), but contends that the truths of arithmetic are analytic. His defence of this contention requires a modification of Kant's conception of analytic truth. Frege (1974: 4[e]) explicates the concept, with respect to mathematical propositions, in terms of features of their proof: "If, in carrying out this process [of following the proof of a proposition], we come only on general logical laws and on definitions, then the truth is an analytic one." The resulting conception of analytic truth is broader than Kant's. It does not restrict such truths to those in which the predicate is contained in the subject. Any mathematical truth whose proof consists solely of general logical laws and definitions qualifies as analytic.

Armed with this broader conception of analyticity, Frege's project is to demonstrate

(F1) All arithmetic truths are analytic.

This project faces a number of formidable technical obstacles. But we will assume that they can be overcome in order to assess its epistemological consequences. A successful demonstration of (F1) has no significant epistemological consequences. A demonstration that all arithmetic truths can be proved

from general logical laws and definitions, taken by itself, tells us little about knowledge of those truths since it is silent with respect to the issue of how one knows the primitive general laws, definitions, and logical principles employed in such proofs. In particular, (F1) is compatible with the claim that the truths of arithmetic are knowable only via intuition.

One might suggest that although (F1) fails to establish that arithmetic knowledge is not grounded in intuition, it does have a significant consequence regarding such knowledge. (F1) establishes that if knowledge of logic and definitions does not have its source in intuition then knowledge of arithmetic does not have its source in intuition. This result is significant since it establishes that there is a uniform explanation of knowledge of logic, definitions, and arithmetic.

The claim that (F1) establishes that there is a uniform explanation of knowledge of logic, definitions, and arithmetic rests on an unsubstantiated assumption: the only route to arithmetic knowledge is through proof from general logical laws and definitions. This assumption has an unwelcome consequence. It entails a wide-ranging scepticism with respect to the elementary truths of arithmetic. If the only route to arithmetic knowledge is through proof from general logical laws and definitions then very few, if any, have such knowledge.

Kant took for granted that most literate adults know *a priori* that 7 + 5 = 12, and set out to provide an account of such knowledge. If most literate adults have such knowledge, then there must be a route to it other than the type of proof envisioned by Frege. Therefore, Frege fails to show that there is a uniform explanation of the typical literate adult's knowledge of logic, definitions, and arithmetic. There are two possible explanations of the typical literate adult's knowledge of arithmetic: either its source is the same as the source of knowledge of logic and definitions, or it is different. If it is the same, then Frege's programme for establishing (F1) is unnecessary. If it is different, then we are still faced with the problem of explaining how such knowledge is possible. If Kant is right about the scope of *a priori* arithmetic knowledge, then Frege fails to provide an explanation of such knowledge.

Ayer (1952: 73) rejects (K4) on the grounds that either we must "accept it as a mysterious inexplicable fact" that there is synthetic *a priori* knowledge, or we must "accept the Kantian explanation which ... only pushes the mystery a stage further back." Instead, he endorses

(LE) All *a priori* knowledge is of analytic truths,

which he regards as epistemologically significant because it provides an explanation of *a priori* knowledge that is free of the mystery that plagues Kant's account.

Ayer (1952: 78) rejects Kant's account of the analytic/synthetic distinction, and offers an alternative: "a proposition is analytic when its validity depends solely on the definitions of the symbols it contains, and synthetic when its validity is determined by the facts of experience." Ayer's conception of analyticity is broader than Kant's since it does not restrict analytic propositions to those whose predicate is contained in the subject. Any proposition that is true in virtue of the definitions of the symbols it contains qualifies as analytic.

Ayer's (1952: 78–9) most explicit defence of (LE) is presented in the context of discussing logical truths:

> [T]he proposition "Either some ants are parasitic or none are" is an analytic proposition. For one need not resort to observation to discover that there either are or are not ants which are parasitic. If one knows what is the function of the words "either", "or", and "not", then one can see that any proposition of the form "Either *p* is true or *p* is not true" is valid, independently of experience.

His argument can be stated as follows:

(AJ1) One need not resort to observation to discover that there either are or are not ants which are parasitic.

(AJ2) Therefore, the proposition "Either some ants are parasitic or none are" is an analytic proposition.

(AJ1) is an epistemological premise: it asserts that the proposition in question can be known *a priori*. (AJ2), however, is a semantic conclusion: it asserts that the proposition in question is analytic. The validity of the argument depends on the following principle, which links the epistemic premise and the semantic conclusion:

(AJ3) All propositions knowable *a priori* are analytic.

Hence, Ayer's defence of (LE) is circular.

Suppose we grant (AJ2). Does (AJ2) provide an explanation of *a priori* knowledge of logical truths? If we return to the previously cited passage, Ayer offers the following premise in support of (AJ1):

(AJ4) If one knows what is the function of the words "either", "or", and "not", then one can see that any proposition of the form "Either *p* is true or *p* is not true" is valid, independently of experience.

Ayer explains *a priori* knowledge of logical truths in terms of an ability to "see" that they are true independently of experience. The sense of "see" invoked by Ayer to explain knowledge of logical truths is not the literal sense. His explanation appeals to a metaphorical sense of "see" that is not further explained. Therefore, Ayer's explanation, like Kant's appeal to intuition, "only pushes the mystery a stage further back".

Quine's rejection of the cogency of the analytic/synthetic distinction has been widely viewed as challenging the existence of *a priori* knowledge. We have examined two lines of argument in support of that view (see pages 239–40) and concluded that both fail. There is, however, another argument, the *Explanatory Argument*, which draws its inspiration from Quine and challenges the existence of *a priori* knowledge:

(E1) A theory of knowledge has two goals: (a) to articulate the sources and extent of human knowledge; and (b) to explain how those sources generate the knowledge in question.

(E2) Therefore, if a theory of knowledge endorses a category of knowledge but cannot explain how that knowledge is possible, then the theory is unacceptable.

(E3) The only available non-mysterious explanation of how *a priori* knowledge is possible involves the analytic/synthetic distinction.

(E4) But Quine has shown that the distinction is incoherent.

(E4) Therefore, a theory of knowledge that endorses the *a priori* is unacceptable.

The Explanatory Argument focuses attention on the explanatory requirements of an adequate theory of knowledge. Its central premise is (E3). Although widely endorsed, there is little support for (E3). As we saw earlier, however, neither Kant's nor Frege's nor Ayer's conception of analyticity offers much in terms of an explanation of *a priori* knowledge.

Nevertheless, the challenge posed by the Explanatory Argument remains. A theory of knowledge endorsing the *a priori* must offer some explanation of how such knowledge is possible. Once we recognize that the concept of analyticity offers little in terms of explanatory power, we are in a position to recognize that the explanatory problem goes beyond the coherence of the analytic/synthetic distinction. The more general problem that must be addressed is how *a priori* knowledge is possible.

Kant's introduction of the analytic/synthetic distinction, along with the assumption that the source of analytic *a priori* knowledge is different from and

more problematic than the source of synthetic *a priori* knowledge, is largely responsible for the continued controversy surrounding the existence of *a priori* knowledge. By focusing attention on the question of how synthetic *a priori* knowledge can be possible, Kant initiated a tradition whose primary preoccupation was with the *semantic* questions of whether there is a cogent analytic/synthetic distinction, how to draw that distinction, and which propositions fall into each category. Providing answers to these semantic questions, however, will not settle the controversy over the existence of *a priori* knowledge. The question that must be addressed to resolve that controversy is the more general *epistemological* question: How is *a priori* knowledge possible?

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