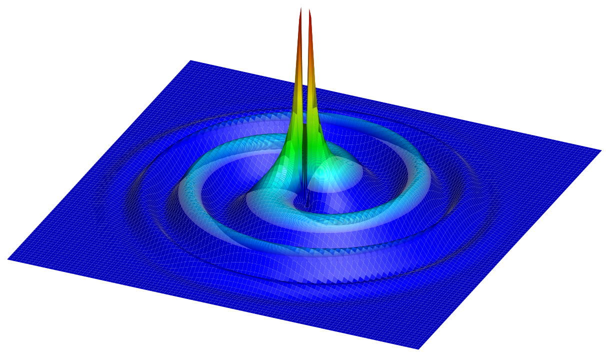
Matter-wave Vortices



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Outline

- ***Background and Motivation***
- ***Electron matter-wave vortex patterns in momentum distribution by circularly-polarized attosecond pulses***
  - He $+(\hbar\omega - \tau - \hbar\omega) \rightarrow$ He^{+}(1s) + e^{-}
  - Predicted using Perturbation Theory
  - Demonstrated numerically by solving the 6-D TDSE
    - Sensitivity to time-delay between the pulses, their relative CEP, handedness, duration, and peak intensity
    - Connection to (i) vortices in the probability distribution, and (ii) optical vortices (wave-particle duality)
- ***Conclusions***

