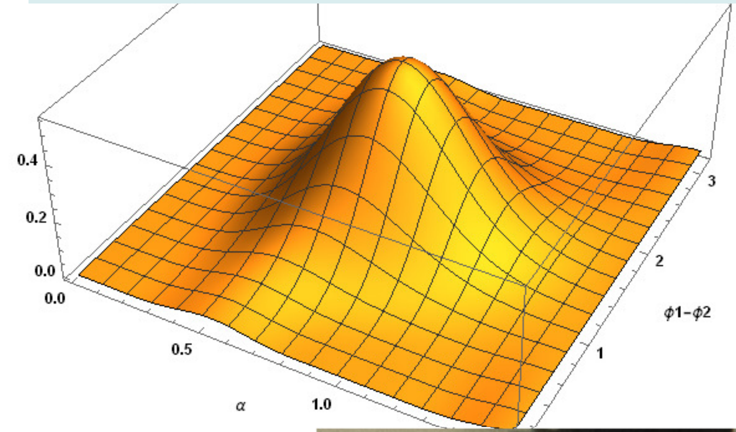
Few-body treatment of the Quantum Hall problem

arXiv:1504.07884

Chris Greene, Kevin Daily and
Rachel Wooten, Purdue University

3 particle Laughlin 1/3 state plotted versus 2 hyperangles



In this talk, we:

- **Formulate the 2D system of electrons on the plane in a B-field using collective hyperspherical coordinates**

- **Show a correlation between fractional quantum Hall states and states of *exceptional degeneracy***

Kevin Daily

Rachel Wooten

Professor Starace wrote to me in 1977 when I joined Ugo Fano's research group in Chicago, with some crucial comments and advice:

1. **You are now going to be part of a close-knit group of theorists, the "Fano School"**

2. **Read everything you can get your hands on; even if you don't understand it at first, this will pay dividends in the future**

3. **Don't be shy about coming up with your own ideas outside of your thesis work with Fano, and publishing it separately**

Tony always made it clear that he hoped I would one day be outstanding in my field, and….

Tony always made it clear that he hoped I would one day be outstanding in my field, and you can see that yesterday I was indeed out standing in my field.

My 3 joint papers with Tony

J. Phys. B: At. Mol. Opt. Phys. 27 (1994) L137–L142. Printed in the UK

LETTER TO THE EDITOR

Parallels between high doubly excited state spectra in H^- and Li^- photodetachment

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PHYSICAL REVIEW A VOLUME 53, NUMBER 2 FEBRUARY 1996

Photodetachment of Li^- from the Li 3*s* threshold to the Li 6*s* threshold

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(Received 18 July 1995)

PRL 94, 229301 (2005) PHYSICAL RE

Comment on "Fano Line Shapes Reconsidered: Symmetric Photoionization Peaks from Pure Continuum Excitation"

← Cooper, CHG, Langhoff, Starace, Winstead

Tony with Shinichi Watanabe, 1999 ICPEAC in Japan

<u>Motivations</u>

arXiv:1504.07884

Microscopic origin of the fractional QHE states

Can they emerge systematically without guessing wavefunctions?

What are quasi-particles?

How many electrons make up a quasi-particle, and how do their fractional charge and unusual statistics emerge?

Do properties of the non-interacting 2D free electron gas with no interactions determine whether a given filling factor yields a measurable FQHE state?

Whereas the full many-body Schroedinger equation is a <u>linear PDE</u>, many-body treatments such as mean-field theory are <u>nonlinear</u>. How can this linear ←→ nonlinear relationship be understood more deeply?

Since the FQHE is heralded as the prototype STRONGLY CORRELATED SYSTEM, can insights emerge from describing the system in COLLECTIVE COORDINATES rather than as independent electrons?

Physics is often about exploring phenomena from different points of view, i.e. different TOOLKITS. One example is the *“few-body hyperspherical toolkit”*

First of all, note that there have been many notable successes of hyperspherical coordinate treatments by Macek, Fano, Lin, and others, especially in the Fano school:

Fano Group theses using hyperspherical coordinates:
Ravi Rau, 1971
Chii-Dong Lin, 1974
C H Greene, 1980
Shinichi Watanabe, 1982
Michael Cavagnero, 1988
John Bohn, 1992

Some recent successes also include the treatment of 3-body and 4-body recombination processes and Efimov physics (CHG, Physics Today 2010)

SINGLE PARTICLE HAMILTONIAN

arXiv:1504.07884

$$H = \frac{1}{2m_e}\left(-\imath\hbar\boldsymbol{\nabla} + e\boldsymbol{A}\right)^2$$

$$\boldsymbol{A} = (B/2)(-y\hat{x} + x\hat{y})$$

$$H = -\frac{\hbar^2}{2m_e}\boldsymbol{\nabla}^2 + \frac{e^2B^2}{8m_e}(x^2+y^2) + \frac{eB}{2m_e}L_z$$

$$E^{(1)} = \frac{1}{2}(2n + m + |m| + 1)$$

← Single particle energy levels

Natural magnetic units we use throughout:

Frequency:

$$\omega_c = eB/(m_e)$$

Energy: $\hbar\omega_c$

Length:

$$\lambda_0 = \sqrt{\frac{\hbar}{m_e\omega_c}}$$

$$H = -\frac{1}{2}\left\{\frac{1}{r}\partial_r r\partial_r - \frac{L_z^2}{\hbar^2 r^2}\right\} + \frac{1}{8}r^2 + \frac{1}{2\hbar}L_z$$

N-BODY RELATIVE HAMILTONIAN

$$H_N = H_{\mathrm{CM}} + H_{\mathrm{rel}}$$

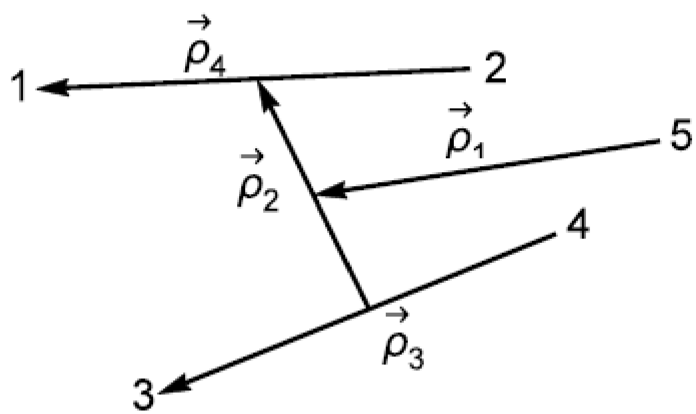
$$H_{\mathrm{rel}} = -\frac{1}{2\mu}\sum_{j=1}^{N_{\mathrm{rel}}} \nabla_j^2 + \frac{\mu}{8}\sum_{j=1}^{N_{\mathrm{rel}}}(x_j^2 + y_j^2) + \frac{1}{2\hbar}\sum_{j=1}^{N_{\mathrm{rel}}} L_{z_j}^{\mathrm{rel}} \qquad N_{\mathrm{rel}} = N - 1$$

$$\mu = \left(\frac{1}{N}\right)^{1/N_{\mathrm{rel}}}$$

← N-body reduced mass

d=2N-2 dimensional space, symmetry group is O(2N-2)

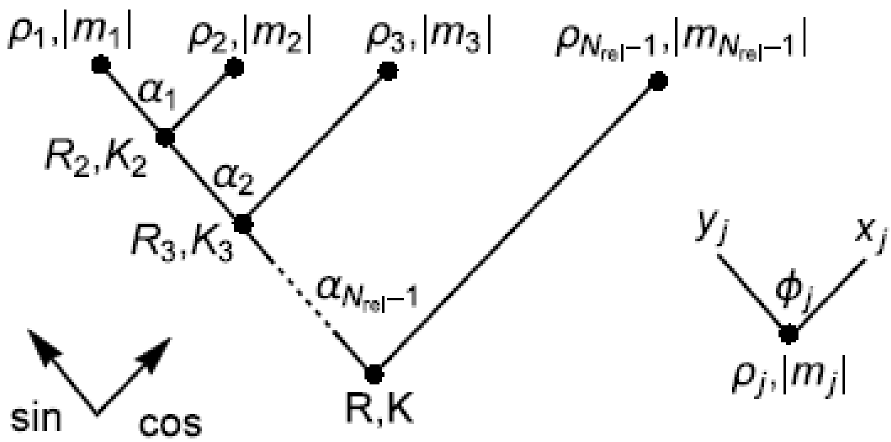
The 4 relative Jacobi vectors that characterize a 5-particle system in 2D:



Linear transformation matrix between the independent particle coordinates and the Jacobi relative+CM coordinates:

$$\begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_{\mathrm{CM}} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{4/5}{\mu}}\times & \{\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -1\} \\ \sqrt{\frac{1}{\mu}}\times & \{\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0\} \\ \sqrt{\frac{1/2}{\mu}}\times & \{0 & 0 & 1 & -1 & 0\} \\ \sqrt{\frac{1/2}{\mu}}\times & \{1 & -1 & 0 & 0 & 0\} \\ \frac{1}{5}\times & \{1 & 1 & 1 & 1 & 1\} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{pmatrix}$$

Hyperspherical coordinate transformation



←The "Jacobi Tree" used to define the hyperangular coordinates

And the squared hyperradius is defined by:

$$R^2 = \sum_{j=1}^{N_{\rm rel}} \rho_j^2$$

$$\tan\alpha_j = \frac{\sqrt{\sum_{k=1}^{j}\rho_k^2}}{\rho_{j+1}}.$$

This can be defined for any N-particle problem, and it is proportional to the trace of the moment of inertia tensor.

arXiv:1504.07884

non-interacting relative Hamiltonian

$$H_{\rm rel} = -\frac{1}{2\mu}\boldsymbol{\nabla}^2_{R,\Omega} + \frac{\mu}{8}R^2 + \frac{1}{2\hbar}L_z^{\rm rel,tot}$$

$$\boldsymbol{\nabla}^2_{R,\Omega} = \frac{1}{R^{2N_{\rm rel}-1}}\partial_R R^{2N_{\rm rel}-1}\partial_R - \frac{\hat{K}^2}{R^2}$$

$\hat{K}$ is called the grand angular momentum operator

The eigenstates of $\hat{K}^2$ are the hyperspherical harmonics, $\Phi^{(M)}_{K\,u}(\boldsymbol{\Omega})$, where

$$\hat{K}^2\Phi^{(M)}_{K\,u}(\boldsymbol{\Omega}) = K(K + 2N_{\rm rel} - 2)\Phi^{(M)}_{K\,u}(\boldsymbol{\Omega})$$

The final quantum number K here is called the "grand angular momentum quantum number", K = |M|, |M|+2, |M|+4, ….

