a) Position of block


$$
\begin{aligned}
F & =-k x \\
\therefore 3 & =50 x \Rightarrow x=3 / 50=6 \mathrm{~cm} \text { to the int. } \\
& =\frac{6}{100}=6 \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

b) Work done on block by applied force

$$
\begin{array}{rlr}
W=\int F^{\prime} \cdot d \bar{x} & =+3 \times .06 & F_{t} d x \text { are } 11 \\
& =0.18 \mathrm{~J}
\end{array}
$$

c) Work done on block by spring force Spuing force is antiparallel to $d x$

$$
\begin{aligned}
F=-k x & =\int(-k x)(t d x)=-\frac{1}{2} k x^{2}=-\frac{1}{2} 50(.06) \\
& =-25 \times .06 \times 06=-25 \times \\
& =-0.09 \mathrm{~J} \\
\text { Vet writ done } & =0.09 \mathrm{~J}
\end{aligned}
$$

d) Net woik done $=0.09 \mathrm{~J}$ block $\frac{1}{2} k x^{2}=+0.09 \mathrm{~J}$
2.

Tooeasy At $t=0 \quad I=300 \mathrm{kgm}^{2}$
Too shait As child walles towadd rum $I=300+(12) r^{2}$

$$
\begin{aligned}
& =300+(12)(.8 t)^{2} \\
I & =300+(12)(0.64) t^{2}
\end{aligned}
$$

Lom consesvation of angular momentum (no enternal torpe is applieal

$$
\begin{aligned}
& \text { Linit }=L\left(t^{\prime}\right) \\
& (300)(2)=\left[300+(12)(0.64) t^{2}\right] \omega \\
& \omega=\frac{600}{\left.300+7.68 t^{2}\right]}
\end{aligned}
$$

$$
V(x)=A / r^{12}-B / r^{6}
$$

a) Equbbrim position is when $F=0 \quad 1 e^{-\partial V / \partial r}=0$

$$
\begin{aligned}
& -\frac{12 A}{r^{13}}+\frac{6 B}{r^{7}}=0, \quad \frac{V Z^{2} A}{r^{136}}=\frac{6 B}{B^{7}} \\
& \therefore r_{0}^{6}=2 A / B \quad B=2 A / r_{60}^{6}
\end{aligned}
$$

b)

$$
\begin{aligned}
& \text { At } r=r_{0}+\delta r \quad V(x)=A / r_{12}-B / r^{6} \\
& F=-\frac{12 A}{\left(r_{0}+\delta_{r}\right)^{1 / 36}}+\frac{6 B}{\frac{\left(r_{0}+\delta_{r}\right.}{2 A}} \\
& =\frac{-12 A+6 B}{\left(r_{0}+\delta r\right)^{6}}=\frac{-12 A}{\left(r_{0}+\delta r\right)^{6}}+\frac{12 A}{r_{0}^{6}} .
\end{aligned}
$$

Drection pulls towads equilituin lie low̃ads ro (muads)
4. There should be a table for moment of inertia in the formula sheet.
a) The difference in the time arises from the different moments of unecha The solid cylinder has a lower moment of inertia + hence higher linear $K, E \rightarrow$ Cylinder A.
b) Cylinder $A \quad I=\frac{1}{2} M_{A} R^{2}$

$$
\begin{aligned}
& M_{A} / g h= \frac{1}{2} M_{A}^{2} V_{A}^{2} \\
&=+\frac{1}{2}\left(\frac{1}{2} M_{A} R^{2}\right) \frac{V_{A}^{2}}{B^{2}} \quad \text { Rot } K \cdot E \\
&=\frac{1}{2} I \omega_{A}^{2} \\
& \quad \text { vel at bottom } \quad V_{A}^{2}=4 / 3 \mathrm{gh} \\
& \hline \mathrm{~s}^{2}
\end{aligned}
$$

Chider $B \quad I=M_{B} R^{2}$

$$
\begin{aligned}
M_{B} g h & =\frac{1}{2} M_{B} V_{B}^{2}+\frac{1}{2}\left(M_{B} R^{2}\right) \frac{V_{B}^{2}}{R^{2}} \\
g h & =V_{B}^{2} \text { vel at bottom } \\
V_{A}-V_{B} & =\sqrt{4 / 3 g h}-\sqrt{g h}
\end{aligned}
$$

c) No difference if then she down plane

Thermodynamics.
a) The slope of the $\mathrm{d} / \mathrm{d}_{\mathrm{m}}$ hin is negative $\therefore V_{l}<V_{s}$ lie $\rho_{e}>\rho_{s}$
We know this from observing that we floats
b) $\quad \frac{d T_{m}}{d P}=\frac{T_{m}}{n L_{m}}\left(V_{n i}-V_{s o l}\right)$

$$
\begin{aligned}
& =\frac{\left.T_{m}\right)\left(9 \times 10^{-5}\right) \quad 273 \mathrm{~K}}{n} 3.3 \times 10^{5} \\
& \text { of ceder } 1-10
\end{aligned}
$$

