UNL - Department of Physics and Astronomy

Preliminary Examination - Day 2
Friday, August 16, 2019

This test covers the topics of *Classical Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Classical Mechanics Group A - Answer only two Group A questions

A1 A block of mass m=1.5 kg lies on a frictionless horizontal surface and is attached to a horizontal spring of spring constant 50 N/m. Initially the spring is at its relaxed length with the block at position x = 0. Then an applied force with a constant magnitude of 3.0 N stretching the spring to the right until the block stops. When that stopping point is reached what are

a. the position of the block?
b. the work that has been done on the block by the applied force?
c. the work that has been done on the block by the spring force?
d. Check that the answers to *b.* and *c.* are consistent with the work-energy theorem.

A2 A child of mass 12 kg is on an *icy* merry-go-round of radius 2 m and mass 150 kg, with a moment of inertia of 300 kg m². Initially, the child is at the center, and the angular speed of the merry-go-round is 2 rad/sec. The child slides on the ice with no friction in a straight line towards the rim of the merry-go-round at a slow speed of 0.8 m/sec. Find an expression for the angular velocity as a function of time. You may assume the child is a point mass, and that she starts sliding towards the rim at $t = 0$.

A3 The interatomic potential between inert atoms such as helium or argon can be accurately represented by the Lennard-Jones potential, which is of the form

$$V(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$$

where r is the distance between them, and A and B are both positive.

(a) What is the equilibrium separation of the atoms, r_0, in terms of A and B?

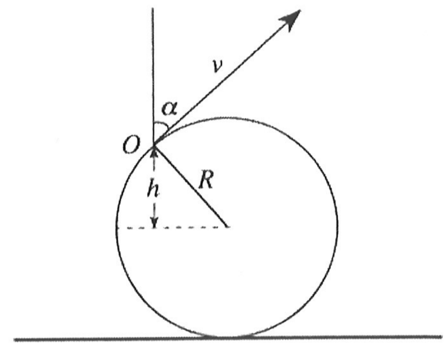
(b) Consider pushing the atoms farther apart from their equilibrium position by an amount Δr, which is not necessarily small. What are the direction and magnitude of the force in terms of A, B, r_0, and Δr?

A4 Consider two cylinders, A and B, of equal dimensions (radius R and length L) rolling down a slope without slipping. The slope makes an angle θ with the horizontal. One of the cylinders is hollow, while the other is solid. (The hollow cylinder is uncapped.) Starting at the same height, h, with zero velocity, cylinder A makes it to the bottom first.

a. Which cylinder is hollow and which is solid, A or B? Explain.
b. Derive an expression for the velocity difference at the bottom of the slope.
c. What happens if instead the slope is frictionless? Which cylinder will get to the bottom first?

Classical Mechanics Group B - Answer only two Group B questions

B1 A car is stuck in the mud. The mud splashes from the rim of a rotating tire of radius R spinning at a speed v where $v^2 > gR$. Neglect air resistance of the air. What is the maximum height the mud can rise above the ground?



B2 A person is riding a bike on a smooth horizontal surface with a speed of 5 m/s. The force of the air resistance is proportional to velocity with the coefficient of proportionality 3.5 N·s/m. The mass of the biker with the bike is 70 kg. At some instant (say, at $t=0$) the biker stops pushing the pedals so that at $t=t_1$ the speed is reduced to 2.5 m/s.

a. Find t_1

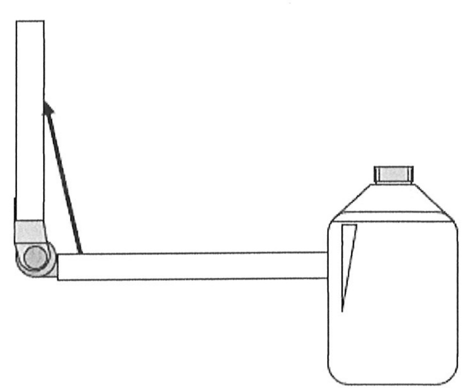
b. What is the distance covered by the bike as a function of time in the time interval from $t=0$ to $t=t_1$?

c. What is the total distance covered by the bike after $t=0$ until it stops?

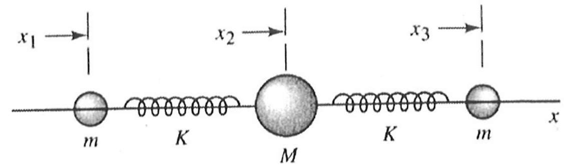
d. Suppose the force of the air resistance is quadratic in velocity with the coefficient of proportionality 2.8 N·s²/m². Find t_1 in this case.