Or for general N, where $N_{\text{rel}} = N - 1$:

$$\prod_{j=1}^{N_{\text{rel}}} (x_j \pm \imath y_j)^{m_j} \prod_{k=1}^{N_{\text{rel}}-1} \left(\sum_{l=1}^{k+1} \rho_l^2 \right) P_{n_k}^{K_k-(k+1),|m_{k+1}|} \left(\frac{\rho_{k+1}^2 - \sum_{l=1}^{k} \rho_l^2}{\sum_{l=1}^{k+1} \rho_l^2} \right),$$

$$\sum_{j=1}^{N_{\text{rel}}} m_j = M$$

and in hyperspherical form

$$\prod_{j=1}^{N_{\text{rel}}} e^{\imath m_j \phi_j} \prod_{k=1}^{N_{\text{rel}}-1} \sin^{K_k} \alpha_k \ \cos^{|m_{k+1}|} \alpha_k \ P_{n_k}^{K_k-(k+1),|m_{k+1}|} (\cos(2\alpha_k))$$

Refs: Smirnov & Shitkova, or see Avery book

where the P are Jacobi polynomials and the K_k are sub-hyperangular 'quantum numbers', defined recursively as

$$K_1 = |m_1|,$$

$$K_k = 2n_{k-1} + K_{k-1} + |m_k|.$$

In the lowest Landau level, the equations simplify in that all of the $n_k = 0$ such that the Jacobi polynomials are all unity.

In Macek's (1968, J Phys B) adiabatic hyperspherical representation, we can transform the d-dimensional Schroedinger equation into motion along a system of coupled 1D potential energy curves $U_i(R)$

This technique has given qualitative insight and quantitative predictive power in many other systems (e.g. universal Efimov physics for 3, 4, or 5 particles, the few-nucleon problem, the Ps_2 system, etc.)

See, e.g. Rittenhouse et al. topical review, J. Phys. B **44**, 172001 (2011)

Strategy of Macek's adiabatic hyperspherical representation: convert the partial differential Schroedinger equation into an infinite set of coupled ordinary differential equations:

To solve: →

$$\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial R^2}+\frac{\Lambda^2}{2\mu R^2}+V(R,\theta,\varphi)\right]\psi_E=E\psi_E$$

First solve the fixed-R Schroedinger equation, for eigenvalues $U_n(R)$:

$$\left[\frac{\Lambda^2}{2\mu R^2}+\frac{15}{8\mu R^2}+V(R,\theta,\varphi)\right]\Phi_\nu(R;\Omega)=U_\nu(R)\Phi_\nu(R;\Omega)$$

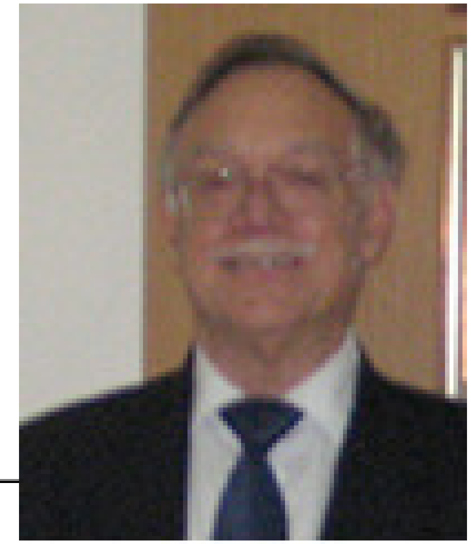
Next expand the desired solution into the complete set of adiabatic eigenfunctions →

$$\psi_E(R,\Omega)=\sum_\nu F_{\nu E}(R)\Phi_\nu(R;\Omega)$$

And the original T.I.S.Eqn. is transformed into the following set which can be truncated on physical grounds, with the eigenvalues interpretable as adiabatic potential curves, in the Born-Oppenheimer sense.

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2}+U_\nu(R)\right]F_{\nu E}(R)-\frac{1}{2\mu}\sum_{\nu'}\left[2P_{\nu\nu'}(R)\frac{d}{dR}+Q_{\nu\nu'}(R)\right]F_{\nu' E}(R)=EF_{\nu E}(R)$$

Joe Macek's (1968 JPB) adiabatic hyperspherical picture gave insight into why only one series of autoionizing states is seen in He photoabsorption near the n=2 threshold, instead of three.



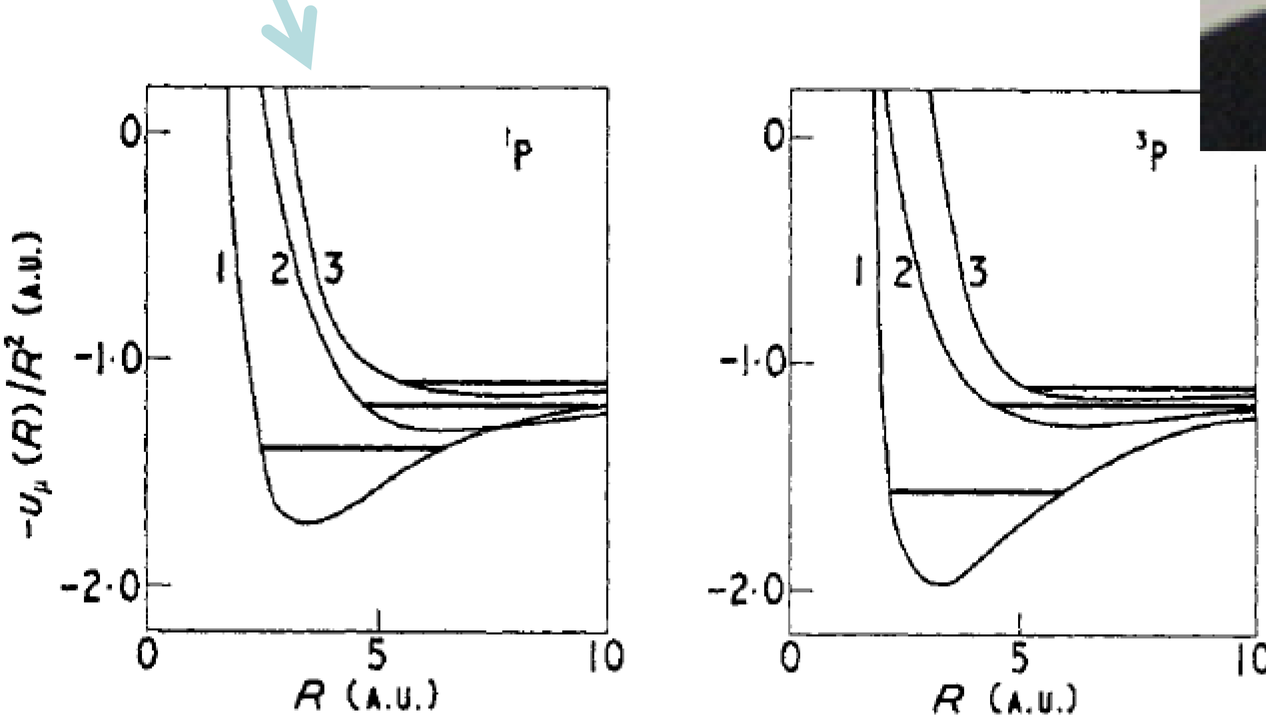
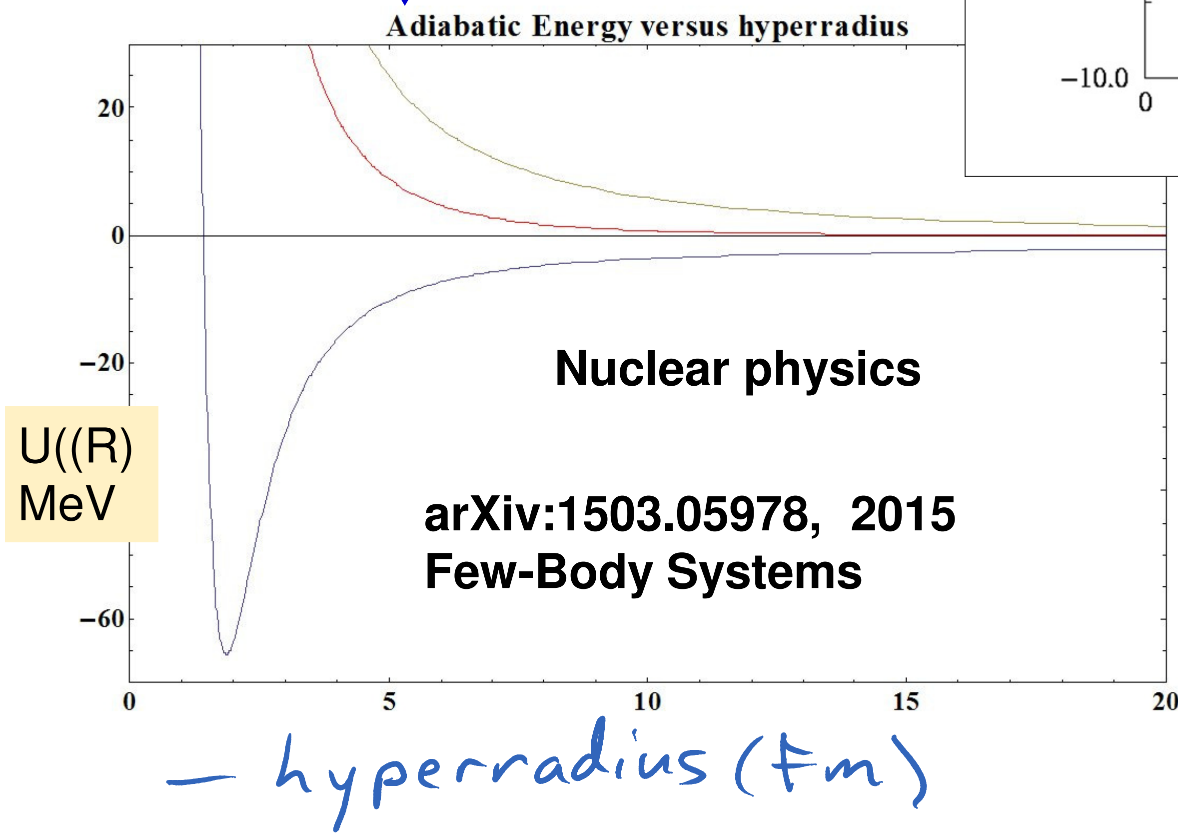
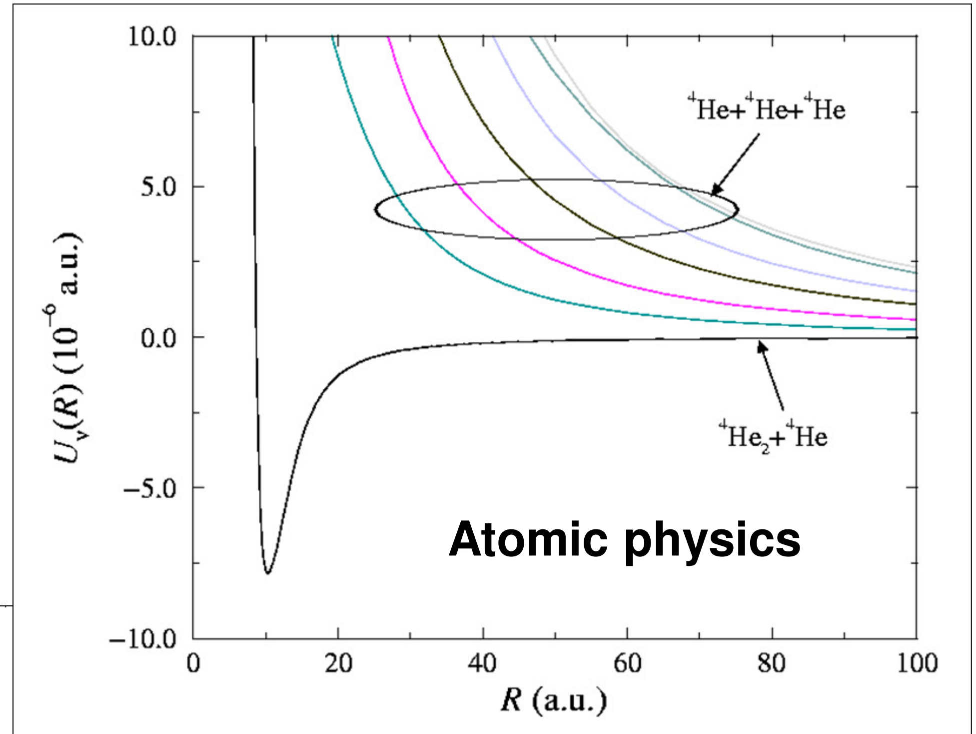