$\therefore \frac{d T_{m}}{d P}$ is very very small

$$
\begin{aligned}
& \left(T_{m}(P)=T_{m}(\text { at } T C)+\left.\left(T_{m}-T_{m c}\right) d T\right|_{F=T_{c}}\right. \\
& T_{m}(P)=\sim 273 K+\Delta T(\text { constant })
\end{aligned}
$$

$T_{m}(P)$ is near in $P$.

## Hard

1. A car is stuck in the mud. The mud splashes from the rim of a rotating tire of radius $R$ spinning at a speed $v$ where $v^{2}>g R$. Neglect the resistance of the air. What is the maximum height the mud can rise above the ground?


Solution:
Energy conservation $m g h_{0}=\frac{m v^{2}(\cos \alpha)^{2}}{2}$. The total height is $H=R+R \sin \alpha+\frac{v^{2}(\cos \alpha)^{2}}{2 g}$.
Minimizing this under assumption $v^{2}>g R$ gives $\sin \alpha_{0}=\frac{g R}{v^{2}}$
We obtain $H=R+\frac{v^{2}}{2 g}+\frac{g R^{2}}{2 v^{2}}$
$C M$

Howd

$$
\begin{aligned}
m \frac{d v}{d t} & =-c v \\
m \frac{d v}{v} & =-c d t \\
m \ln \frac{v}{v_{0}} & =-c t \quad
\end{aligned}
$$

(a) for $\frac{v}{v_{0}}=\frac{1}{2} \quad t_{1}=\frac{m}{c} \ln 2=\frac{70 \cdot 0.301}{3.5}=6.02 \mathrm{~s}$
(6) for $t=t_{1} e^{-\frac{c_{1} t}{m}}=\frac{1}{2}$

$$
x_{1}=\frac{m v_{0}}{2 c_{1}}=\frac{70.5}{2.3 .5}=50 \mathrm{~m}
$$

(c) $x=\frac{m v_{0}}{c_{1}}=100 \mathrm{~m}$
(d)

$$
\begin{aligned}
& m \frac{d V}{d t}=-c_{2} V^{2} \\
& m \frac{d^{2}}{v^{2}}=-c_{2} d t \\
& m\left(\frac{1}{v_{0}}-\frac{1}{v}\right)=-c_{2} t \quad \frac{c_{2} t}{m}=\frac{1}{v}-\frac{1}{v_{0}} \\
& v=\frac{1}{\frac{1}{v_{0}}+\frac{c_{2} t}{m}} \\
& \text { For } v=\frac{1}{2} v_{0} \\
& \frac{c_{2} t}{m}=\frac{1}{v_{0}} \\
& t_{1}=\frac{m}{c_{2} v_{0}}=\frac{70}{2.8 \cdot 5}=\frac{70}{14}=5 \mathrm{~s}
\end{aligned}
$$

## Easy

1. A capacitor is made from two hollow, coaxial copper cylinders, one inside the other. There is air in the space between the cylinders. The inner cylinder has net positive charge and the outer cylinder has net negative charge. The inner cylinder has radius $r_{1}=2.30 \mathrm{~mm}$, the outer cylinder has radius $r_{2}=2.90 \mathrm{~mm}$, and the length of each cylinder is $\mathrm{L}=36.0 \mathrm{~cm}$.
A. If the potential difference between the surfaces of the two cylinders is 60.0 V , what is the magnitude of the electric field at a point between the two cylinders that is a distance of 2.60 mm from their common axis and midway between the ends of the cylinders?
B. What is the capacitance of this capacitor?