1. Background and Motivation

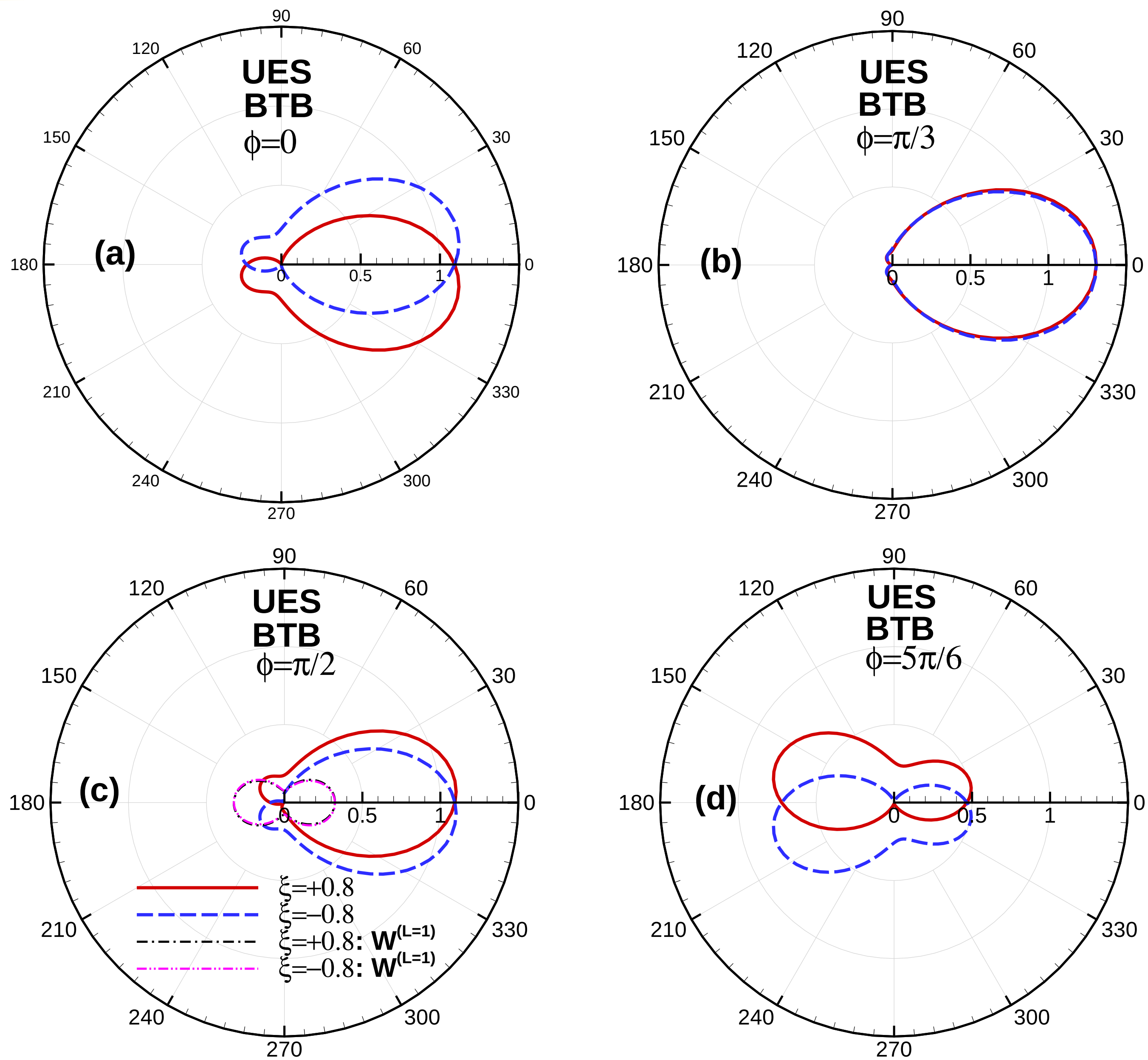
Background and Motivation

J.M. Ngoko Djiokap et al., *Phys. Rev. Lett.* **113**, 223002 (2014).

- **TDSE**: $i\partial_t \Phi(\mathbf{r}_1, \mathbf{r}_2, t) = H(t)\Phi(\mathbf{r}_1, \mathbf{r}_2, t)$
- Linear polarization: 5-D problem as M is conserved
  - FE-DVR + Split-operator
- **Elliptical polarization**: 6-D problem (M-mixing problem)
  - H.G. Muller, Laser Physics **9**, 138 (1999)
  - T. K. Kjeldsen *et al.*, Phys. Rev. A **75**, 063427 (2007)
  - The electric field seen by an observer in the rotating frame is always linearly-polarized
  - At each τ: Atomic int in Lab frame - Rotate - Laser int in Rot frame - Rotate Back - Atomic int in Lab frame

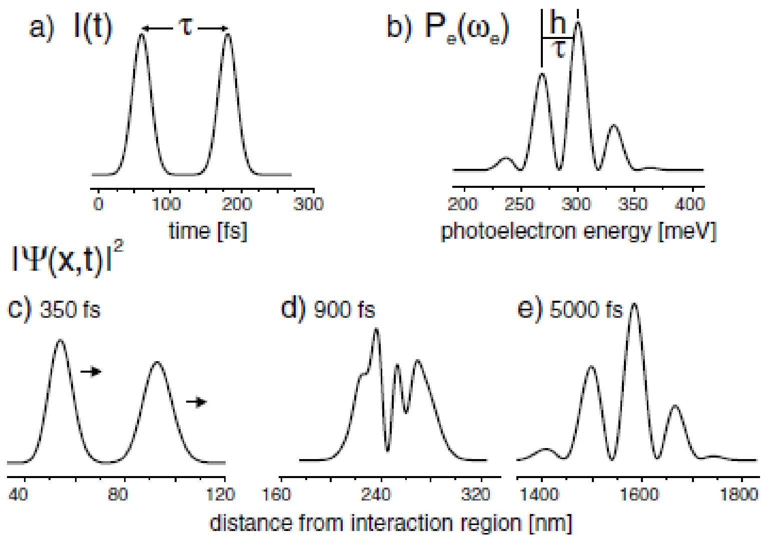
Angular distributions for $\xi = \pm 0.8$ vs. CEP at 2 PW/cm^2