Figure 1. Graphs of $-U_\mu(R)/R^2$ against R for $^1S^e$, $^3S^e$, $^1P^o$ and $^3P^o$ cases. The 0th curve (ground state) is not shown. Positions of the lowest member of the Rydberg series of autoionizing states for each curve are marked by a horizontal line.

Universality, from nuclear scale energies to the chemical

Adiabatic potential curves for n+n+p, in collaboration with Alejandro Kievsky and Kevin Daily, nuclear physics on 10^6 eV scale (FBS 2015)





3-atom hyperspherical potential curves for He+He+He on a 10^{-3} eV scale, looks very similar to the 3-nucleon potentials

Extensively used to understand universal Efimov physics

arXiv:1504.07884

Now apply this adiabatic hyperspherical method to the quantum Hall problem

How to define the "filling factor" $$\nu = \frac{\rho h}{eB} = \frac{N\phi_0}{BA}$$

$$\langle R^2 \rangle_{N,r_c} = \frac{(N-1)r_c^2}{2\mu}$$

$\phi_0 = h/e$ in S.I. units
fundamental flux quantum

Typical GaAs: $\rho = 2.4 \times 10^{11} cm^{-2}$

$$\nu = \frac{N(N-1)}{2K}$$

=HYPERSPHERICAL FILLING FACTOR

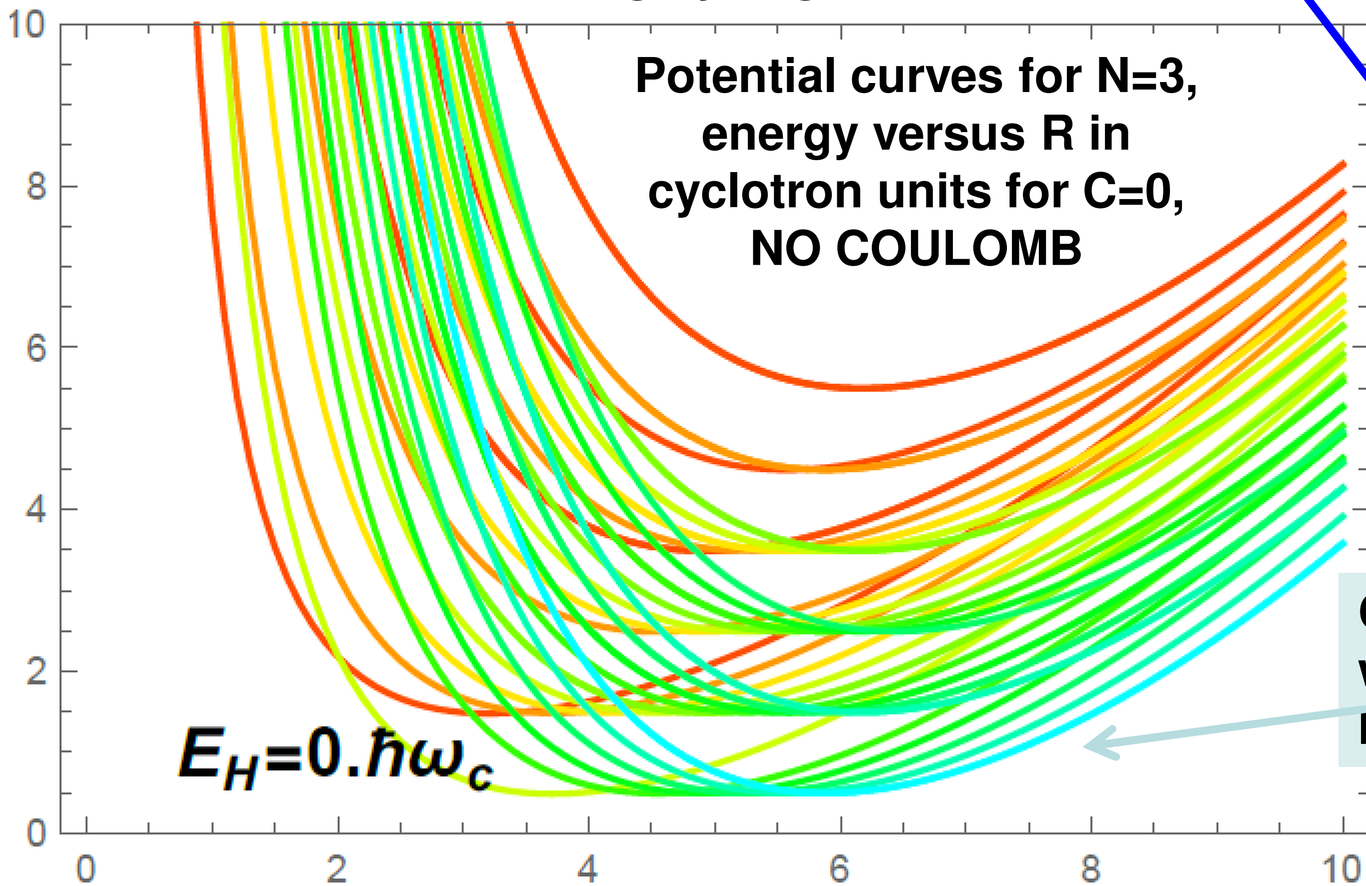
the $\nu = 1$ quantum Hall state is found at a magnetic field near $B = 10$T and the $\nu = 1/3$ state occurs around the much higher field $B \approx 29$T.

