Alya F.

QM

QUANTUM A1

easy

photon energy $E = \frac{hc}{\lambda} = \frac{1240\ eV \cdot nm}{500\ nm} = 2.48\ eV$

$= (1.6 \times 10^{-19} \cdot 2.48)\ J = 3.97 \times 10^{-19}\ J$

$$\text{flux} = \frac{P}{4\pi r^2 E} = \frac{40\ W}{12.56 \cdot (4 \times 10^3)^2\ 3.97 \times 10^{-19}\ (J \cdot m^2)}$$

$$= 5.01 \times 10^{11}\ \frac{\text{photons}}{m^2} = 5.01 \times 10^{7}\ \frac{\text{photons}}{cm^2}$$

QUANTUM A2

easy

(a) $c\, e^{\pm i \sqrt{2mE}\, x/\hbar - iEt/\hbar}$

(b) $c \sin\left(\frac{\sqrt{2mE}\, x}{\hbar}\right) e^{-iEt/\hbar}$ (parity -1)

$c \cos\left(\frac{\sqrt{2mE}}{\hbar} x\right) e^{-iEt/\hbar}$ (parity $+1$)

sets (a) and (b) are different because $p = -i\hbar \frac{\partial}{\partial x}$ is parity odd

or p and P don't commute: $[p, P] \neq 0$

Ilya F.

Q 4

QUANTUM B1

(hard)

(a) $\sin\theta \sin\phi = \text{const}\,(Y_{11} + Y_{1-1})$,

therefore the state is a superposition of $m=1$ and $m=-1$ with equal weights

$\langle L_z \rangle = 0$

(b) possible values of L_z: $\pm\hbar$ with 50% probability each

(c) $L_x = \frac{1}{2}(L_+ + L_-)$

$$\langle L_x \rangle = (\langle 11| + \langle 1-1|)(L_+ + L_-)(|11\rangle + |1-1\rangle)$$

$$= (\langle 11| + \langle 1-1|)(|10\rangle + |10\rangle) = 0$$

~~this also follo~~

(d) $\langle \vec{L}^2 \rangle = 2\hbar^2$ (eigenvalue).

(e) $C^2 \int \sin^2\theta \sin^2\phi \sin\theta \, d\theta \, d\phi = 1$

$$\int_0^{2\pi} \sin^2\phi \, d\phi \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta = \frac{2\pi}{2} \int_{-1}^{1} (1-s^2)\, ds \qquad s = \cos\theta$$

$$= \pi\left(2 - \frac{2}{3}\right) = \frac{4}{3}\pi \qquad C = \sqrt{\frac{3}{4\pi}}$$

$\langle \cos\theta \rangle = 0$ because $s(1-s^2)$ is an odd function

$$\langle \cos^2\theta \rangle = \frac{3}{4\pi} \int_0^{2\pi} \sin^2\phi \, d\phi \int_{-1}^{1} s^2(1-s^2)\, ds$$

$$= \frac{3}{4\pi} \cdot \pi \cdot \left(\frac{s^3}{3} - \frac{s^5}{5}\right)_{-1}^{1} = \frac{3}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{3}{2} \cdot \frac{2}{15} = \frac{1}{5}$$

Ilya F.

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QUANTUM B2 hard

(a) $\delta(x-x_0) = \sum_n \psi_n(x_0)\,\psi_n(x)$

probability amplitudes are $\psi_n(x_0)$, therefore

$$P_n = |\psi_n(x_0)|^2 = \frac{2}{a}\sin^2\frac{\pi x_0}{a}$$

with possible outcomes $E_n = \dfrac{\pi^2 n^2 \hbar^2}{2ma^2}$ $n=1,2,\ldots$

(б) at $t>t_0$ the particle is in the ground state $\psi_1(x)$, therefore

$$P = \int_{a/4}^{3a/4} |\psi_1(x)|^2 dx = \frac{2}{a}\int_{a/4}^{3a/4} \sin^2\frac{\pi x}{a}\,dx$$

$$= \frac{1}{a}\int_{a/4}^{3a/4}\left(1-\cos\frac{2\pi x}{a}\right)dx$$

$$= \frac{1}{a}\left[\frac{a}{2} - \frac{a}{2\pi}\sin\frac{2\pi x}{a}\Big|_{a/4}^{3a/4}\right) = \frac{1}{a}\left(\frac{a}{2} - \frac{a}{2\pi}(-2)\right)$$

$$P = \frac{1}{2} + \frac{1}{\pi}$$

QUANTUM A3

ANSWER

write $Gg(x) = \lambda g(x)$

$GG - 3G + 2E = 0$

$(GG - 3G + 2E)g(x) = 0g(x) = 0$

Using the linearity of G:

$(\lambda^2 - 3\lambda + 2)g(x) = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 1$ or $\lambda = 2$

QUANTUM A4

ANSWER

Denote the parity operator by $\hat{P}$, then

$$\langle P \rangle = \frac{\langle \psi | \hat{P} | \psi \rangle}{\langle \psi | \psi \rangle}$$

$$\langle \psi | P | \psi \rangle = \int_{-\infty}^{\infty} \psi(x) \hat{P} \psi(x) dx = \int_{-b}^{b} C^2 dx = 2bC^2$$

$$\langle \psi | \psi \rangle = \int_{-\infty}^{\infty} \psi(x) \psi(x) dx = \int_{-b}^{2b} C^2 dx = 3bC^2$$

$$\langle P \rangle = \frac{\langle \psi | \hat{P} | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{2bC^2}{3bC^2} = \frac{2}{3}$$

QUANTUM B3

ANSWERS

Part a

$$a^\dagger | n \rangle = a^\dagger \left\{ \frac{(a^\dagger)^n}{\sqrt{n!}} | 0 \rangle \right\} = \frac{\sqrt{n+1}}{\sqrt{n+1}} \frac{(a^\dagger)^{n+1}}{\sqrt{n!}} | 0 \rangle = \sqrt{n+1} \frac{(a^\dagger)^{n+1}}{\sqrt{(n+1)!}} | 0 \rangle = \sqrt{n+1} | n+1 \rangle$$

Part b

$$\hat{a} = \tfrac{1}{2}\sqrt{2}\,\beta \left(\hat{x} + \frac{i}{m\omega_0} \hat{p} \right) \quad \text{and} \quad \hat{a}^\dagger = \tfrac{1}{2}\sqrt{2}\,\beta \left(\hat{x} - \frac{i}{m\omega_0} \hat{p} \right) \quad \Rightarrow \quad \hat{a} + \hat{a}^\dagger = \sqrt{2}\,\beta \hat{x}$$

so $\sqrt{2}\,\beta\langle n|x|n\rangle \propto \langle n|(\hat{a}+\hat{a}^\dagger)|n\rangle = \langle n|\hat{a}|n\rangle + \langle n|\hat{a}^\dagger|n\rangle = \sqrt{n}\langle n|n-1\rangle + \sqrt{n+1}\langle n|n+1\rangle = 0$

<u>Part c</u>

$(\sqrt{2}\,\beta\hat{x})^2 = 2\beta^2\hat{x}^2 = (\hat{a}+\hat{a}^\dagger)^2 = \hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}$

Now

$$\begin{aligned}
&\langle n|\hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}|n\rangle = \langle n|\hat{a}\hat{a}|n\rangle + \langle n|\hat{a}^\dagger\hat{a}^\dagger|n\rangle + \langle n|\hat{a}\hat{a}^\dagger|n\rangle + \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \\
&= 0 + 0 + \langle n|\hat{a}\hat{a}^\dagger|n\rangle + \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \sqrt{n+1}\langle n|\hat{a}|n+1\rangle + \sqrt{n}\langle n|\hat{a}^\dagger|n-1\rangle = \\
&= \sqrt{n+1}\sqrt{n+1}\langle n|n\rangle + \sqrt{n}\sqrt{n}\langle n|n\rangle = n+1+n = 2n+1
\end{aligned}$$

$$2\beta^2\langle n|\hat{x}^2|n\rangle = 2n+1 \quad\Rightarrow\quad \langle x^2\rangle = \frac{1}{2\beta^2}(2n+1) = \frac{1}{\beta^2}(n+\tfrac{1}{2})$$

<u>Part d</u>

We have $(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2 = \dfrac{1}{\beta^2}(n+\tfrac{1}{2}) - 0 \quad\Rightarrow\quad \Delta x = \dfrac{1}{\beta}\sqrt{n+\tfrac{1}{2}}$