Ngoko et al., Phys. Rev. Lett. **113**, *223002 (2014)*



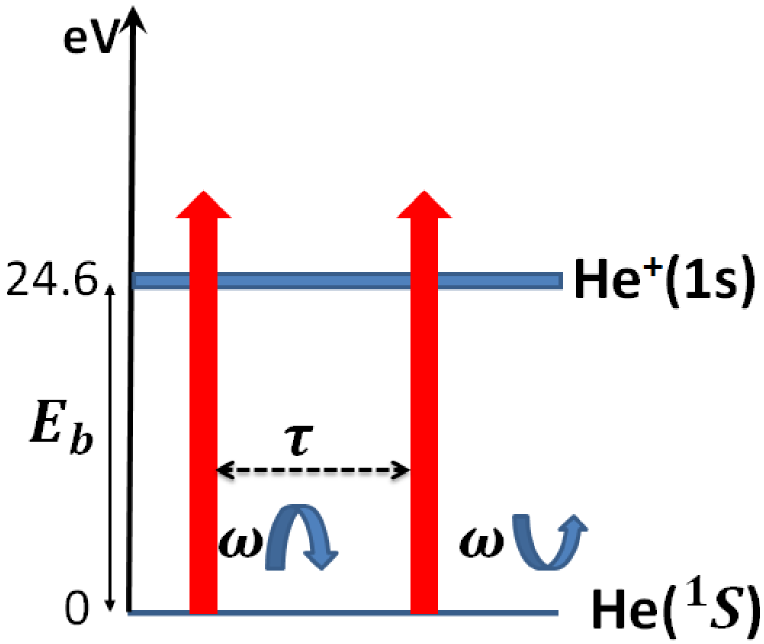
Background and Motivation

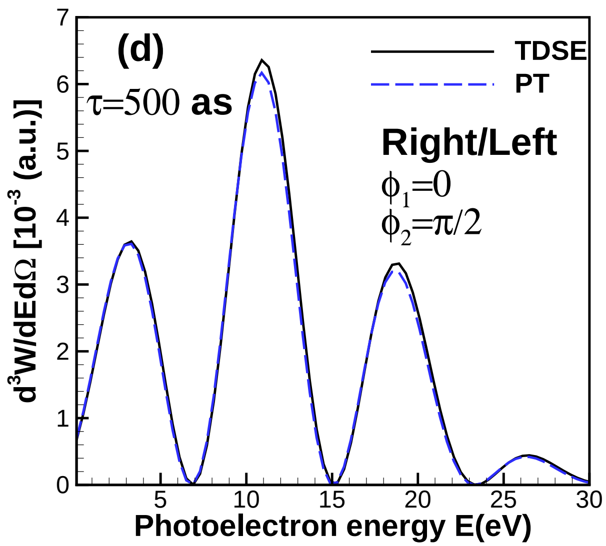
N. F. Ramsey, *Phys. Rev.* **78**, 695 (1950). Ramsey interference of laser-produced electron wave packets has been investigated



- in the continuum
  1. M. Wollenhaupt *et al.*, PRL **89**, 173001 (2002)
  2. Suxing and Starace, PRA **68**, 043407 (2003)

- in the Rydberg states
  1. L. D. Noordam, D. I. Duncan, and T. F. Gallagher, Phys. Rev. A **45**, 4734 (1992)
  2. M. Strehle, U. Weichmann, and G. Gerber, Phys. Rev. A **58**, 450 (1998)





2. Electron matter-wave vortex patterns in momentum distribution by circularly-polarized attosecond pulses

Parameterization of the Electric Field

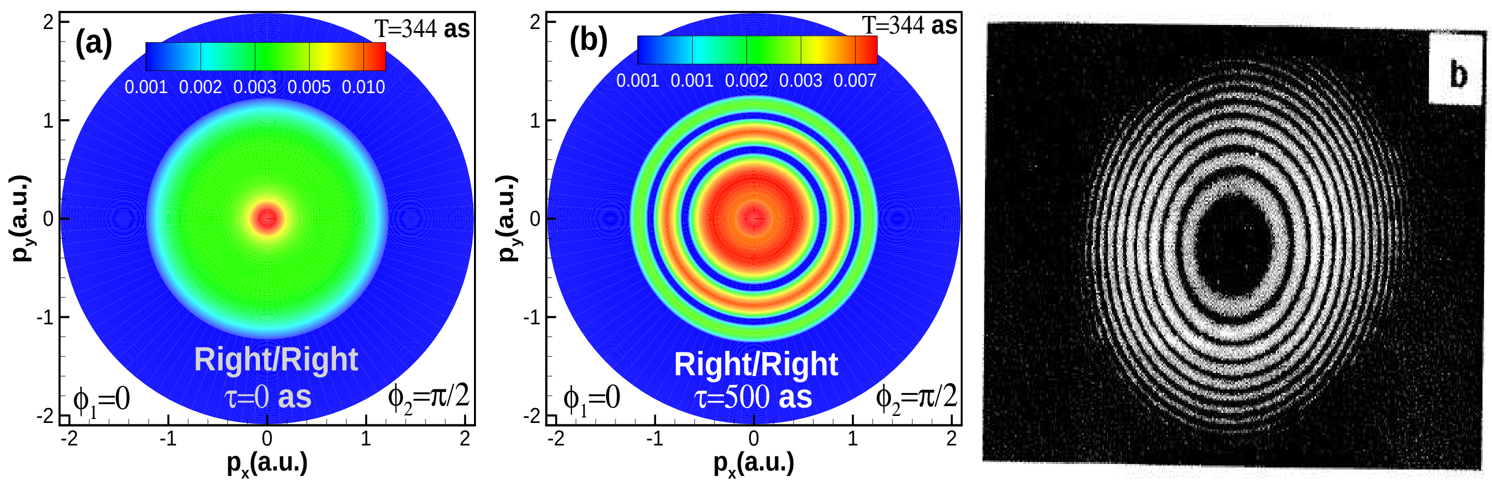
- **Electric field**: $\boldsymbol{F}(t) =$
  $F_0(t)\,\mathrm{Re}\left[\mathbf{e}_1 e^{-i(\omega t+\phi_1)}\right] + F_0(t-\tau)\,\mathrm{Re}\left[\mathbf{e}_2 e^{-i(\omega(t-\tau)+\phi_2)}\right]$
- **Polarization vector** of the jth pulse:
  $\mathbf{e}_j \equiv (\hat{\boldsymbol{\epsilon}} + i\eta_j \hat{\boldsymbol{\zeta}})/\sqrt{1+\eta_j^2}$
  - **Polarization plane** is defined by: major axis $\hat{\boldsymbol{\epsilon}}$ and minor axis $\hat{\boldsymbol{\zeta}} \equiv \hat{\mathbf{k}} \times \hat{\boldsymbol{\epsilon}}$
  - Ellipticity: $-1 \leq \eta_j \leq +1$
  - carrier frequency: $\omega = 36$ eV $> E_b = 24.6$ eV
  - Intensity: $I = 10^{14}$ W/cm^2 or lower