Potential energy curves for N noninteracting electrons in 2D in a B-field

$$U_{KM\gamma}(R) = \frac{(K+N-\frac{3}{2})(K+N-\frac{5}{2})}{2\mu R^2} + \kappa\frac{C_{KM\gamma}}{R} + \frac{1}{8}\mu R^2 + \frac{1}{2}M$$

Antisymmetrization has been carried out, and most curves shown are highly degenerate

This term vanishes if no Coulomb interactions. These C are eigenvalues of the Coulomb interaction within a degenerate K,M-space

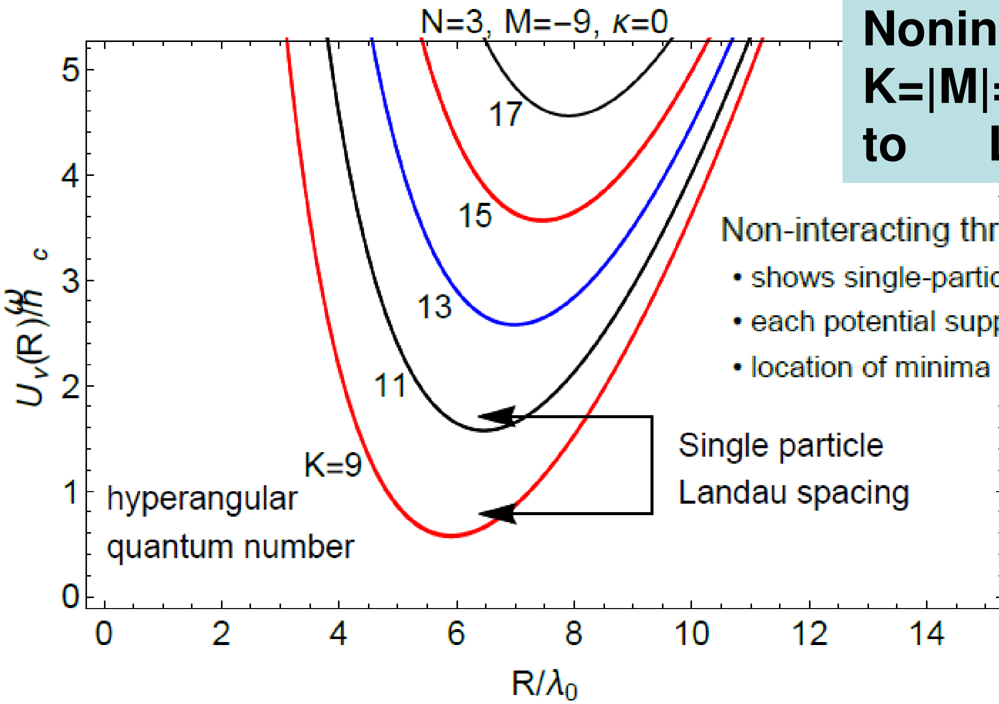


$$\kappa = \frac{e^2}{4\pi\epsilon\lambda_0}\frac{1}{\hbar\omega_c}$$

Channels associated with the lowest Landau level, K=-M=|M|

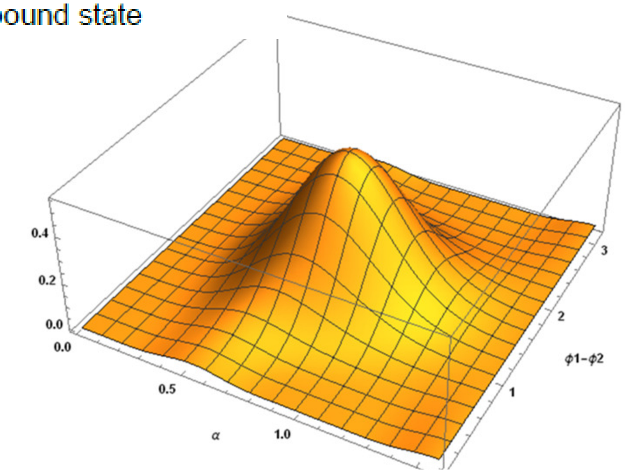
← R, hyperradius in cyclotron units

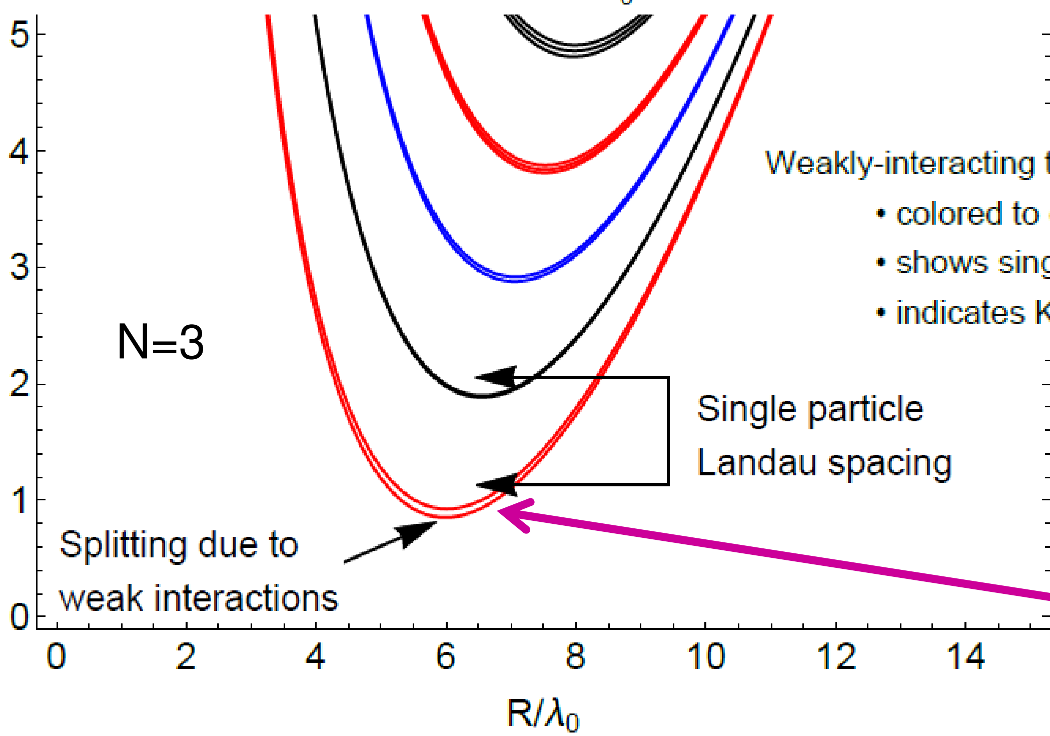
Noninteracting states for K=|M|=9,11,13,15,…, corresponding to LL = 0, 1, 2, 3, … resp.



Non-interacting three-body system in magnetic units.

- shows single-particle energy spacing
- each potential supports an infinite number of bound states
- location of minima indicate “size” of bound state





Weakly-interacting three-body system in magnetic units.

- colored to distinguish different K-manifolds
- shows single-particle energy spacing is larger than that of interactions
- indicates K-manifolds are, in general, degenerate

The lower hyperangular wavefunction here has a 99% overlap with the 1/3 Laughlin wfn

PHYSICAL REVIEW B VOLUME 27, NUMBER 6 15 MARCH 1983

Quantized motion of three two-dimensional electrons in a strong magnetic field

R. B. Laughlin
University of California, Lawrence Livermore National Laboratory, Livermore, California 94550

We have found a simple, exact solution of the Schrödinger equation for three two-dimensional electrons in a strong magnetic field, given the assumption that they lie in a single Landau level. We find that the interelectronic spacing has characteristic values, not dependent on the form of the interaction, which change discontinuously as pressure is applied, and that the system has characteristic excitation energies of approximately $0.03e^2/a_0$, where a_0 is the magnetic length.

TABLE I. Coulomb matrix elements across the states $|m,n\rangle$ defined by Eq. (18) in units of $(3/\sqrt{2})/(e^2/a_0)$. Quantum numbers m,n are indicated in parenthesis. $M=3m+2n$ is the total angular momentum. There are no states of $M=0$, 1, 2, or 4.

$M=3$	(1,0)	$5.679\,0797\times10^{-1}$	
$M=5$	(1,1)	$4.978\,3743\times10^{-1}$	
$M=6$	(2,0)	$4.201\,1726\times10^{-1}$	
$M=7$	(1,2)	$4.471\,2999\times10^{-1}$	
$M=8$	(2,1)	$4.032\,3072\times10^{-1}$	
$M=9$	(3,0)	$3.401\,7834\times10^{-1}$	$1.306\,1401\times10^{-2}$
	(1,3)	$1.306\,1401\times10^{-2}$	$4.087\,2620\times10^{-1}$

Next Laughlin diagonalizes this matrix, i.e. applies degenerate perturbation theory in all coordinates

TESTING ADIABATICITY

Comparison of energy level calculations in the adiabatic hyperspherical approximation with the Laughlin method (1983 Phys. Rev. B first row) which does degenerate perturbation theory in all degrees of freedom

	N, $-M$	3, 9	3, 15	4, 18	5, 30
1	ΔE, Perturbation Theory	0.716527	0.55248	1.30573	2.02725
2	ΔE, Degenerate fixed-K	0.704637	0.54792	1.28552	1.99742
3	ΔE, Born-Oppenheimer (lower bound*)	0.70198	0.54722	1.28086	1.99226
4	ΔE, Adiabatic (upper bound)	0.70204	0.54723	1.28092	1.99230

Row 1: degenerate perturbation theory in all coordinates (as in Laughlin, 1983, PRL; agrees with his numbers to all 8 digits; and Jain et al. 2006 arXiv for N=4,5)

Row 2: degenerate perturbation theory in the hyperangular degrees of freedom only, followed by exact solution in R

Row 3: full Born-Oppenheimer calculation, treating R adiabatically, giving lower bound (if converged) to the ground state energy

Row 4: full adiabatic approximation including repulsive "diagonal correction term" (d^2/dR^2), giving an upper bound to the ground state energy

Filling factor, nu →	1/3	1/5	1/3	1/3	1/3
N,$-M$	3,9	3,15	4,18	5,30	6,45
ΔE, Haldane sphere, fit, extrapolation	0.71656	0.5526	1.310	2.04	≈ 3
ΔE, Planar calculations [47, 48]	0.716527	0.55248	1.30573	2.02725	2.86015
ΔE, Perturbation Theory	0.716527	0.55248	1.30573	2.02725	2.86015
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ΔE, Adiabatic (upper bound)	0.70204	0.54723	1.28092	1.99230	–