<u>Part e</u>

$$\hat{a} = \tfrac{1}{2}\sqrt{2}\,\beta\left(\hat{x} + \frac{i}{m\omega_0}\hat{p}\right) \quad\text{and}\quad \hat{a}^\dagger = \tfrac{1}{2}\sqrt{2}\,\beta\left(\hat{x} - \frac{i}{m\omega_0}\hat{p}\right) \quad\Rightarrow\quad \hat{a} - \hat{a}^\dagger = i\frac{\sqrt{2}\,\beta}{m\omega_0}\hat{p}$$

so $i\dfrac{\sqrt{2}\,\beta}{m\omega_0}\langle n|p|n\rangle \propto \langle n|(\hat{a}-\hat{a}^\dagger)|n\rangle = \langle n|\hat{a}|n\rangle - \langle n|\hat{a}^\dagger|n\rangle = \sqrt{n}\langle n|n-1\rangle - \sqrt{n+1}\langle n|n+1\rangle = 0$

<u>Part f</u>

$$\left(i\frac{\sqrt{2}\,\beta}{m\omega_0}\hat{p}\right)^2 = -\frac{2\beta^2}{m^2\omega_0{}^2}\hat{p}^2 = (\hat{a}-\hat{a}^\dagger)^2 = \hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}$$

Now

$$\begin{aligned}
&\langle n|\hat{a}\hat{a} + \hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}|n\rangle = \langle n|\hat{a}\hat{a}|n\rangle + \langle n|\hat{a}^\dagger\hat{a}^\dagger|n\rangle - \langle n|\hat{a}\hat{a}^\dagger|n\rangle - \langle n|\hat{a}^\dagger\hat{a}|n\rangle = \\
&= 0 + 0 - \langle n|\hat{a}\hat{a}^\dagger|n\rangle - \langle n|\hat{a}^\dagger\hat{a}|n\rangle = -\sqrt{n+1}\langle n|\hat{a}|n+1\rangle - \sqrt{n}\langle n|\hat{a}^\dagger|n-1\rangle = \\
&= -\sqrt{n+1}\sqrt{n+1}\langle n|n\rangle - \sqrt{n}\sqrt{n}\langle n|n\rangle = -(n+1) - n = -2n-1
\end{aligned}$$

$$-\frac{2\beta^2}{m^2\omega_0{}^2}\langle n|\hat{p}^2|n\rangle = -2n-1 \quad\Rightarrow\quad \langle p^2\rangle = \frac{m^2\omega_0{}^2}{\beta^2}(n+\tfrac{1}{2})$$

Part g

We have $(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2 = \dfrac{m^2 \omega_0{}^2}{\beta^2}(n + \tfrac{1}{2}) - 0 \quad \Rightarrow \quad \Delta p = \dfrac{m\omega_0}{\beta}\sqrt{n + \tfrac{1}{2}}$

Part h

We have $\Delta x \Delta p = \left(\dfrac{1}{\beta}\sqrt{n + \tfrac{1}{2}}\right)\left(\dfrac{m\omega_0}{\beta}\sqrt{n + \tfrac{1}{2}}\right) = \dfrac{m\omega_0}{\beta^2}(n + \tfrac{1}{2}) = \hbar(n + \tfrac{1}{2})$ (note that $\beta^2 = m\omega_0 / \hbar$)

This satisfies Heisenberg's uncertainty relation, $\Delta x \Delta p \geq \tfrac{1}{2}\hbar$. The uncertainty is smallest for the ground state ($n = 0$); then, $\Delta x \Delta p = \tfrac{1}{2}\hbar$

QUANTUM B4

ANSWERS

Part a.

$\hat{\Pi}\varphi(x) = \Pi\varphi(x) \quad \Rightarrow \quad \hat{\Pi}\hat{\Pi}\varphi(x) = \varphi(x) = \Pi^2\varphi(x) \quad \Rightarrow \quad \Pi^2 = 1 \quad \Rightarrow \quad \Pi = \pm 1$.

Part b.