Parameterization of the Observable

- Triply differential probability (TDP) for single ionization:
  $d^3W/d^3\mathbf{p} = |\langle\Theta_{1s}^{(-)}(\mathbf{p})|\Psi(T+\tau)\rangle|^2$, $\mathcal{W}_{\xi_2}^{\xi_1}(\mathbf{p}) = \mathcal{C}|A(\mathbf{p})|^2$
- 1st-order amplitude for single ionization:
  $A(\mathbf{p}) = -i\int_{-\infty}^{\infty}\langle\Psi_{1s\mathbf{p}}^{(-)}|\boldsymbol{F}(t)\cdot\boldsymbol{d}|i\rangle\, e^{i(E+E_b)t}dt$
- 1st-order amplitude in terms of vectors of the problem:
  $A(\mathbf{p}) = -e^{-i\phi_1}\alpha(p)A_\gamma(\hat{\mathbf{p}})$
  - Kinematic factor: $A_\gamma(\hat{\mathbf{p}}) = \hat{\mathbf{p}}\cdot\left(\mathbf{e}_1 + \mathbf{e}_2 e^{i\Phi}\right)$
  - Dynamical parameter:
    $\alpha(p) = \langle\Psi_{\nu\mathbf{p}}^{(-)}|\boldsymbol{F}(t)\cdot\boldsymbol{d}|i\rangle\hat{F}_0(E+E_b-\omega)$
  - Relative phase: $\Phi = (E+E_b)\tau + (\phi_1-\phi_2)$
- **Dynamical vortex**: $\alpha(p) = 0$. **Kinematical vortex**: $A_\gamma(\hat{\mathbf{p}}) = 0$ is absent in $(e,2e)$ amplitude [PRA **90**, 062709 (2014)]

Two Identical Pulses

- Two identical pulses: $\mathbf{e}_1 = \mathbf{e}_2 \equiv \mathbf{e}$ or $\xi_1 = \xi_2 \equiv \xi = +1$
  - TDP is: $\mathcal{W}_{\xi}^{\xi}(\mathbf{p}) = \frac{3W_p}{2\pi} \sin^2\theta \cos^2(\Phi/2)$
  - For CP pulses in the polarization plane ($\theta = \pi/2$), the TDP is independent of φ
  - Relative phase: $\Phi = (E + E_b)\tau + (\phi_1 - \phi_2)$
  - Harris *et al.*, Opt. Commun. **106**, 161 (1994).