## Solution:

From Gauss's law $E=\frac{Q}{L 2 \pi r \epsilon_{0}}$. Integrating we can find the potential difference $V=\frac{Q}{L 2 \pi \epsilon_{0}} \ln \frac{r_{2}}{r_{1}}$.
The answer for A. becomes $E=\frac{V}{\operatorname{rln}\left(\frac{r_{2}}{r_{1}}\right)}$
The answer for B. becomes $C=\frac{Q}{V}=\frac{L 2 \pi \epsilon_{0}}{\ln \frac{r_{2}}{r_{1}}}$
2. A parallel-plate capacitor has square plates that are $a=5.00 \mathrm{~cm}$ on each side and $d=3.50 \mathrm{~mm}$ apart. The space between the plates is completely filled with two square slabs of dielectric, each 5.00 cm on a side and $d_{1}=2.00, d_{2}=1.50 \mathrm{~mm}$ thick. One slab is pyrex glass and the other is polystyrene.
If the potential difference between the plates is $V=90.0 \mathrm{~V}$, how much electrical energy is stored in the capacitor?

Solution:
The problem is equivalent to having two capacitors in series. $U=\frac{C V^{2}}{2}=\frac{V}{2} \frac{1}{\frac{1}{C_{1}}+\frac{1}{C_{2}}}$ where $C_{i}=\frac{\epsilon_{0} a^{2}}{d_{i}} \kappa_{i}$

## EM-easy-1

A circular coil 0.0500 meter in radius, with 30 turns of wire, lies in a horizontal plane. It carries a current of 5.00 A in a counterclockwise sense when viewed from above. The coil is in a uniform magnetic field directed toward the right, with magnitude 1.20 T .
Find
(a) The magnitudes of the magnetic moment
(b) The torque on the coil.
(c) The magnitudes of force on the coil


## Magnetic torque on a circular coil

A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a current of 5.00 A in a counterclockwise sense when viewed from above. The coil is in a uniform magnetic field directed toward the right, with magnitude 1.20 T . Find the magnitudes of the magnetic moment and the torque on the coil.

## SOLUTION

IDENTIFY: This problem uses the definition of magnetic moment and the expression for the torque on a magnetic dipole in a magnetic field.


The area of the coil is

$$
A=\pi r^{2}=\pi(0.0500 \mathrm{~m})^{2}=7.85 \times 10^{-3} \mathrm{~m}^{2}
$$

The magnetic moment of each turn of the coil is

$$
\mu=I A=(5.00 \mathrm{~A})\left(7.85 \times 10^{-3} \mathrm{~m}^{2}\right)=3.93 \times 10^{-2} \mathrm{~A} \cdot \mathrm{~m}^{2}
$$

and the total magnetic moment of all 30 turns is

$$
\mu_{\text {total }}=(30)\left(3.93 \times 10^{-2} \mathrm{~A} \cdot \mathrm{~m}^{2}\right)=1.18 \mathrm{~A} \cdot \mathrm{~m}^{2}
$$

The angle $\phi$ between the direction of $\overrightarrow{\boldsymbol{B}}$ and the direction of $\overrightarrow{\boldsymbol{\mu}}$ (which is along the normal to the plane of the coil) is $90^{\circ}$.

The magnitude $\tau$ of the torque

$$
\begin{aligned}
\tau & =\mu_{\text {total }} B \sin \phi=\left(1.18 \mathrm{~A} \cdot \mathrm{~m}^{2}\right)(1.20 \mathrm{~T})\left(\sin 90^{\circ}\right) \\
& =1.41 \mathrm{~N} \cdot \mathrm{~m}
\end{aligned}
$$

## The force is zero

## EM-easy-2

## A bridge circuit is shown in the Figure.

(a) Find the current in each resistors and the equivalent resistance of the network of five
resistors
(b) Find the potential difference $\mathrm{V}_{\mathrm{ab}}\left(\mathrm{V}_{\mathrm{ab}}=\mathrm{V}_{\mathrm{a}}-\mathrm{V}_{\mathrm{b}}\right)$


## Example 26.6 A complex network

Figure 26.12 shows a "bridge" circuit of the type described at the beginning of this section (see Fig. 26.6b). Find the current in each resistor and the eauivalent resistance of the network of five resistors. find the potential dif-firence $V_{a b}$.

## SOLUTION

IDENTIFY: This network cannot be represented in terms of series and parallel combinations. Hence we must use Kirchhoff's rules to find the values of the target variables.
SET UP: There are five different currents to determine, but by applying the junction rule to junctions $a$ and $b$, we can represent them in terms of three unknown currents, as shown in the figure. The current in the battery is $I_{1}+I_{2}$.
26.12 A network circuit with several resistors.