$|\psi\rangle = c_{-1}|\Pi = -1\rangle + c_{+1}|\Pi = +1\rangle \quad \Rightarrow \quad \langle\psi|\hat{\Pi}|\psi\rangle = \langle\Pi\rangle = |c_{-1}|^2(-1) + |c_{+1}|^2(+1) =$
$(1 - |c_{+1}|^2)(-1) + |c_{+1}|^2(+1) = -1 + 2|c_{+1}|^2 \quad \Rightarrow \quad |c_{+1}|^2 = \tfrac{1}{2}\left[\langle\Pi\rangle + 1\right]$

So, $0 \leq |c_{+1}|^2 \leq 1 \quad \Rightarrow \quad 0 \leq \tfrac{1}{2}\left[\langle\Pi\rangle + 1\right] \leq 1 \quad \Rightarrow \quad 0 \leq \left[\langle\Pi\rangle + 1\right] \leq 2 \quad \Rightarrow \quad -1 \leq \langle\Pi\rangle \leq +1$

Part c

$$[\hat{H},\hat{\Pi}] = [\hat{p}\hat{p},\hat{\Pi}]/2m + \tfrac{1}{2}m\omega_0{}^2[\hat{x}\hat{x},\hat{\Pi}]$$

$$\hat{x}\hat{x}\hat{\Pi}\varphi(x) = xx\varphi(-x)$$
$$\hat{\Pi}\hat{x}\hat{x}\varphi(x) = \hat{\Pi}\left\{\hat{x}\hat{x}\varphi(x)\right\} = \hat{\Pi}\left\{xx\varphi(x)\right\} = (-x)(-x)\varphi(-x) = xx\varphi(-x) = \hat{x}\hat{x}\hat{\Pi}\varphi(x)$$
$$\Rightarrow \quad \hat{\Pi}\hat{x}\hat{x} = \hat{x}\hat{x}\hat{\Pi} \quad \Rightarrow \quad [\hat{x}\hat{x},\hat{\Pi}] = 0$$

$$\left(\frac{i}{\hbar}\right)\hat{p}\hat{\Pi}\varphi(x) = \frac{d}{dx}\varphi(-x) = (-1)\frac{d}{d(-x)}\varphi(-x)$$
$$\left(\frac{i}{\hbar}\right)^2\hat{p}\hat{p}\hat{\Pi}\varphi(x) = \frac{d}{dx}\left\{(-1)\frac{d}{d(-x)}\varphi(-x)\right\} = (-1)\frac{d}{d(-x)}\left\{(-1)\frac{d}{d(-x)}\varphi(-x)\right\} = \frac{d^2}{d(-x)^2}\varphi(-x)$$

$$\left(\frac{i}{\hbar}\right)^2\hat{\Pi}\hat{p}\hat{p}\varphi(x) = \hat{\Pi}\left\{\frac{d^2}{dx^2}\varphi(x)\right\} = \frac{d^2}{d(-x)^2}\varphi(-x) = \left(\frac{i}{\hbar}\right)^2\hat{p}\hat{p}\hat{\Pi}\varphi(x) \quad \Rightarrow \quad \hat{\Pi}\hat{p}\hat{p} = \hat{p}\hat{p}\hat{\Pi}$$
$$\Rightarrow \quad [\hat{p}\hat{p},\hat{\Pi}] = 0$$
So $\quad [\hat{H},\hat{\Pi}] = [\hat{p}\hat{p},\hat{\Pi}]/2m + \tfrac{1}{2}m\omega_0{}^2[\hat{x}\hat{x},\hat{\Pi}] = \hat{0} + \hat{0} = \hat{0}$

<u>Part d</u>

$$|\psi\rangle = \tfrac{1}{2}\sqrt{3}\,|0\rangle - c\,|3\rangle \quad \Rightarrow \quad \langle\psi|\psi\rangle = \left|\tfrac{1}{2}\sqrt{3}\right|^2 + |c|^2 = \tfrac{3}{4} + c^2 = 1 \quad \Rightarrow \quad c = \tfrac{1}{2}$$

<u>Part e</u>

$$|\psi\rangle(t=0) = (\tfrac{1}{2}\sqrt{3})\,|n=0,\Pi=+1\rangle - (\tfrac{1}{2})\,|n=3,\Pi=-1\rangle$$
$$\langle\Pi\rangle(t=0) = \left|\tfrac{1}{2}\sqrt{3}\right|^2(+1) + \left|\tfrac{1}{2}\right|^2(-1) = \tfrac{3}{4} - \tfrac{1}{4} = \tfrac{1}{2}$$

<u>Part f</u>

Because $[\hat{H},\hat{\Pi}] = \hat{0}$, $\dfrac{d}{dt}\langle\Pi\rangle = 0$: $\langle\Pi\rangle$ is a conserved quantity.