B3 An artificial elbow joint should allow a patient to hold a gallon of milk (3.76 liters, specific mass 1.035 kg/L) with the lower arm horizontal. The "bicep muscle" attaches to the lower arm bone 1/6 the distance from the elbow joint (the full distance is 36 cm), making an angle of 80° with the horizontal bone. The lower arm bone has a mass of 1.4 kg.

You should design the artificial joint to withstand how strong a force?



B4 Consider the motion of a three-particle system in which all the particles lie in a straight line, such as the carbon dioxide molecule CO_2. We consider motion only in one dimension, along the x-axis (as shown in the figure.). The two end particles, each of mass m, are bound to the central particle, mass M, via a potential function that is equivalent to that of two spring of stiffness K, as shown in the figure. The coordinates expressing the displacements of each mass are x_1, x_2, and x_3.



Find

a. The Lagrangian of the system.
b. The three equations of motion.
c. The normal mode frequencies.

Electrodynamics Group A - Answer only two Group A questions

A1 A capacitor is made from two hollow, coaxial copper cylinders, one inside the other. There is air in the space between the cylinders. The inner cylinder has net positive charge and the outer cylinder has net negative charge. The inner cylinder has radius $r_1 = 2.30$ mm, the outer cylinder has radius $r_2 = 2.90$ mm, and the length of each cylinder is $L = 36.0$ cm.

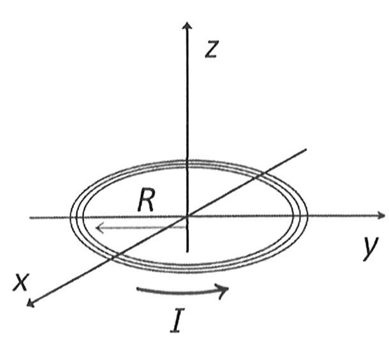
a. If the potential difference between the surfaces of the two cylinders is 60.0 V, what is the magnitude of the electric field at a point between the two cylinders that is a distance of 2.60 mm from their common axis and midway between the ends of the cylinders?

b. What is the capacitance of this capacitor?

A2 A parallel-plate capacitor has square plates that are $a = 5.00$ cm on each side and $d = 3.50$ mm apart. The space between the plates is completely filled with two square slabs of dielectric, each 5.00 cm on a side and $d_1 = 2.00, d_2 = 1.50$ mm thick. One slab is pyrex glass and the other is polystyrene, with dielectric constants κ_1 and κ_2, respectively.

If the potential difference between the plates is $V = 90.0$ V, how much electrical energy is stored in the capacitor?

A3 A circular coil of radius $R = 5.0$ cm, with 30 turns of wire, lies in the xy-plane, concentric with the origin. It carries a current $I = 5.0$ A in the direction shown in the figure. A uniform magnetic field is present, with magnitude 1.20 T and pointing in the $+y$ direction.



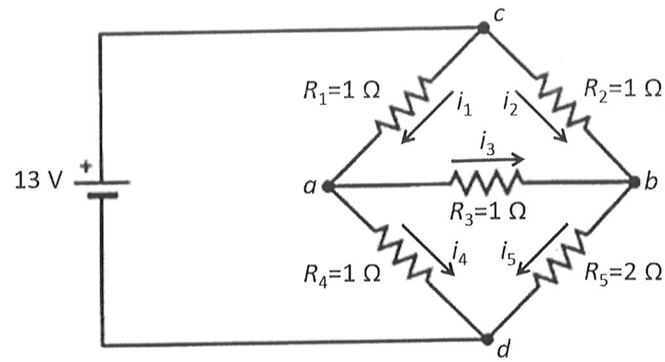
Find

a. The force acting on the coil (give a vector).

b. The torque acting on the coil (give a vector).

A4 The figure shows a bridge circuit.

Find the current in each resistor (magnitude and sign) and the equivalent resistance of the network of five resistors.

