UNL - Department of Physics and Astronomy

Preliminary Examination - Day 2
Friday, August 10, 2018

This test covers the topics of *Classical Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Classical Mechanics Group A - Answer only two Group A questions

A1 The position of a bird flying in the xy-plane is $\mathbf{r}(t) = -\alpha t\ \hat{\mathbf{i}} + (3.0\ \mathrm{m} - \beta t^2)\ \hat{\mathbf{j}}$, where $\alpha = 2.4\ \mathrm{m/s}$ and $\beta = 1.2\ \mathrm{m/s^2}$. At time $t = 2\ \mathrm{s}$, is the bird's *speed* increasing, decreasing, or constant?

A2 You throw a ball from your window, which is 8.0 m above the ground. When the ball leaves your hand, it is moving at 10 m/s at an angle of $20°$ below the horizontal. How far horizontally from your window will the ball hit the ground? Ignore air resistance. Use $g = 9.8\ \mathrm{m/s^2}$.

A3 Two demo carts, A and B, move without friction on a straight, level track. Their masses are $m_A = 2.0\ \mathrm{kg}$ and $m_B = 4.0\ \mathrm{kg}$. Initially, cart B is stationary. Cart A approaches cart B with a speed of 3 m/s, and the two carts collide elastically. What is the speed of cart B after the collision?

A4 Consider a big tank filled with liquid of mass density ρ_1. The depth of the tank is h. A small ball is released from rest from the bottom of the tank. The ball has volume V and uniform mass density ρ_2, with $\rho_2 < \rho_1$. How much time will it take the ball to reach the surface? Ignore any fluid resistance.

Classical Mechanics Group B - Answer only two Group B questions

B1 A particle with mass m, energy E and angular momentum L is moving in a central field described by the force function

$$F(r) = -kr.$$

(a) Find the radial distances r_1 and r_2 at which the particle is closest to and furthest from the center;

(b) Suppose the particle is staying on a circular orbit. Find its energy E and the orbit's radius, if m, k and L are known;

(c) Find the velocity and the angular velocity of the particle on the circular orbit.

B2 A very long, straight, thin line emits sound uniformly in all directions. The line is surrounded by nothing but uniform atmospheric air. The line's acoustic power per unit of length is λ.

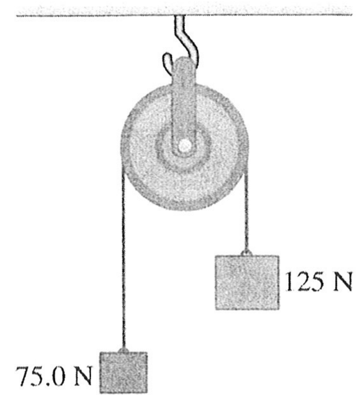
a. Suppose the sound level at a distance r_0 from the line is L_0. Find $L(r)$, the sound level as a function of distance. *Note: See cheat sheet for the definition of sound level.*

For a particular line, the sound level is measured to be 30 dB at a distance of 5 m.

b. What is the acoustic power per unit length of this line?

c. How far away from this line can its sound no longer be heard?

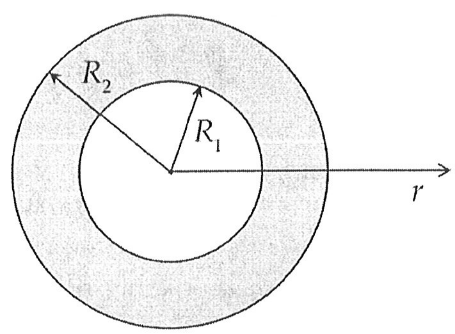
B3 Two weights are connected by a very light, flexible, and unstretchable cord that passes without slipping over an 80.0-N pulley of radius 0.300 m. The pulley is a solid uniform disk and is supported by a hook connected to the ceiling. It rotates without friction about its symmetry axis.



a. What is the acceleration of the blocks?

b. What force does the ceiling exert on the hook?

B4 Calculate the gravitational field and gravitational potential everywhere in space due to the thick spherical shell shown in the diagram (inner radius R_1, outer radius R_2). The mass density of the shell is $\rho(r) = A / r \;\; (R_1 < r < R_2)$.