Oppositely Circularly-Polarized Pulses

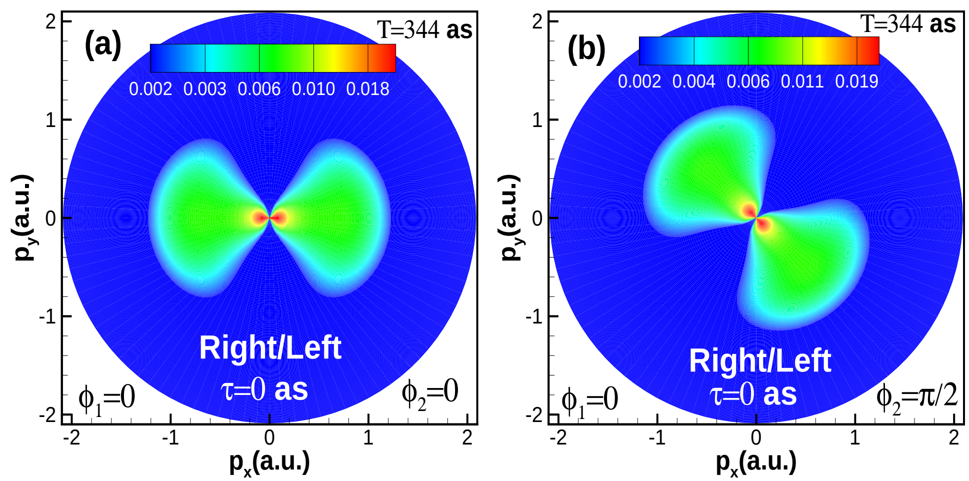
- Oppositely circularly-polarized pulses: $\mathbf{e}_1^* = \mathbf{e}_2$, or $\xi_1 = -\xi_2 = \pm 1$
  - TDP is: $\mathcal{W}_{\xi_2}^{\xi_1}(p,\theta,\varphi) = \frac{3W_p}{2\pi} \sin^2\theta \, \cos^2(\Phi/2 - \xi_1\varphi)$
  - Optical fringe intensity: $I = I_0(01^*) \cos^2(k^2 r^2 + \varphi)$, Harris *et al.*, Opt. Commun. **106**, 161 (1994).
  - Relative phase: $\Phi = (E + E_b)\tau + (\phi_1 - \phi_2)$
  - **Two-start (**$n = 0, 1$**) Fermat (or Archimedean) spirals (or helixes)** are defined by the maximum and zero values of the TDP:

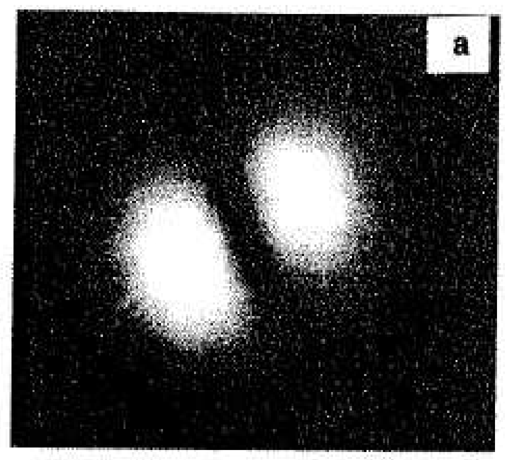
$$\varphi_n^{max}(p) = \xi_2 \left[\pi n - (\tau E_b + \phi_{12})/2 - \tau\, p^2/4\right],$$

$$\varphi_n^{zero}(p) = \xi_2 \left[\pi/2 + \pi n - (\tau E_b + \phi_{12})/2 - \tau\, p^2/4\right]$$

Oppositely Circularly-Polarized Pulses: Sensitivity to the relative CE phase

- For $\tau = 0$, superposing two oppositely circularly-polarized pulses gives a linearly-polarized pulse.
  - TDP in the polarization plane: $\mathcal{W}_{\xi_2}^{\xi_1}(p, \theta, \varphi) \propto \cos^2(\phi_{12}/2 - \xi_1\varphi)$; Optical fringe intensity: $I = I_0(01^*)\cos^2(k^2r^2 + \varphi)$