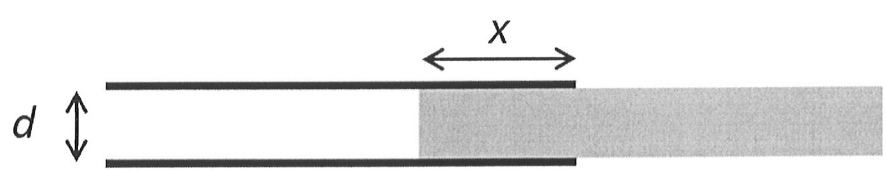


Electrodynamics Group B - Answer only two Group B questions

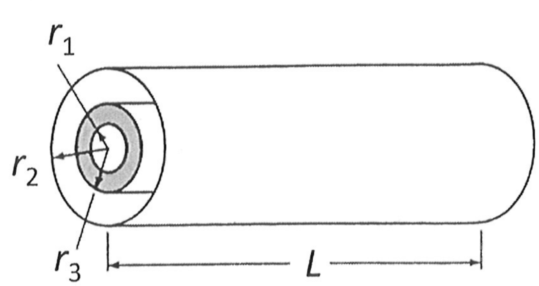
B1 An AC parallel RCL circuit consists of a voltage source of frequency f, a resistor R, a capacitor of capacitance C, and an inductor L.

a. What must the current amplitude I through the inductor be for the average electrical power consumed in the resistor to be W?
b. Draw the energy dissipation in R, C and L as a function of time.
c. Calculate the power factor.

B2 Two square metal plates of side L are separated by a distance d much smaller than L. A dielectric slab of size $L \times L \times d$ slides between the plates. It is inserted at a distance x (parallel to one side of the squares) and held there (see figure). *Before* this is done, the plates were charged using a battery with constant emf V which was then disconnected.
a. Find the electrical force exerted on the slab. Be careful and explicit about its direction.
b. How does the situation change if the battery is left connected?

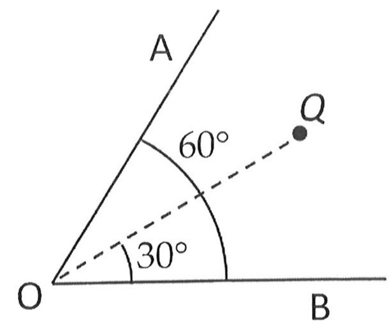


B3 A capacitor is made of two concentric cylinders of radius r_1 and r_2 ($r_1 < r_2$) and length $L \gg r_2$. The region between r_1 and $r_3 = \sqrt{r_1 r_2}$ is filled with a circular cylinder of length L and dielectric constant K (the remaining volume is an air gap).



What is the capacitance?

B4 Two semi-infinite conductive plates A and B are grounded and connected at O, with a point charge Q placed in the acute angle between two plates, as shown in the figure. The distance from point O to charge Q is R. Find the electric potential in the space between the two plates.



Physical Constants

speed of light .... $c = 2.998 \times 10^{8}$ m/s

Planck's constant .... $h = 6.626 \times 10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055 \times 10^{-34}$ J·s

Boltzmann constant ...... $k_B = 1.381 \times 10^{-23}$ J/K

elementary charge .... $e = 1.602 \times 10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257 \times 10^{-6}$ H/m

molar gas constant .... $R = 8.314$ J / mol·K

Avogadro constant .... $N_A = 6.022 \times 10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}$ m/F

electron mass .... $m_{el} = 9.109 \times 10^{-31}$ kg

electron rest energy ..... 511.0 keV

Compton wavelength .. $\lambda_C = h/m_{el}c = 2.426$ pm

proton mass .... $m_p = 1.673 \times 10^{-27}\,\text{kg} = 1836 m_{el}$

1 bohr .... $a_0 = \hbar^2 / ke^2 m_{el} = 0.5292$ Å

1 hartree (= 2 rydberg) .... $E_h = \hbar^2 / m_{el} a_0^{\,2} = 27.21$ eV

gravitational constant ... $G = 6.674 \times 10^{-11}$ m^3 / kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

ELECTROSTATICS

$$\oint\!\!\oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{encl}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2/2C$

Relative permittivity: $\varepsilon_r = 1 + \chi_e$

Bound charges

$\rho_b = -\nabla \cdot \mathbf{P}$
$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$

Capacitance in vacuum

Parallel-plate: $C = \varepsilon_0 \dfrac{A}{d}$

Spherical: $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

Cylindrical: $C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$ (for a length L)

MAGNETOSTATICS

Relative permeability: $\mu_r = 1 + \chi_m$

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

Current densities: $I = \int \mathbf{J} \cdot d\mathbf{A}$, $I = \int \mathbf{K} \cdot d\boldsymbol{\ell}$