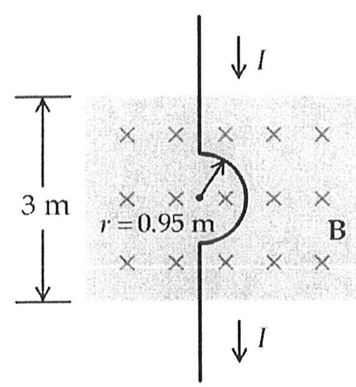


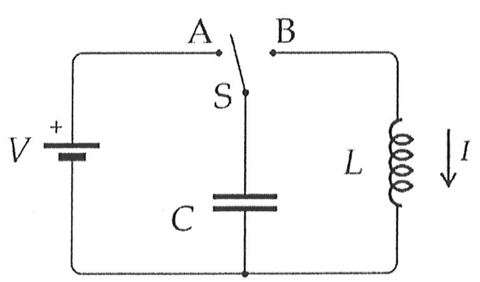
Electrodynamics Group A - Answer only two Group A questions

$\boxed{\text{A1}}$ The resistance of a galvanometer coil is $25\ \Omega$, and the current required for full-scale deflection is $500\ \mu\text{A}$.

a. Show in a diagram how to convert the galvanometer to an ammeter reading 20 mA full scale, and compute the shunt resistance.
b. Show in a diagram how to convert the galvanometer to a voltmeter reading 500 mV full scale, and compute the series resistance.

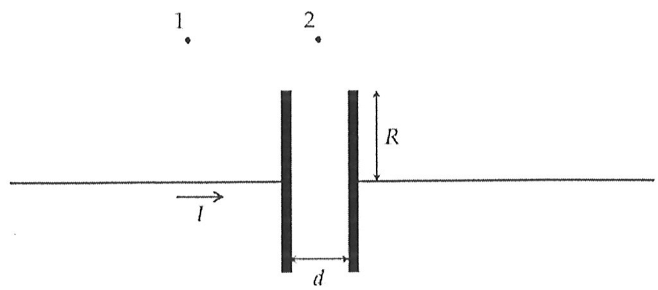
$\boxed{\text{A2}}$ A long, straight wire containing a semicircular section of radius 0.95 m is placed in a uniform magnetic field of magnitude 2.2 T as show in the figure. What is the net magnetic force acting on the wire when it carries a current of 3.4 A?





$\boxed{\text{A3}}$ Switch S has been in position A for a long time, causing the capacitor to be completely charged. At $t = 0$, the switch is thrown to position B, and we measure the maximum current I_{max}. Give an expression for L in terms of the quantities V, C, and I_{max}.

A4 A constant current I is supplied for a brief time to charge a parallel-plate capacitor using infinitely long straight lines, as shown. The plates are circular with radius R. Point 1 is at a distance ($R+d$) from the wire, and point 2 is at the same distance ($R+d$) from the center of the capacitor.



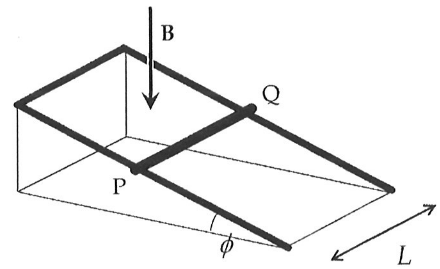
a. During the time interval that the constant current I is flowing, a magnetic field is generated. What is the relationship between the magnetic fields at points 1 and 2 ($B_1 > B_2$, $B_1 = B_2$, or $B_1 < B_2$)?

b. What is the magnetic field between the plates after the capacitor has been charged to a final charge Q, and $I = 0$?

c. Point 2 can now be at any distance (r) away from the center of the capacitor (but in a plane parallel to the plates). Calculate B_2 as a function of r. Constant current is assumed, and the plates are circular.