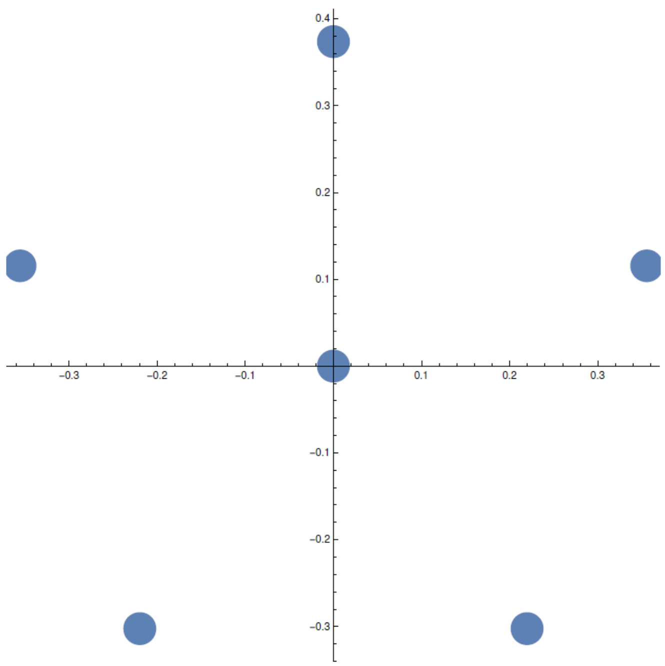
Energy level calculations in our hyperspherical coordinate picture, compared with previous calculations of quantum Hall effect pioneers Laughlin (1983 PRB) and Jain(arXiv:2006)

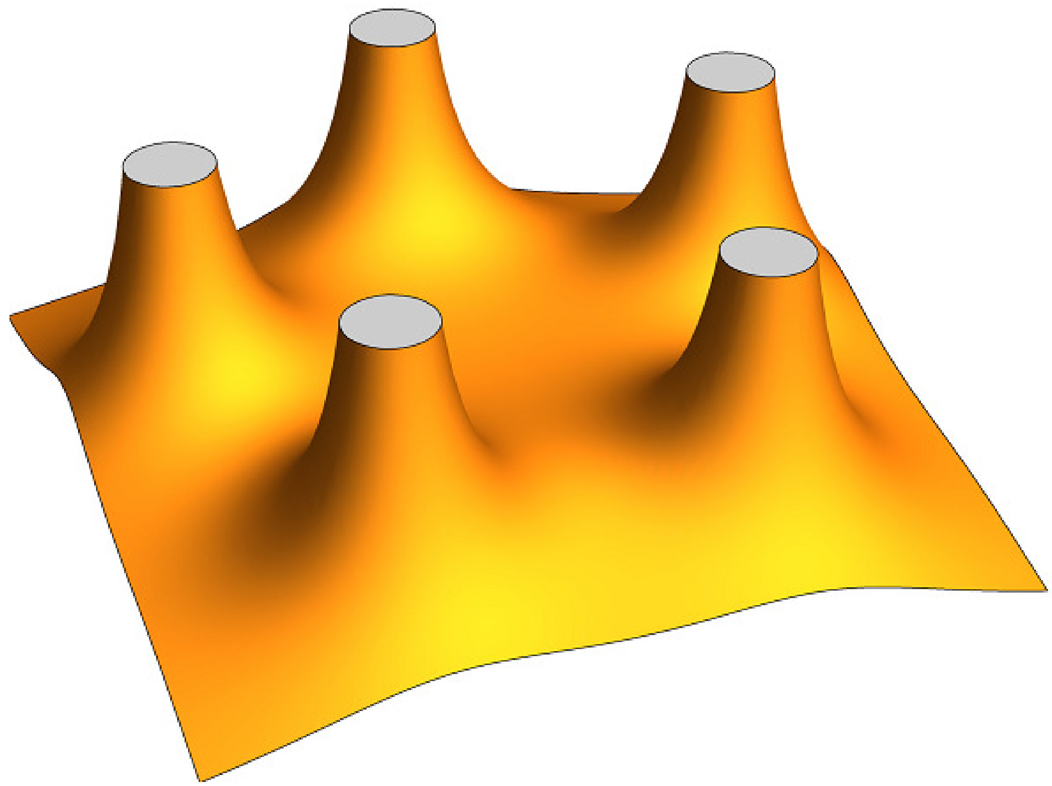
The lower bound calculations neglect the diagonal adiabatic correction term, which as shown by Starace and Webster (1979) must bound each exact energy level from below.

The upper bound calculations conform to the usual Rayleigh-Ritz variational principle and are guaranteed to give energies higher than or equal to the exact energy levels.

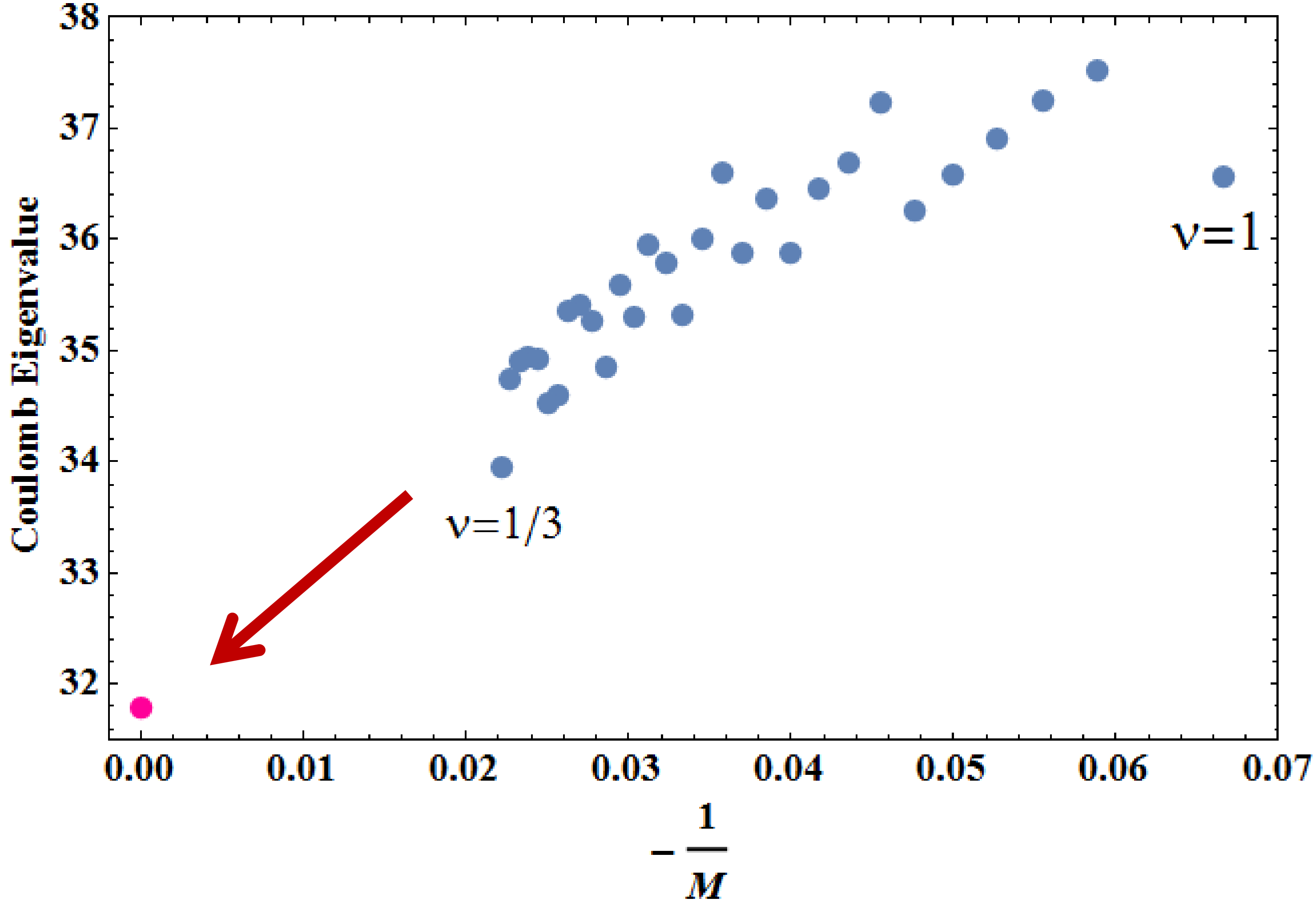
Potential energy landscape at fixed hyperradius for 6 particles, in a configuration that minimizes the classical potential energy (left)

After minimization, this (right) figure shows the potential energy as the 6th particle is allowed to move throughout the plane at fixed R