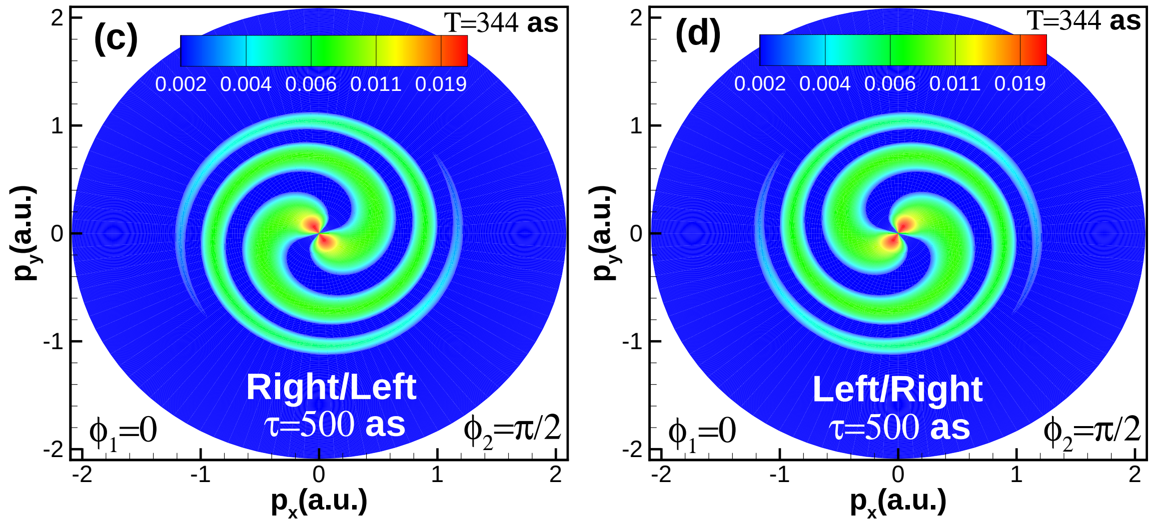




  - For $\phi_{12} \neq 0$, a change in sign of ξ_1 will change the angular distribution, unlike when $\phi_{12} = 0$.

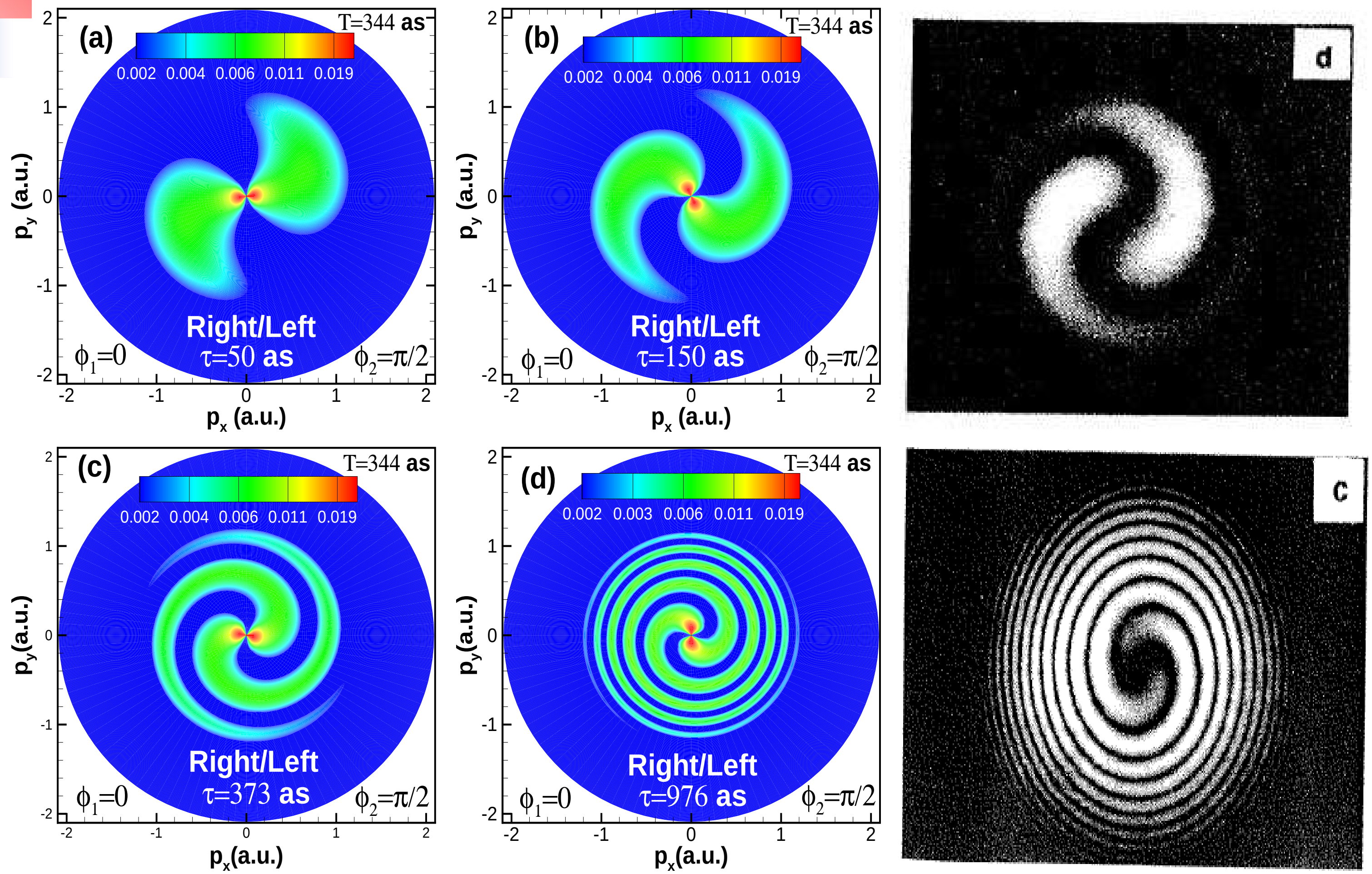
Oppositely Circularly-Polarized Pulses: Sensitivity to the handedness of the pulses