Electrodynamics Group B - Answer only two Group B questions

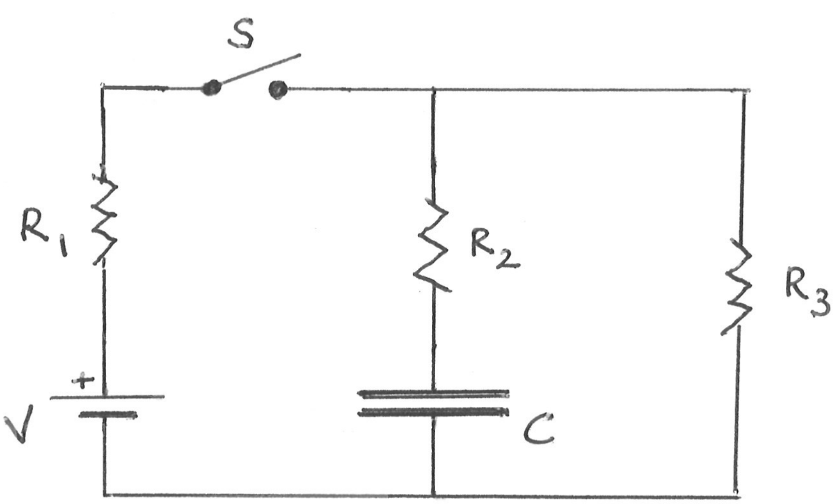
B1 A metal bar of length L, mass m, and resistance R is placed on frictionless metal rails that make an angle ϕ with the horizontal. The rails are connected at the top by a third rail, forming one conducting Π -shaped system. This whole rail system has negligible resistance. A uniform magnetic field of magnitude B is directed downward, as shown. The bar is released from rest and slides down the rails.



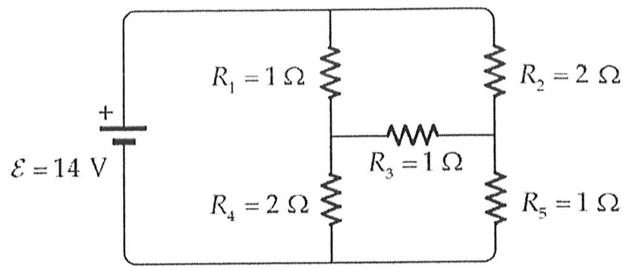
a. What is the direction of the current in the bar (P→Q or Q→P)?
b. What is the bar's terminal speed?
c. What is the induced current in the bar as a function time?

B2 In the circuit, assume V, C, and R_1, R_2, R_3 are known. The capacitor is initially uncharged, and then switch S is closed.

a. What is τ_C, the charging time constant?
b. What is τ_C when the battery is shorted and the capacitor is discharged?



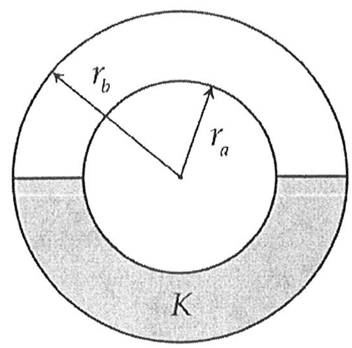
B3 The adjacent diagram shows a dc circuit with one battery and five resistors.



a. Find the current through the battery and through each resistor.
b. Calculate the equivalent resistance of the resistor network.

B4 An isolated spherical capacitor has charge $+Q$ on its inner conductor (radius r_a) and charge $-Q$ on its outer conductor (radius r_b). Half of the volume between the two conductors filled with a liquid dielectric of dielectric constant K, as shown in the figure.

a. Find the capacitance of the half-filled capacitor.
b. Find the magnitude of **E** in the volume between the two conductors as a function of the distance r from the center of capacitor. Give answers for both the upper and lower halves of this volume.
c. Find the surface density of free charge on the upper and lower halves of the inner and outer conductors.
d. Find the surface density of bound charge on the inner ($r = r_a$) and outer ($r = r_b$) surfaces of the dielectric.
e. What is the surface density of bound charge on the flat surface of the dielectric? Explain.