XECUTE: We apply the loop rule to the three loops shown, thaining the following three equations:

$$
\begin{array}{r}
13 \mathrm{~V}-I_{1}(1 \Omega)-\left(I_{1}-I_{3}\right)(1 \Omega)=0 \\
-I_{2}(1 \Omega)-\left(I_{2}+I_{3}\right)(2 \Omega)+13 \mathrm{~V}=0 \\
-I_{1}(1 \Omega)-I_{3}(1 \Omega)+I_{2}(1 \Omega)=0 \tag{3}
\end{array}
$$

This is a set of three simultaneous equations for the three unknown arents. They may be solved by various methods; one straightforuard procedure is to solve the third equation for $I_{2}$, obtaining $h=I_{1}+I_{3}$, and then substitute this expression into the second quation to eliminate $I_{2}$. When this is done, we are left with the mo equations

$$
\begin{aligned}
& 13 \mathrm{~V}=I_{1}(2 \Omega)-I_{3}(1 \Omega) \\
& 13 \mathrm{~V}=I_{1}(3 \Omega)+I_{3}(5 \Omega)
\end{aligned}
$$

## SOLUTION

IDENTIFY: Our target variable is $V_{a b}=V_{a}-V_{b}$, which is the polential at point $a$ with respect to point $b$.
SET UP: To find $V_{a b}$, we start at point $b$ and follow a path to point a, adding potential rises and drops as we go. We can follow any of sereral paths from $b$ to $a$; the value of $V_{a b}$ must be independent of which path we choose, which gives us a natural way to check our pesult.
EXECUTE: The simplest path to follow is through the center $1-\Omega$ resistor. We have found $I_{3}=-1 \mathrm{~A}$, showing that the actual curtent direction in this branch is from right to left. Thus, as we go

Now we can eliminate $I_{3}$ by multiplying Eq. ( $1^{\prime}$ ) by 5 and adding the two equations. We obtain

$$
78 \mathrm{~V}=I_{1}(13 \Omega) \quad I_{1}=6 \mathrm{~A}
$$

We substitute this result back into Eq. ( $1^{\prime}$ ) to obtain $I_{3}=-1 \mathrm{~A}$, and finally, from Eq. (3) we find $I_{2}=5 \mathrm{~A}$. The negative value of $I_{3}$ tells us that its direction is opposite to our initial assumption.

The total current through the network is $I_{1}+I_{2}=11 \mathrm{~A}$, and the potential drop across it is equal to the battery emf-namely, 13 V . The equivalent resistance of the network is

$$
R_{\mathrm{eq}}=\frac{13 \mathrm{~V}}{11 \mathrm{~A}}=1.2 \Omega
$$

from $b$ to $a$, there is a drop of potential with magnitude $I R=(1 \mathrm{~A})(1 \Omega)=1 \mathrm{~V}$, and $V_{a b}=-1 \mathrm{~V}$. That is, the potential at point $a$ is 1 V less than that at point $b$.

EM-easy-3 (could be used for EM-hard)
A rod with length $L$, mass $m$, resistance $R$ on a fixed rail on a horizontal work station under a uniform magnetic field. It starts to move with an initial velocity of $v_{o}$, as shown in the Fig. (Ignore friction and resistance of the rail).
How far will the rod go?


uniform magmatic field ignore friction d resistance of rail
fixed rail on a horizontal work station
a rod with length $L$, mass $m$. resotanee $R$ starts to move with

$$
\begin{aligned}
& \text { now far w. ill the pool go? }
\end{aligned}
$$

$$
\text { solution: } \begin{aligned}
\varepsilon & =\frac{d \Phi}{d t}=B L v \\
I & =\frac{\varepsilon}{R}=\frac{B L V}{R} \\
|F| & =I B L=\frac{B^{2} L^{2} v}{R} \\
a & =-\frac{|F|}{m}=-\frac{B^{2} L^{2} v}{m R} \\
\because a & =\frac{d v}{d t} \\
\therefore d t & =\frac{1}{a} d v=-\frac{m R}{B^{2} L^{2} v} d v \\
\therefore S & =\int_{0}^{t_{s}} v o l t=\int_{v_{0}}^{0} v \cdot\left(-\frac{m R}{B^{2} L^{2} v}\right) d v=-\frac{m R}{B^{2} L^{2}} \cdot\left(-v_{0}\right) \\
& =\frac{m R v}{B^{2} L^{2}}
\end{aligned}
$$

## EM-hard-1

A capacitor is made of two concentric cylinders of radius $r_{1}$ and $r_{2}\left(r_{1}<r_{2}\right)$ and length $L \gg r_{2}$. The region between $r_{1}$ and $r_{3}=\left(r_{1} * r_{2}\right)^{1 / 2}$ is filled with a circular cylinder of length $L$ and dielectric constant K (the remaining volume is an air gap).
What is the capacitance?