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi} \displaystyle\int \frac{I d\boldsymbol{\ell} \times \hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

Infinitely long solenoid: B-field inside is $B = \mu_0 n I$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{encl}$

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I \int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi} \dfrac{1}{r^3} \left[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m} \right]$

Bound currents

$J_b = \nabla \times \mathbf{M}$
$K_b = \mathbf{M} \times \hat{\mathbf{n}}$

Maxwell's Equations in vacuum

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1. $\nabla \cdot \mathbf{D} = \rho_{\mathrm{f}}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{f}} + \dfrac{\partial \mathbf{D}}{\partial t}$ Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\dfrac{d\Phi_B}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \frac{1}{2} LI^2 = \frac{1}{2} \oint \mathbf{A} \cdot \mathbf{I} d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2}	$\dfrac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2}	$\dfrac{1}{2a}$
$x^2e^{-ax^2}$	$\dfrac{\sqrt{\pi}}{4a^{3/2}}$
$x^3e^{-ax^2}$	$\dfrac{1}{2a^2}$
$x^4e^{-ax^2}$	$\dfrac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5e^{-ax^2}$	$\dfrac{1}{a^3}$
$x^6e^{-ax^2}$	$\dfrac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+bx^2}dx = \pi/2b^{1/2}$$

$$\int_0^\infty x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$$\int (x^2+b^2)^{-1/2}dx = \ln\left(x+\sqrt{x^2+b^2}\right)$$

$$\int (x^2+b^2)^{-1}dx = \frac{1}{b}\arctan(x/b)$$

$$\int (x^2+b^2)^{-3/2}dx = \frac{x}{b^2\sqrt{x^2+b^2}}$$

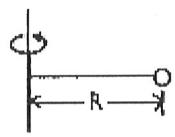
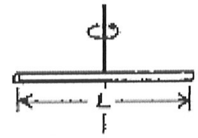
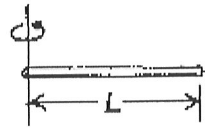
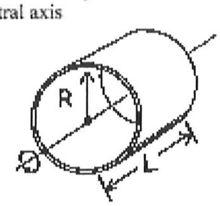
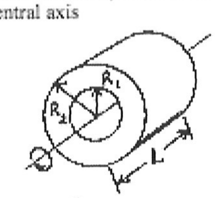
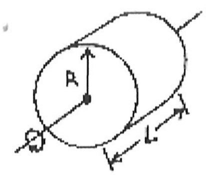
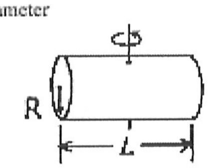
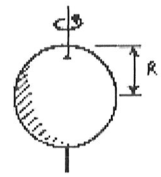
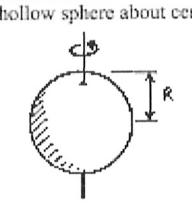
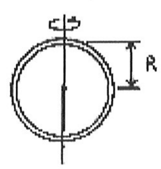
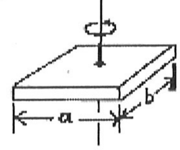
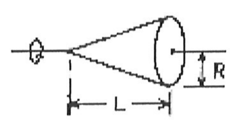
$$\int (x^2+b^2)^{-2}dx = \frac{\dfrac{bx}{x^2+b^2}+\arctan(x/b)}{2b^3}$$

$$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$$

$$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$
$$= -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$$

Table of Selected Moments of Inertia

Point mass at a radius R  $I = MR^2$	Thin rod about axis through center perpendicular to length  $I = \frac{1}{12}ML^2$	Thin rod about axis through end perpendicular to length  $I = \frac{1}{3}ML^2$
Thin-walled cylinder about central axis  $I = MR^2$	Thick-walled cylinder about central axis  $I = \frac{1}{2}M(R_1^2 + R_2^2)$	Solid cylinder about central axis  $I = \frac{1}{2}MR^2$
Solid cylinder about central diameter  $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$	Solid sphere about center  $I = \frac{2}{5}MR^2$	Thin hollow sphere about center  $I = \frac{2}{3}MR^2$
Thin ring about diameter  $I = \frac{1}{2}MR^2$	Slab about perpendicular axis through center  $I = \frac{1}{12}M(a^2 + b^2)$	Cone about central axis  $I = \frac{3}{10}MR^2$

Note: All formulas shown assume objects of uniform mass density.