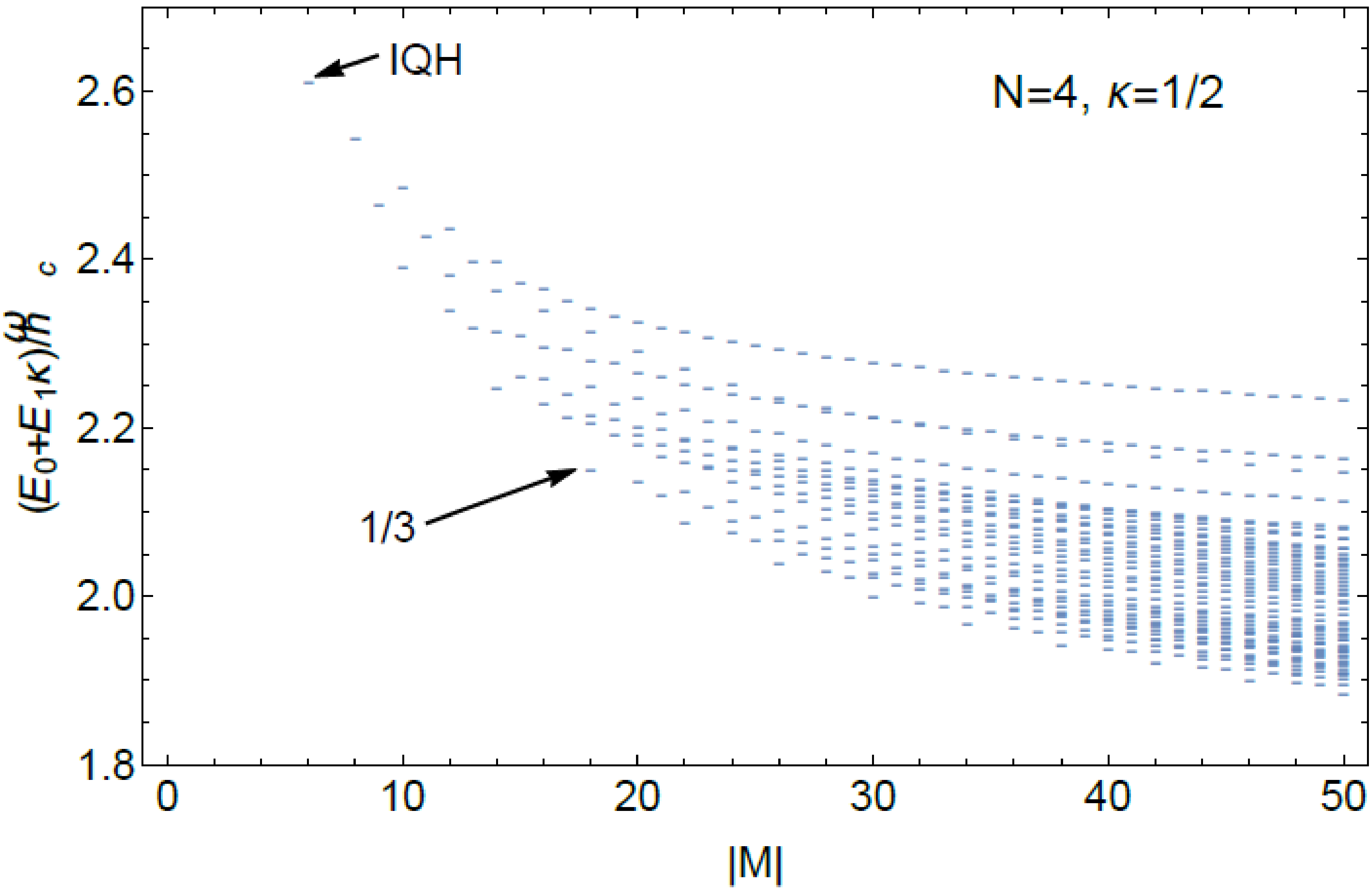


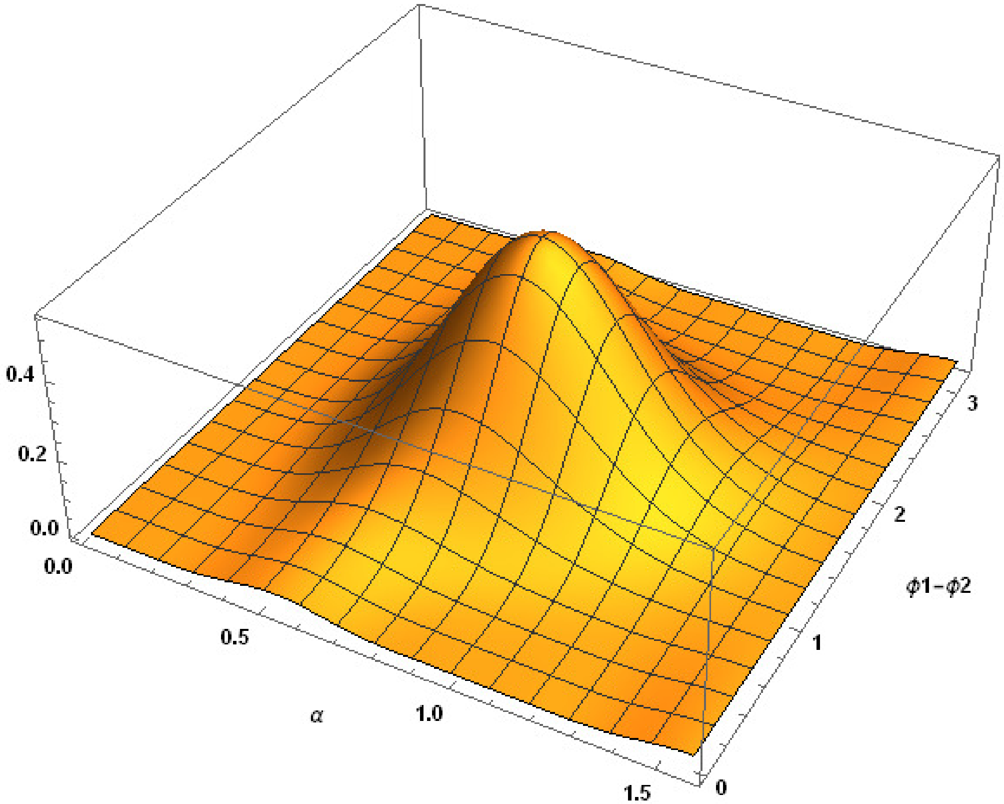


Minimum quantum Coulomb potential eigenvalues for lowest K=|M| (lowest Landau level) for 6 particles, showing their trend towards the classical minimum potential energy (magenta point) as K increases



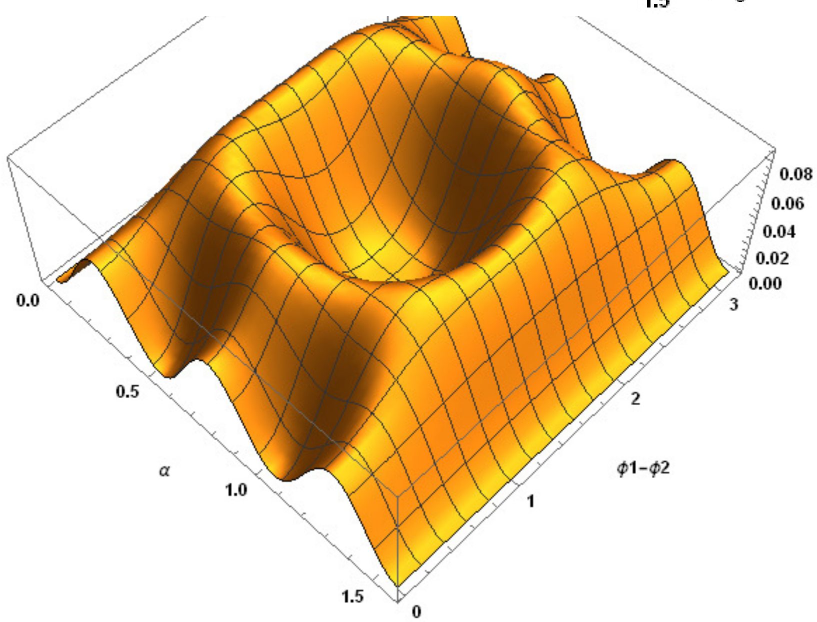
Eigenenergies for 4 particles after quantizing also in the hyperradius R





<u>K= -M=9 for N=3</u>

This 1/3 Laughlin eigenstate has a strong peak at an equilateral triangle configuration, where electrons can stay as far apart as possible, minimize repulsion

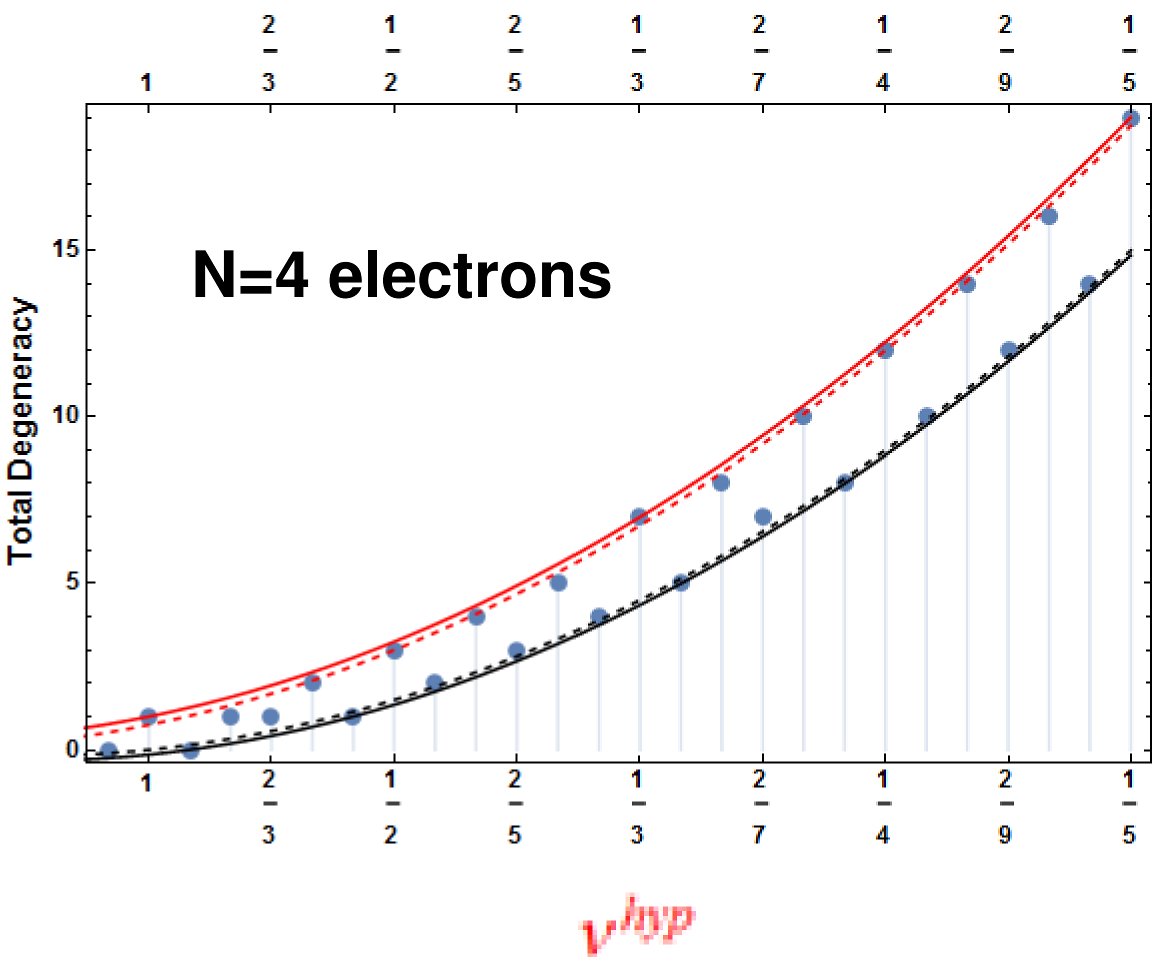


<u>K= -M=10 for N=3</u>

This non-FQHE eigenstate has a deep minimum at an equilateral triangle configuration