Let $\lambda$ be the charge per unit length on the cylinder with radius $r_{1}$. From Gauss's law we have
$\iint E_{r} d A=\frac{\lambda}{\varepsilon} L$
where the surface $S$ is defined to be a cylinder of radius $r$ and of unit length. Since $E_{r}$ is a function of $r$ only, (1) immediately leads to
$\mathrm{E}_{\mathrm{r}}=\frac{\lambda}{2 \pi \epsilon \mathrm{r}}$.
The potential difference between the two cylinders is
$\mathrm{V}=\int_{\mathrm{r}_{1}}^{\mathrm{r}_{2}} \mathrm{E}_{\mathrm{r}} \mathrm{dr}=\frac{\lambda}{2 \pi \varepsilon_{\mathrm{o}}}\left(\frac{1}{\mathrm{~K}}\left\langle\mathrm{n} \frac{\mathrm{r}_{3}}{\mathrm{r}_{1}}+\ell \mathrm{n} \frac{\mathrm{r}_{2}}{\mathrm{r}_{3}}\right)\right.$
from which we find the capacitance $C$
$C=\frac{\lambda L}{V}=\frac{2 \pi \varepsilon_{0} L}{\frac{1}{K} \ell n \frac{r_{3}}{r_{1}}+\ln \frac{r_{2}}{r_{3}}}$

## EM-hard-2

Two semi-infinite conductive plates A and B are grounded and connected at O , with a point charge Q placed in the acute angle between two plates, as shown below. The distance from point $O$ to charge Q is R . Find the electric potential in the space between the two plates.


$$
\begin{aligned}
& \text { 3. Two semi-infinite conductive plates } \mathrm{A} \text { and } \mathrm{B} \text { are grounded and connected at } \mathrm{O} \text {, with a point charge } \mathrm{Q} \\
& \text { placed in the acute angle between two plates, as shown below. Find the electric potential in the space } \\
& \text { between the two plates }
\end{aligned}
$$

$$
\begin{aligned}
& q_{5}=-\dot{Q} \quad \cdot q_{1}=-Q \\
& \text { - } q_{4}=+Q \\
& \text { Find image point charges } q_{1}, q_{2}, q_{3}, q_{4}, q_{5} \text { to make } \\
& \text { the boundary condition satisfied } \\
& Q\left(\frac{\sqrt{3}}{2} R, \frac{1}{2} R\right), q_{1}\left(\frac{\sqrt{3}}{2} R,-\frac{1}{2} R\right) \\
& q_{2}(0, R), q_{3}\left(-\frac{\sqrt{3}}{2} R, \frac{1}{2} R\right) \\
& q_{4}(0,-R), \quad q_{5}\left(-\frac{\sqrt{3}}{2} R,-\frac{1}{2} R\right) \\
& \therefore \phi(x, y)=\frac{1}{4 \pi \varepsilon_{0}}\left\{\frac{Q}{\left[\left(x-\frac{\sqrt{3}}{2} R\right)^{2}+\left(y-\frac{1}{2} R\right)^{2}\right]^{1 / 2}}-\frac{Q}{\left[\left(x-\frac{\sqrt{3}}{2} R\right)^{2}+\left(y+\frac{1}{2} R\right)^{2}\right]^{1 / 2}}\right. \\
& -\frac{Q}{\left[x^{2}+(y-R)^{2}\right]^{1 / 2}}+\frac{Q}{\left[\left(x+\frac{\sqrt{3}}{2} R\right)^{2}+\left(y-\frac{1}{2} R\right)^{2}\right]^{1 / 2}} \\
& \left.+\frac{Q}{\left[x^{2}+(y+R)^{2}\right]^{1 / 2}}-\frac{Q}{\left.\left[\left(x+\frac{\sqrt{3}}{2} R\right)^{2}+\left(y+\frac{1}{2} R\right)^{2}\right]^{2}\right]^{2}}\right\}
\end{aligned}
$$

## ME-hard-1

Let us consider the motion of a three-particle system in which all the particles lie in a straight line, such as the carbon dioxide molecule $\mathrm{CO}_{2}$. We consider motion only in one dimension, along the x -axis (as shown in the Fig.). The two end particles, each of mass m , are bound to the central particle, mass M , via a potential function that is equivalent to that of two spring of stiffness K , as shown in Fig. The coordinated expressing the displacements of each mass are $\mathrm{x}_{1}, \mathrm{x}_{2}$, and $\mathrm{x}_{3}$.