So $\langle\Pi\rangle(t>0) = \langle\Pi\rangle(t=0) = \tfrac{1}{2}$.

There's no need to write this out explicitly, as in:

$$|\psi\rangle(t) = (\tfrac{1}{2}\sqrt{3})e^{iE_0t/\hbar}\,|0,+1\rangle - (\tfrac{1}{2})e^{iE_3t/\hbar}\,|3,-1\rangle$$
$$\langle\Pi\rangle(t) = \left|(\tfrac{1}{2}\sqrt{3})e^{iE_0t/\hbar}\right|^2(+1) + \left|-(\tfrac{1}{2})e^{iE_3t/\hbar}\right|^2(-1) = \left|\tfrac{1}{2}\sqrt{3}\right|^2(+1) + \left|-\tfrac{1}{2}\right|^2(-1) = \langle\Pi\rangle(t=0) = \tfrac{1}{2}$$

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Electricity & Magnetism A1

$$V(0) = \int_0^R dV = \int_0^R \frac{k\,dQ}{r} = \int_0^R \frac{\frac{1}{\epsilon_0} r^2 dr\, r^2 50 \frac{mC}{m^5}}{r}$$

$$= \frac{50 \frac{mC}{m^5}}{\epsilon_0} \int_0^{5cm} r^3 dr$$

$$= \frac{5 \times 10^{-2} C}{\epsilon_0} \left. \frac{r^4}{4} \right|_0^{0.05m}$$

$$= \frac{5 \times 10^{-2}}{8.85 \times 10^{-12}} \cdot \frac{1}{4} \cdot (5 \times 10^{-2})^4 \, V$$

$$= 88.3 \times 10^2 = \boxed{+8.83 \times 10^3 \, V}$$

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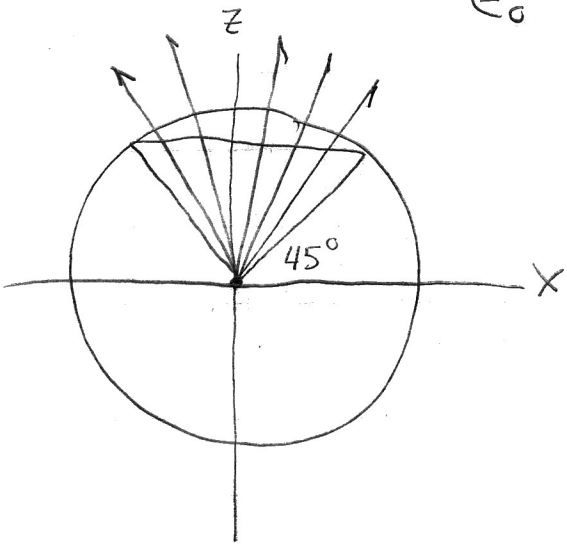
Electricity & Magnetism A2

$$\Phi_e = \frac{Q_{encl}}{\epsilon_0}$$



$$\Phi_e = \frac{\text{Solid } \angle \text{ of cone}}{4\pi} \frac{Q_{encl}}{\epsilon_0}$$

Solid $\angle$ of cone

$$= \int_0^{2\pi} d\phi \int_0^{\pi/4} \sin\theta \, d\theta$$

$$= 2\pi \cos\theta \Big|_{\pi/4}^{0}$$

$$= 2\pi \left[1 - \frac{\sqrt{2}}{2} \right]$$

$$\frac{\text{Solid } \angle \text{ of cone}}{4\pi} = \frac{1}{2}\left[1 - \frac{\sqrt{2}}{2}\right] = 0.146$$