On the role of **exceptional degeneracy**: e.g., from group theory, the number of antisymmetric states for 4 particles in states with K=|M| turns out to be the following:

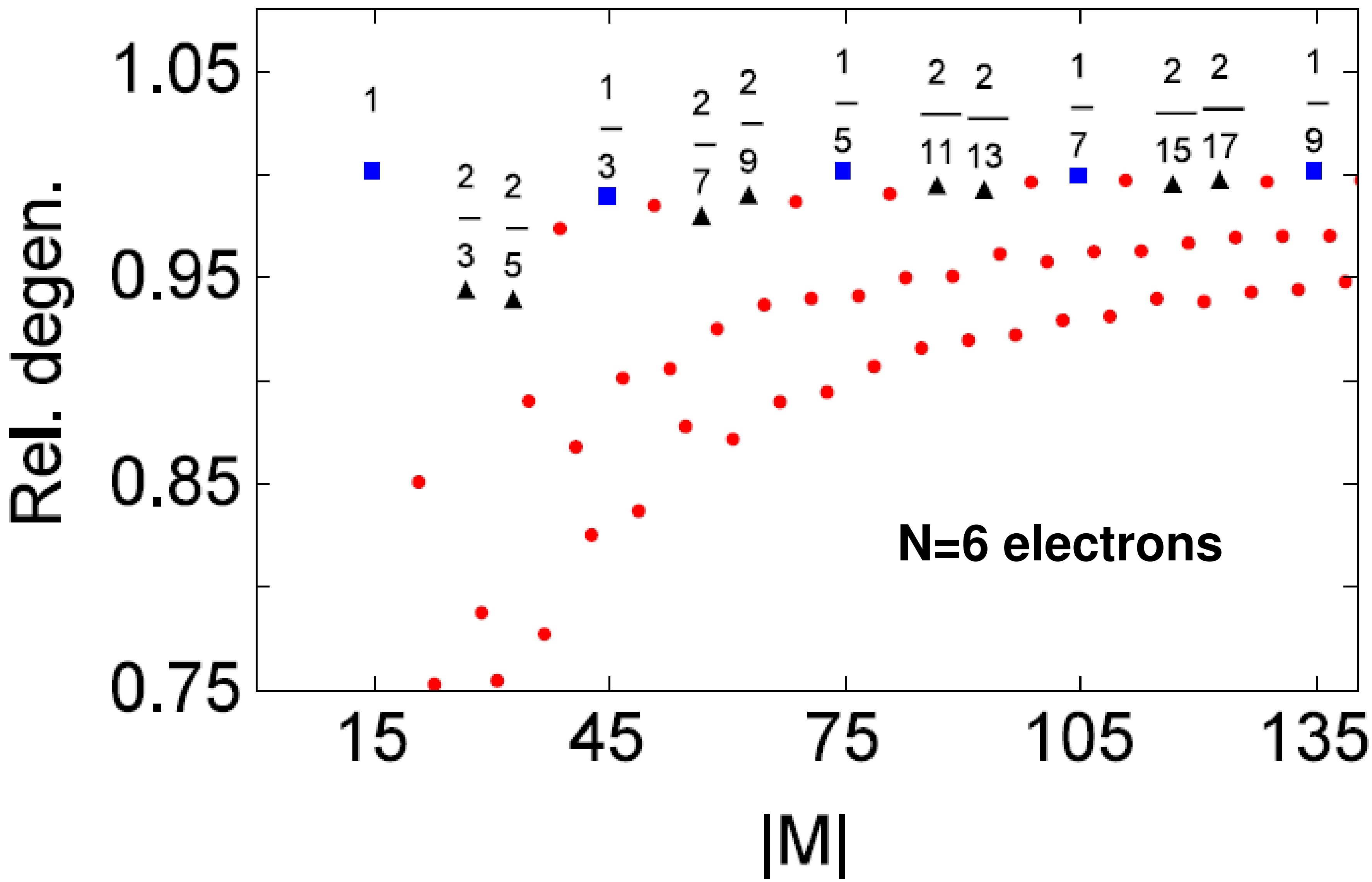
$$\frac{|M|^2}{48}+\frac{1}{16}\left((-1)^{|M|}-1\right)|M|+\frac{1}{288}\left(64\cos\left(\frac{2\pi|M|}{3}\right)-9(-1)^{|M|}\left(4\sin\left(\frac{\pi|M|}{2}\right)+4\cos\left(\frac{\pi|M|}{2}\right)+3\right)-1\right)$$



Note: the "hyperspherical filling factor", which agrees with the usual definition for integer QHE and the Laughlin FQHE states, is given by

$$\nu^{hyp}=\frac{N(N-1)}{2K}$$

Connection between the high relative degeneracy states having known filling factors seen experimentally and in theory (Laughlin, Jain, etc.)



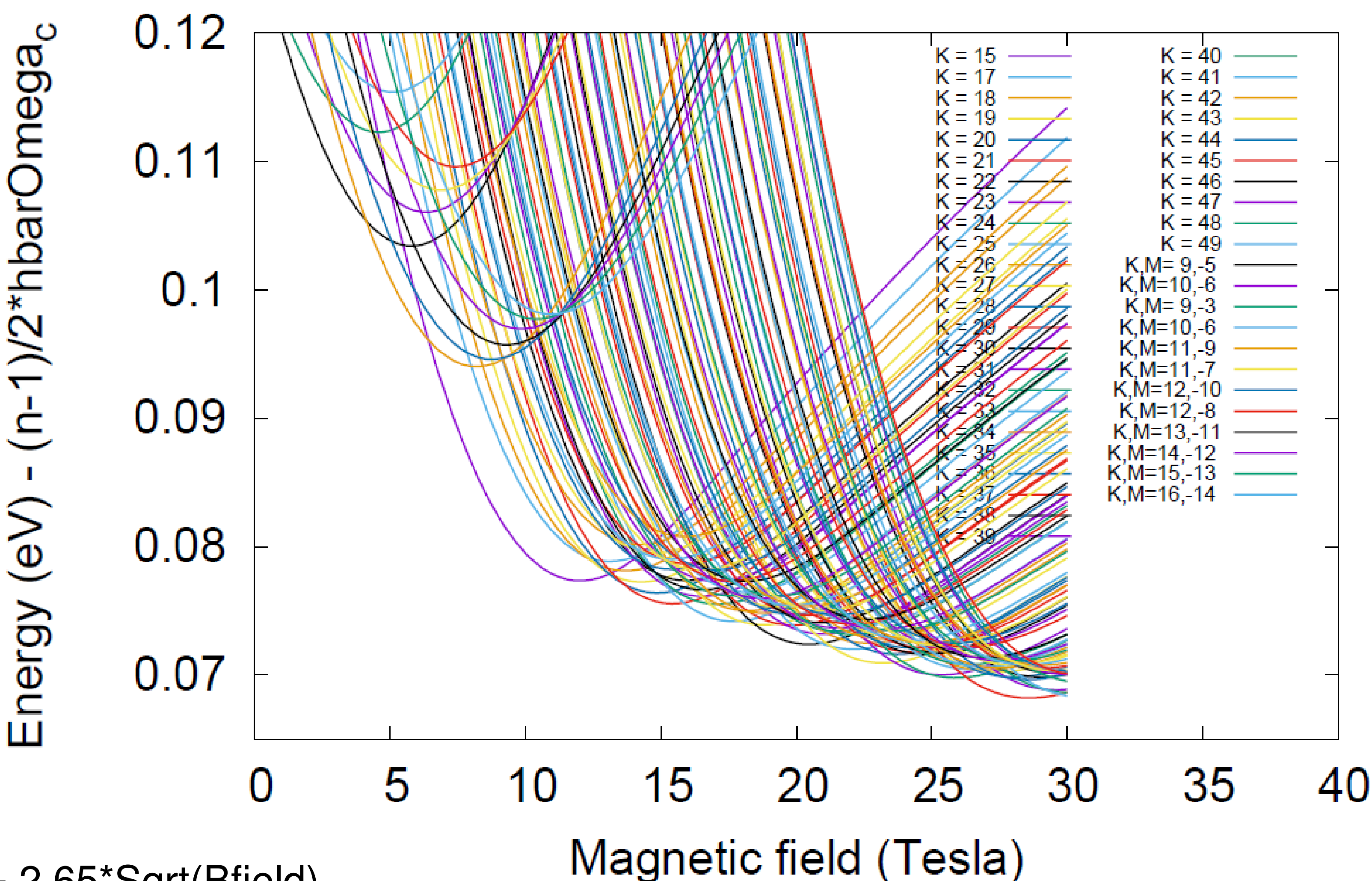
N	$-M$	ν_{CF}	ν_{HS}	$(\frac{1}{\nu_{CF}} - \frac{1}{\nu_{HS}})$
3	3	1	1	0
	9	$\frac{1}{3}$	$\frac{1}{3}$	0
	15	$\frac{1}{5}$	$\frac{1}{5}$	0
4	6	1	1	0
	12	$\frac{2}{5}$	$\frac{1}{2}$	$-\frac{1}{2}$
	18	$\frac{1}{3}$	$\frac{1}{3}$	0
	24	$\frac{2}{7}$	$\frac{1}{4}$	$-\frac{1}{2}$
	30	$\frac{1}{5}$	$\frac{1}{5}$	0
6	15	1	1	0
	27	$\frac{2}{3}$	$\frac{5}{9}$	$-\frac{3}{10}$
	33	$\frac{2}{5}$	$\frac{5}{11}$	$\frac{3}{10}$
	45	$\frac{1}{3}$	$\frac{1}{3}$	0
	57	$\frac{2}{7}$	$\frac{5}{19}$	$-\frac{3}{10}$
	75	$\frac{1}{5}$	$\frac{1}{5}$	0

TABLE I. Sample list of identified N-body quantum Hall states in the lowest Landau level. M is the total relative azimuthal quantum number of Laughlin and Jain states identified by exact numerical diagonalization in a spherical geometry [6]. ν_{CF} gives the filling factor of identified QH states according to the Jain composite fermion picture, including a correction that accounts for the finite size shift associated with the spherical geometry. ν_{HS} is the calculated hyperspherical filling factor, given by Eq.(34). The final column gives a finite size correction to the hyperspherical filling factor.

