Thermodynamics solutions

TH A1

Easy: First Law

You do 25 kJ of work on a system consisting of 3.0 kg of water by stirring it with a paddle wheel. During this time, 63 kJ of heat is released by the system due to poor thermal insulation. What is the change in the internal energy of the system?

ΔU is found using the First Law of thermodynamics

$$\Delta U = Q_{in} + W_{on}$$

Heat is released by the system: $Q_{in} = -63\,kJ$

The work is done on the system: $W_{on} = +25\,kJ$

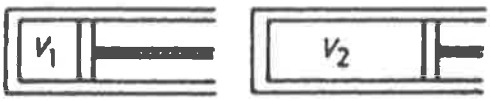
Then $\Delta U = (-63\,kJ) + 25\,kJ = -38\,kJ$

(overall system is losing internal energy)

TH A2

Easy: Entropy

N atoms of a perfect gas are contained in a cylinder with insulating walls, closed at one end by a piston. The initial volume is V_1 and the initial temperature T_1. Find the change in entropy that would occur if the volume were suddenly increased to V_2 by withdrawing the piston.



The gas does no work when the piston is withdrawn rapidly. Also, the walls are thermally insulating, so the internal energy of the gas does not change, i.e. $\Delta U = 0$. Since the internal energy of an ideal gas is only dependent upon temperature T, the change in temperature is 0, i.e. $T_2 = T_1$. As for the pressure, $P_2/P_1 = \frac{V_2}{V_1}$. The increase in entropy then is

$$S_2 - S_1 = \int_{V_1}^{V_2} \frac{p\,dV}{T} = Nk \ln \frac{V_2}{V_1}$$

TH A3

Easy: Adiabatic process

Compression in a diesel engine occurs quickly enough so that very little heating of the environment occurs, and this the process may be considered adiabatic. If a temperature of 500 °C is required for ignition, what is the compression ratio? Assume that the air can be treated as an ideal gas with γ=1.4, and the temperature is 20 °C before compression.

For an adiabatic process, $PV^{\gamma} = \text{const}$, or $TV^{\gamma-1} = \text{const}$.

We use the latter one.

Denote T_1, V_1 and T_2, V_2 to be the temperature and volume at the beginning and the end of the piston stroke. Then:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

The compression ratio V_1/V_2 is

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1} = \left(\frac{773\,K}{293\,K}\right)^{1/0.4} = 11$$

Thermodynamics and Statistical Mechanics 1

Monday, July 24, 2017 12:10 AM

TH A4

P1

easy

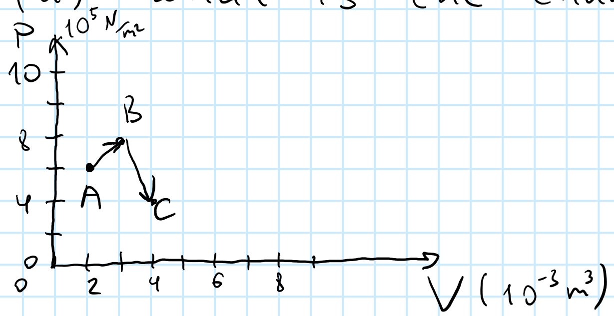
An internal energy of an ideal gas is given by $E = \frac{5}{2} nRT$. A mole of this gas is taken quasistatically from state A to state B, and then from state B to state C along the line paths shown in the figure.

(a) What is the molar heat capacity at constant volume?

(b) What is the work done by the gas in the process $A \to B \to C$?

(c) What is the heat absorbed by the gas in the process?

(d) What is the change of entropy in the process?



Solution.

(a) $$C_v = \frac{\partial E}{\partial T} = \frac{5}{2} R$$

(b) Integrate area under the curve,

$$W = \int P \cdot dV = 1300 \text{ joules.}$$

(c) First, lets find the energy change:

$$\Delta E = C_v \Delta T = \frac{5}{2} R (T_c - T_A) = \frac{5}{2} (P_c V_c - P_A V_A) = 1500 \text{ joules}$$

$$Q = \Delta E + W = 1500 + 1300 = 2800 \text{ joules}$$

Thermodynamics and Statistical Mechanics 2

Thursday, May 04, 2017 12:10 AM

(d) $$\Delta S = \int \frac{dQ}{T} = \int C_v \frac{dT}{T} + \int \frac{P\,dV}{T} = C_v \ln \frac{T_c}{T_A} + R \ln \ln \frac{V_c}{V_A}$$

$PV = nRT$

$$= \frac{5}{2} R \ln \frac{12 \times 10^2 / R}{6 \times 10^2 / R} + R \ln \frac{3 \times 10^3}{10^3}$$

$$= 23.6 \text{ joules}/K$$

TH B4

P2 hard

The free expansion of a gas is a process where the total energy E is constant. Find the following quantities:

(a) $(\partial T / \partial V)_E$ in terms of p, T, $(\partial p / \partial T)_v$, C_v.