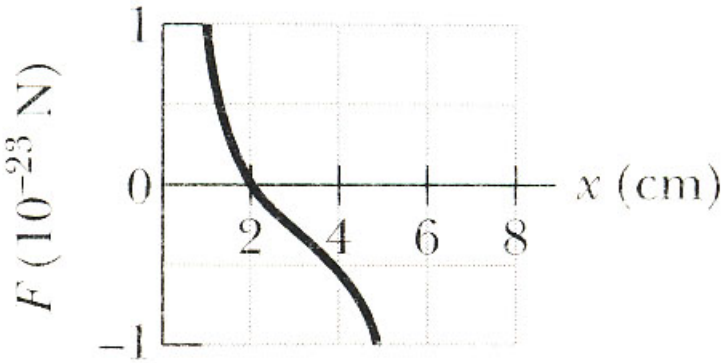
so $$\Phi_e = \frac{-15 \times 10^{-6}\,C \times 0.146}{8.85 \times 10^{-12}\,C^2} \cdot Nm^2$$

$$= \boxed{-2.47 \times 10^5 \ \frac{Nm^2}{C}}$$

EM A3 SL

In Fig.1(a) charge particle q1 and charge particle q2 are fixed in place on x axis 8.00 cm apart. Charge particle q3 =8.00 x10-19 C is to placed on the line (x-axis) between charge particle q1 and charge particle q2 so that produce net electrostatic force F3, net on it. Fig. 1 (b) gives the x component of that force versus the coordinate x at which charge particle q3 is placed.
What are (a) the sign of charge particle q1 and (b) the ratio q2/q1?

(a)



(b)

EM A3 SL

12. (a) According to the graph, when q_3 is very close to q_1 (at which point we can consider the force exerted by particle 1 on 3 to dominate) there is a (large) force in the positive x direction. This is a repulsive force, then, so we conclude q_1 has the same sign as q_3. Thus, q_3 is a positive-valued charge.

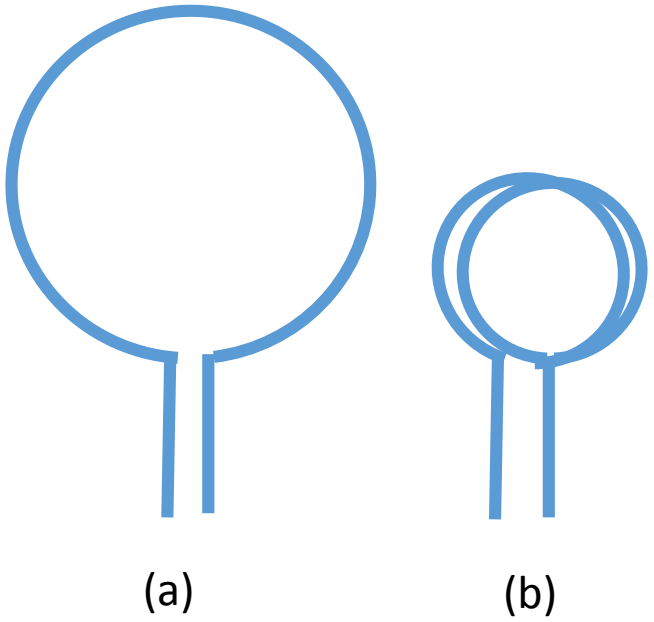
(b) Since the graph crosses zero and particle 3 is *between* the others, q_1 must have the same sign as q_2, which means it is also positive-valued. We note that it crosses zero at $r = 0.020$ m (which is a distance $d = 0.060$ m from q_2). Using Coulomb's law at that point, we have

$$\frac{q_1 q_3}{4\pi\varepsilon_o r^2} = \frac{q_3 q_2}{4\pi\varepsilon_o d^2} \quad \Rightarrow \quad q_2 = \left(\frac{d^2}{r^2}\right) q_1 = 9.0\, q_1 ,$$

or $q_2/q_1 = 9.0$.

EM A4 SL

Fig 2 (a) shows a length of wire carrying a current I and bent into a circular coil of one turn. In Fig. 2 (b) the same length of wire has been bent to give a coil of two turns, each of half the original radius.



(a) If B_a and B_b are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio Bb/Ba?

(b) What is the rato μ_b/μ_a of the dipole moment magnitudes of the coils?