Connections between hyperspherical and conventional filling factors for known FQHE states for 3,4, and 6 electrons

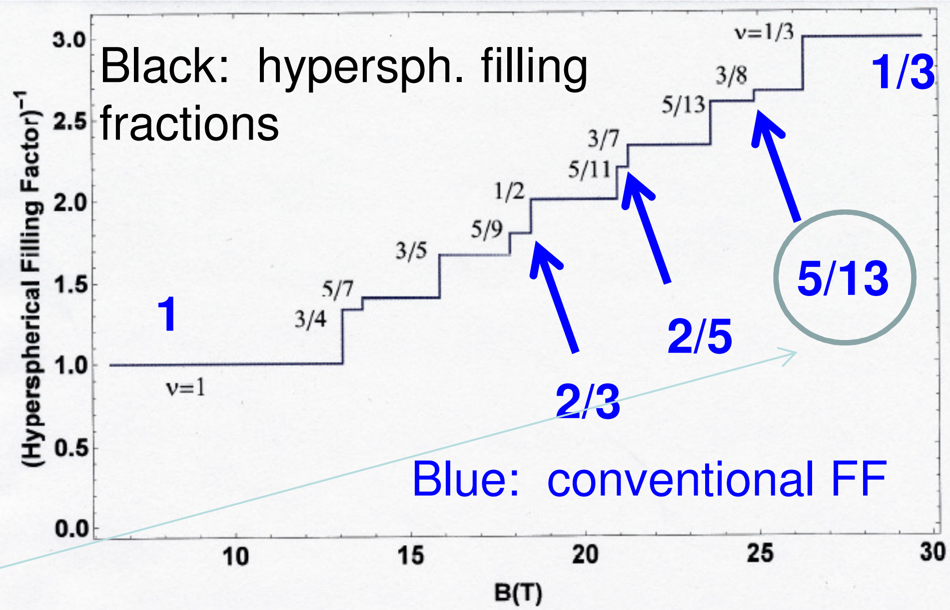
Energy spectrum after solving for the hyperradial vibrational degree of freedom, as a function of magnetic field. The B-field magnitude correlates with the maximum hyperradius used in the radial calculation according to the formula

Energy vs. Magnetic field

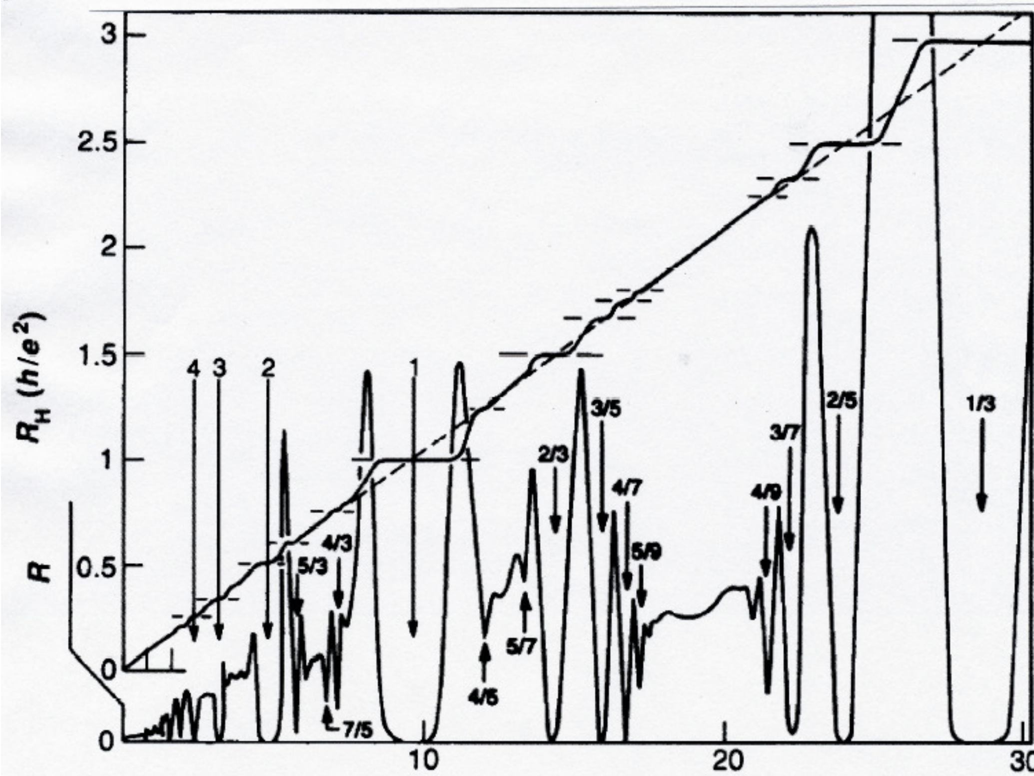


Rmax = 2.65*Sqrt(Bfield)

“Devil’s Staircase” showing lowest energy state for 6 electrons with density, effective mass, and dielectric constant parameters appropriate for a typical GaAs experiment in the fractional quantum Hall effect.



Interestingly, the 5/13 state that emerges from the 6 electron calculation (M=-39) is one state in particular that does not emerge naturally in the Jain composite fermion picture. On the Haldane sphere (for experts) it corresponds to 2Q=13, with 1 completely filled composite fermion Landau level 0 + a partially filled Landau level 1 that holds the extra quasi electrons, which interact to form pairs. See Quinn&Quinn, SSCommun 2006



Fractional Quantum Hall Effect of Composite Fermions

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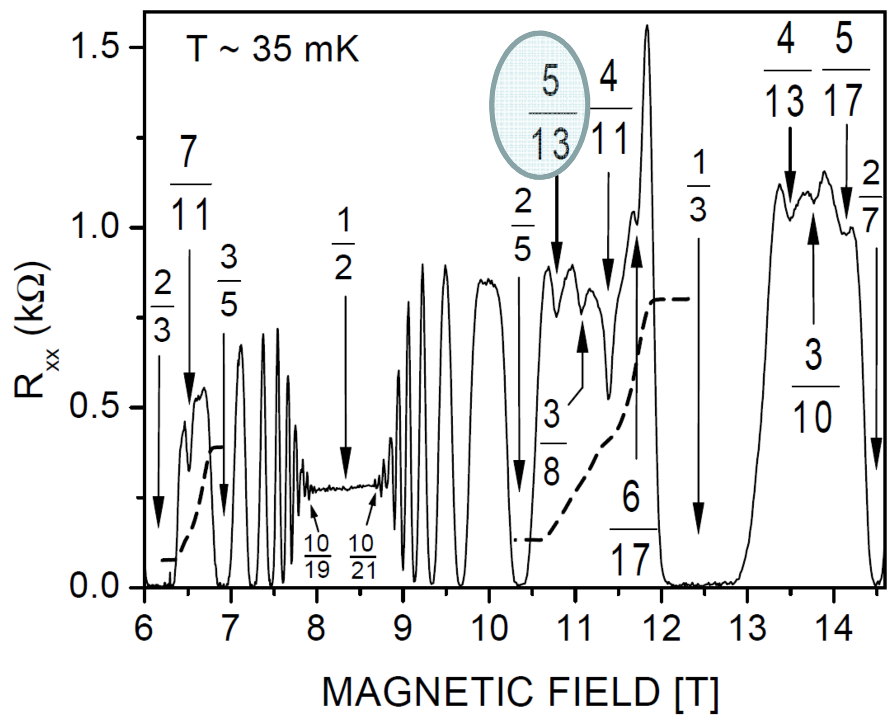
In a GaAs/AlGaAs quantum well of density 1×10^{11} cm^{-2} we observed a fractional quantum Hall effect at $\nu = 4/11$ and $5/13$, and weaker states at $\nu = 6/17, 4/13, 5/17$, and $7/11$. These sequences of fractions do not fit into the standard series of integral quantum Hall effects (IQHE) of composite fermions (CF) at $\nu = p/(2mp \pm 1)$. They rather can be regarded as the FQHE of CFs attesting to residual interactions between these composite particles. In tilted magnetic fields the $\nu = 4/11$ state



Experimental observation of some states that challenge the first-order composite fermion theory, in which the CF's are noninteracting; condmat/0303429

Conclusions

1. A hyperspherical mapping provides a systematic, microscopic way to tackle fractional quantum Hall states
2. The usual Laughlin and Jain states (and possibly others) correlate closely with the symmetries having an exceptional degeneracy for noninteracting fermions
3. Since these states are identifiable by a property of noninteracting electrons, it should be possible to probe these exceptional degeneracy states in other ways, e.g. without a magnetic field, or with neutral, ultracold polarized fermionic (or bosonic atoms)
4. One can use the approximate separability of the hyper-radial coordinate to predict a class of excitation frequencies, almost trivially.
5. Further study is needed to understand more detailed properties of the eigenstates, such as the fractional nature of charge carriers, the nature of quasi-particles, etc.

Thanks, Tony, for playing such a crucial and supportive role for me over the years, both personally and professionally! And HAPPY BIRTHDAY!!

Tony with Shinichi Watanabe, Keystone 2015

Shin also sends his happy celebratory greeting to this occasion!