- For $\tau = 500\ as$, $\phi_{12} = -\pi/2$, $T = 344\ as$



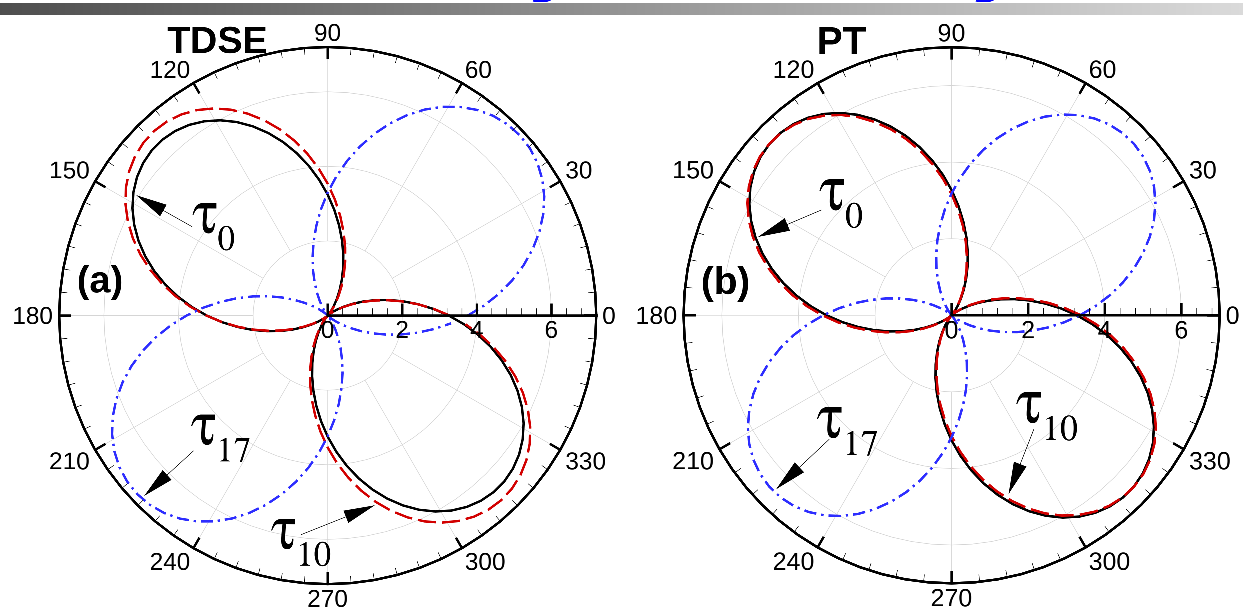
- $\mathcal{W}_{\xi_2}^{\xi_1}(p,\theta,\varphi) \propto \cos^2[(E+E_b)\tau/2 + \phi_{12}/2 - \xi_1\varphi]$
- The handedness of the vortex patterns depends upon the ordering of the pulses. There is a circular dichroic effect.
- The **two spiral arms** of the vortex pattern are clearly visible.