EM A4 SL

48. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

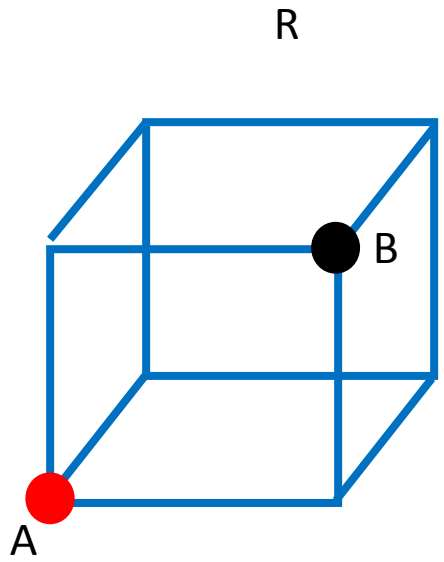
$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50.$$

EM B3 SL

The edges of a cube consist of equal resistors of resistance R, which are jointed at corners. Let a battery be connected to two opposite corners of a face of the cube (A and B). What is the effective resistance.

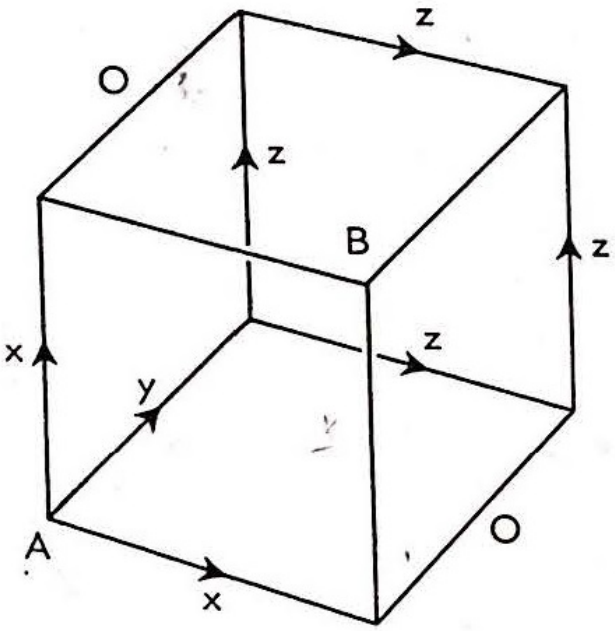


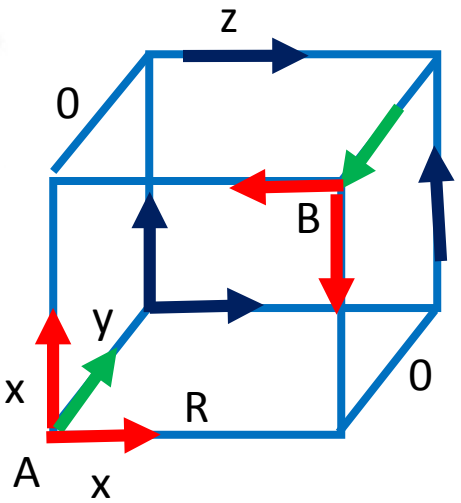
EM B3 SL

1. From the symmetry of the cube the currents in the resistors must be as shown in the figure. Conservation of current at the corners requires:

$$I = 2x + y, \qquad \text{and} \qquad y = 2z,$$

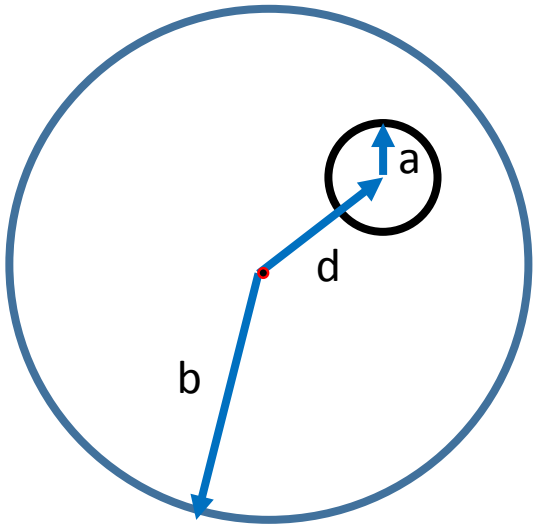
where I is the input current. The requirement that the voltage between A and B be independent of path yields the additional equation $2xR = 2(y + z)R$. These three equations have the solution $x = 3I/8$; $y = I/4$; and $z = I/8$. The resistance between A and B is $(2xR/I)$; therefore $R_{AB} = 3R/4$.