Physical Constants

speed of light .... $c = 2.998\times10^{8}$ m/s

Planck's constant .... $h = 6.626\times10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055\times10^{-34}$ J·s

Boltzmann constant ...... $k_{B} = 1.381\times10^{-23}$ J/K

elementary charge .... $e = 1.602\times10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854\times10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257\times10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

Avogadro constant .... $N_A = 6.022\times10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988\times10^{9}$ m/F

electron mass .... $m_{el} = 9.109\times10^{-31}$ kg

electron rest energy...... 511.0 keV

Compton wavelength .. $\lambda_C = h/m_{el}c = 2.426$ pm

proton mass .... $m_p = 1.673\times10^{-27}\,\text{kg} = 1836 m_{el}$

1 bohr............................ $a_0 = \hbar^2 / ke^2 m_{el} = 0.5292$ Å

1 hartree (= 2 rydberg) ... $E_h = \hbar^2 / m_{el} a_0^{\,2} = 27.21$ eV

gravitational constant ... $G = 6.674\times10^{-11}$ m^3 / kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

MECHANICS

Gauss's Law for gravitation: $\oint \mathbf{g}\cdot\hat{\mathbf{n}}\, da = -4\pi G m_{\text{encl.}}$

The sound level L (in decibel, dB) of sound of acoustic intensity I is given by $L = 10\cdot\log_{10}(I/I_{\text{ref}})$. The quantity I_{ref} is the standard reference acoustic intensity of 10^{-12} W/m^2; this is also the limit of sensitivity of the human ear.

ELECTROSTATICS

$$\oint\!\!\oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{\text{encl}}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2/2C$

Relative permittivity: $\varepsilon_r = 1 + \chi_e$

Bound charges

$\rho_b = -\nabla\cdot\mathbf{P}$

$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$

Capacitance in vacuum

Parallel-plate: $C = \varepsilon_0 \frac{A}{d}$

Spherical: $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$

Cylindrical: $C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$ (for a length L)

MAGNETOSTATICS

Relative permeability: $\mu_r = 1 + \chi_m$

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$ Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\boldsymbol{\ell}$

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\boldsymbol{\ell}\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

Infinitely long solenoid: B-field inside is $B = \mu_0 n I$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{encl}}$

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I\int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}\right]$

Bound currents

$J_\mathrm{b} = \nabla \times \mathbf{M}$
$K_\mathrm{b} = \mathbf{M} \times \hat{\mathbf{n}}$

Maxwell's Equations in vacuum

1.	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$	Gauss' Law
2.	$\nabla\cdot\mathbf{B} = 0$	no magnetic charge
3.	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's Law
4.	$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \varepsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t}$	Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1.	$\nabla\cdot\mathbf{D} = \rho_\mathrm{f}$	Gauss' Law
2.	$\nabla\cdot\mathbf{B} = 0$	no magnetic charge
3.	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's Law
4.	$\nabla\times\mathbf{H} = \mathbf{J}_\mathrm{f} + \frac{\partial\mathbf{D}}{\partial t}$	Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E}\cdot d\ell = -\frac{d\Phi_B}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21}I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1}\int_V B^2 d\tau = \frac{1}{2}LI^2 = \frac{1}{2}\oint \mathbf{A}\cdot\mathbf{I}d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2}	$\frac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2}	$\frac{1}{2a}$
$x^2 e^{-ax^2}$	$\frac{\sqrt{\pi}}{4a^{3/2}}$
$x^3 e^{-ax^2}$	$\frac{1}{2a^2}$
$x^4 e^{-ax^2}$	$\frac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5 e^{-ax^2}$	$\frac{1}{a^3}$
$x^6 e^{-ax^2}$	$\frac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+bx^2}dx = \pi/2b^{1/2}$$
$$\int_0^\infty x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$
$$\int (x^2+b^2)^{-1/2}dx = \ln\left(x+\sqrt{x^2+b^2}\right)$$
$$\int (x^2+b^2)^{-1}dx = \frac{1}{b}\arctan(x/b)$$
$$\int (x^2+b^2)^{-3/2}dx = \frac{x}{b^2\sqrt{x^2+b^2}}$$
$$\int (x^2+b^2)^{-2}dx = \frac{\frac{bx}{x^2+b^2}+\arctan(x/b)}{2b^3}$$
$$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$$
$$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$$
$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right) = -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$$