Oppositely Circularly-Polarized Pulses: Sensitivity to the time delay



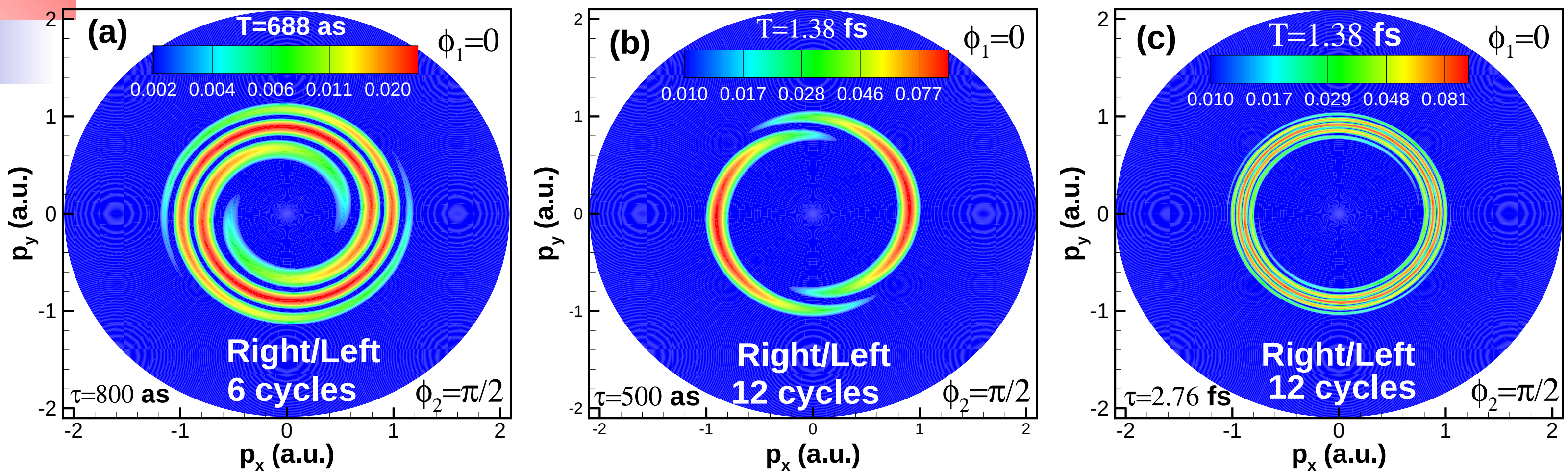
- **Time delays of several hundred attoseconds** are necessary to observe well-defined vortex patterns.
- Dramatic example of wave-particle duality.

Oppositely Circularly-Polarized Pulses: Sensitivity to Time delay



- For electron energy $E = \omega - E_b$, the angular distribution $\mathcal{W}_{\xi_2}^{\xi_1}(p,\theta,\varphi) \propto \cos^2[(E+E_b)\tau/2 + \phi_{12}/2 - \xi_1\varphi]$ is periodic with period $\tau_n = n\pi/\omega$, where n is even.
- Photoelectron angular distributions for $\tau = \tau_0$ and τ_{10} are (or nearly) identical.
- Ability to control the direction of ionization of electrons, by adjusting the time delay τ.

Oppositely Circularly-Polarized Pulses: Sensitivity to pulse bandwidth



- The spiral pattern widths decrease as the pulse bandwidths decrease
- The spiral arms of the vortex pattern for the 6-cycle pulses are compressed compared to 3-cycle pulses.
- For longer pulses, the two spiral arms are clearly discernible for the shorter τ, whereas for longer τ it cannot be discerned as the ring-like spiral pattern is tightly-wound.

3. Conclusions

Conclusions

- **Electron matter-wave vortex patterns** can be produced by photoionization by **oppositely circularly-polarized** pulses, with full control of the time-delay and relative CEPs.
- In the polarization plane, our **two-start spiral or helical vortex pattern** has a **counterpart in optics**: wave-particle duality.
- Experimental observation of these patterns requires the **large bandwidth** characteristic of few-cycle attosecond pulses.
- He atom and other light s-atoms such as H, Li, and Be are ideal targets.
- Being a **linear process**, it requires **low peak pulse intensities**.
- Circularly-polarized attosecond pulse operating at low intensity is a reality. **Velocity-map-imaging** technique can be used to measure the photoelectron momentum distributions.

Acknowledgments

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