EM B4 SL

An eccentric hole of radius a is bored parallel to the axis of a right circular cylinder of radius b (b>a).The two axes are at a distance d apart. A current of I amperes flows in the cylinder. What is the magnetic field at the center of the hole.



EM B4 SL

The linearity of Maxwell's equations allows us to think of the magnetic field as arising from two current densities:

(1) A current density $j = I/\pi(b^2-a^2)$, carried by the cylinder of radius b, and

(2) a current density $-j$ carried by a cylinder of radius a.

The sum of the current densities (1) and (2) is the current distribution of the bored-out cylinder. From Ampère's circuital law $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \iint \mathbf{J}\cdot d\mathbf{S}$, one finds that (1) produces a magnetic field $B = \mu_0 Id/2\pi(b^2-a^2)$ at the center of the hole, while (2) produces no magnetic field at the center of the hole. The resultant magnetic field is thus given by $B = \mu_0 Id/2\pi(b^2-a^2)$.

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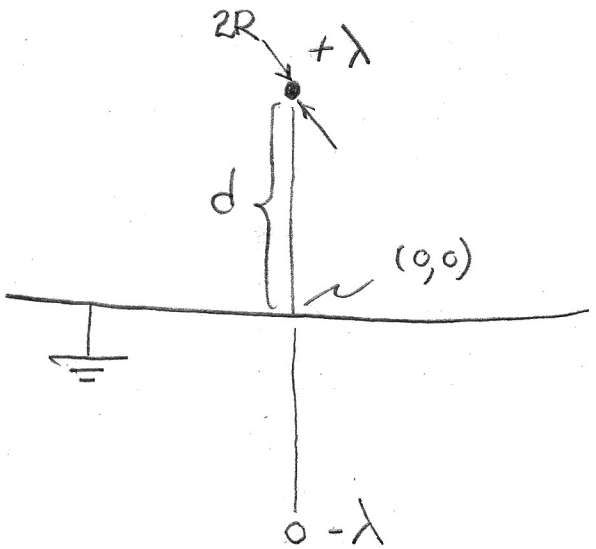
Electricity & Magnetism B1



The field between the wire & the plate equals the field due to the wire & its image charge below the plane.

$$E = \frac{-2k\lambda}{(d-z)} - \frac{2k\lambda}{(d+z)} = -2k\lambda\left(\frac{1}{d-z} + \frac{1}{d+z}\right)$$

$$V = 2k\lambda \int_0^{d-R} \frac{dz}{(d-z)} + 2k\lambda \int_0^{d-R} \frac{dz}{(d+z)}$$

$$= 2k\lambda\left(-\ln(d-z) + \ln(d+z)\right)\Big|_0^{d-R}$$

$$= 2k\lambda \ln\left(\frac{d+z}{d-z}\right)\Big|_0^{d-R}$$

$$= 2k\lambda \ln\left(\frac{2d-R}{R}\right) \simeq 2k\lambda \ln\left(\frac{2d}{R}\right)$$

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Electricity & Magnetism B2

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

$$V = -\int_{0.02m}^{0.01m} \frac{kQ}{\epsilon r^2}\,dr = \frac{k}{\epsilon}Q\left(\frac{1}{r}\right)\Big|_{0.02m}^{0.01m}$$

$$= \frac{8.99\times10^9}{2}\,Nm^2/C^2 \times Q \times 50m^{-1}$$

$$= 2.25\times10^{11}\,Q\,\frac{Nm}{C^2}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{\epsilon}\cdot 4.5\times10^{11}\,Q\,\frac{Nm}{C^2}}$$

$$= \frac{\epsilon}{4.5}\times10^{-11}\,F$$

$$U = \frac{1}{2}Q^2/C$$

$$U_{full} - U_{empty} = \frac{1}{2}\times25\times10^{-12}C^2\left(\frac{1}{\frac{2}{4.5}\times10^{-11}} - \frac{1}{\frac{1}{4.5}\times10^{-11}}\right)$$

$$= 1.25\left(\frac{4.5}{2} - 4.5\right) = -2.81\,J$$

$$W = -\Delta U = \boxed{+2.81\,J}$$

- To remove the oil requires pulling polarized pluses away from the outer wall and polarized minuses away from the inner wall. This requires positive work because we're separating charges that like each other.
- NO $W \propto Q^2$