Find
(a) The Lagrangian of the system
(b) The three equations of motion
(c) The normal mode frequencies


The Lagrangian of the system is

$$
\begin{aligned}
L & =T-V \\
& =\left(\frac{m}{2} \dot{x}_{1}^{2}+\frac{M}{2} \dot{x}_{2}^{2}+\frac{m}{2} \dot{x}_{3}^{2}\right)-\left[\frac{K}{2}\left(x_{2}-x_{1}\right)^{2}+\frac{K}{2}\left(x_{3}-x_{2}\right)^{2}\right]
\end{aligned}
$$

and Lagrange's three equations of motion read

$$
\begin{array}{ccr}
m \ddot{x}_{1}+K x_{1} & -K x_{2} & \\
-K x_{1} & +M \ddot{x}_{2}+2 K x_{2} & -K x_{3} \\
& -K x_{2} & =0 \\
& +m \ddot{x}_{3}+K x_{3} & =0
\end{array}
$$

If a solution of the form $x_{1}=a_{1} \cos \omega t, x_{2}=a_{2} \cos \omega t, x_{3}=a_{3} \cos \omega t$ exists, then

$$
\left(\begin{array}{ccc}
K-m \omega^{2} & -K & 0  \tag{11.4.17}\\
-K & 2 K-M \omega^{2} & -K \\
0 & -K & K-m \omega^{2}
\end{array}\right)\left(\begin{array}{c}
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right)=0
$$

The secular equation is thus,

$$
\left|\begin{array}{ccc}
K-m \omega^{2} & -K & 0 \\
-K & 2 K-M \omega^{2} & -K \\
0 & -K & K-m \omega^{2}
\end{array}\right|=0
$$

(11.4.18a)
which, on expanding the determinant and collecting terms, fortuitously becomes the product of three factors

$$
\omega^{2}\left(-m \omega^{2}+K\right)\left(-m M \omega^{2}+K M+2 K m\right)=0
$$

Equating each of the three factors to zero gives the three normal frequencies of the system:

$$
\omega_{1}=0 \quad \omega_{2}=\left(\frac{K}{m}\right)^{1 / 2} \quad \omega_{3}=\left(\frac{K}{m}+2 \frac{K}{M}\right)^{1 / 2}
$$

## Hard

1. An ac parallel RLC circuit consists of a voltage source of frequency $f$, a resistor $R$, a capacitor of capacitance $C$, and inductor $L$.
A. What must the current amplitude $I$ through the inductor be for the average electrical power consumed in the resistor to be $W$ ?
B. Draw the energy dissipation in $\mathrm{R}, \mathrm{L}$ and C as a function of time.
C. Calculate the power factor.

## Solution:

We have $X_{L}=\omega L$. The current amplitude $I=V_{0} / X_{L}$. We also have $W=\frac{V_{0}^{2}}{2 R}$ for the resistor $->I=$ $\frac{\sqrt{2 R W}}{X_{L}}$.

The power factor $=\frac{W}{V_{0}^{2} / 2 Z}=\frac{Z}{R}=\frac{1}{\sqrt{1+\left(R \omega C-\frac{R}{\omega L}\right)^{2}}}$
2. Two square metal plates of side $L$ are separated by a distance $d$ much smaller than $L$. A dielectric slab of size $L \times L \times d$ slides between the plates. It is inserted at a distance $x$ (parallel to one side of the squares) and held there (see Figure)
A. Find the force exerted electrically on the slab. Be careful and explicit about its direction.
B. How does the situation change if the battery is left connected?


## Solution:

The energy is given by $U=\frac{Q^{2}}{2 C}=\frac{C V^{2}}{2}$. The capacitance $C=\frac{\epsilon_{0} L}{d}(\kappa x+L-x)$
A. $F=-\frac{\partial U}{\partial x}=\frac{\epsilon_{0} L V^{2}}{2 d}(\kappa-1)$ Drawn further between the plates.
B. $F=-\frac{\Delta\left(\frac{C V^{2}}{2}\right)-V \Delta Q}{\Delta x}=\frac{\epsilon_{0} L V^{2}}{2 d}(\kappa-1)$