(b) $(\partial S / \partial V)_E$ in terms of p and T.

(c) Using (a) and (b) calculate the temperature change in free expansion from V_1 to V_2 for van der Waals gas.

Hint: use $dF = -S\,dT - p\,dV$; $p = \dfrac{\nu R T}{V - \nu b} - \dfrac{\nu^2 a}{V^2}$

Solution.

(a) $$dE = \left(\frac{\partial E}{\partial V}\right)_T dV + \left(\frac{\partial E}{\partial T}\right)_V dT$$

$$\left(\frac{\partial E}{\partial T}\right)_V = C_v, \quad \begin{cases} dE = T\,dS - p\,dV \\ dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \\ \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \end{cases} \Rightarrow \left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{(\partial E/\partial V)_T}{(\partial E/\partial T)_V} = -\frac{T\left(\frac{\partial P}{\partial T}\right)_V - P}{C_v}$$

TH B1

Difficult: Probability

This probability question is known to be asked at Wall Street interviews. The original formulation is kept. “Let’s play a game of Russian roulette. You are tied to your chair. Here’s a gun, a revolver. Here’s the barrel of the gun, six chambers, all empty. Now watch me as I put two bullets into the barrel, into two adjacent chambers. I close the barrel and spin it. I put a gun to your head and pull the trigger. Click. Lucky you! Now I’m going to pull the trigger one more time. Which would you prefer: that I spin the barrel first or that I just pull the trigger?”

In case if the barrel is spun again, the (unconditional) probability of a bad outcome is $\frac{2}{6} = \boxed{\frac{1}{3}}$

If the barrel is not spun again, the probability becomes conditional. Let us introduce:

A: event that slot X has a bullet.

B: event that slot next to X, counterclockwise, is empty.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

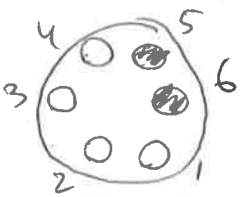
$P(B) = 4/6$ (general probability of a free slot)

$P(A \cap B) = 1/6$ (only one slot out of all has a bullet and an empty slot next to it counter clockwise)

Then $P(A|B) = 1/6 \,/\, 4/6 = \boxed{1/4}$ of bad outcome chance.

i.e., do not spin

Alternative: ~~After~~ In first attempt, we are in 1, 2, 3, or 4. If we do not spin, only one case out of (1)–(4) leads to bad outcome, $P = \boxed{1/4}$ on the next trigger pull.



TH B2

Difficult: Heat engines

Given are a 1.0 kg of water at 100 °C and a very large block of ice at 0 °C. A reversible heat engine absorbs heata from the water and expels heat to the ice until work can no longer be extracted from the system. At the completion of the process, how much work has been done by the engine?

Because the block of ice is large, we can assume its temperature to be constant. In the process the temperature of the water gradually decreases. When work can no longer be extracted from the system, the efficiency of the cycle is zero.

$$\eta = 1 - \frac{T_{ice}}{T} = 0\% \Rightarrow T = T_{ice} = 0°C$$

i.e. the final temperature of the water is 0 °C.

The heat absorbed by the ice block is

$$Q_2 = \int (1 - \eta(T)) dQ = m C_v \int_{273}^{373} \frac{273}{T} dT = m C_v 273 \ln \frac{373}{273}$$

$m = 1 kg$, $C_v = 4184$ J/kg for water $\quad Q_2 \approx 358$ kJ

The heat lost by the water:

$$Q_1 = m C_v \Delta T = 418 \text{ kJ}$$

The work done by the engine

$$W = Q_1 - Q_2 = 418 \text{ kJ} - 358 \text{ kJ} \approx \underline{60 \text{ kJ}}$$

TH B3

Difficult: Thermodynamic potentials

An experimentalist determines the heat capacity of a substance to obey the empirical relation $C_V(T,V) = \alpha T^2 V^3$, where α is a constant. The experimentalist also finds the entropy and energy to be zero at absolute zero for all volumes. Find the expression for the Helmholtz free energy F(T, V) for a system with a fixed number of particles.

Definition of constant volume heat capacity.

$$c_V = \left(\frac{\partial U}{\partial T}\right)_V \qquad (1)$$

Generally, $dU = TdS - pdV$, ~~but~~ but at $V = \text{const}$
$$dU = TdS.$$

so
$$c_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad (2)$$

Integrate (1) and (2):

$$U = \int c_V\, dT = \int \alpha T^2 V^3 dT = \left(\frac{\alpha}{3}\right) T^3 V^2 + \text{const}(V)$$

$$S = \int \frac{c_V}{T} dT = \left(\frac{\alpha}{2}\right) T^2 V^2 + \text{const}(V)$$

Since $U\big|_{T=0} = 0$ and $S\big|_{T=0} = 0$, the "const" above are zero.

$$\boxed{U(T,V) = \frac{\alpha}{3} T^3 V^2}, \quad \boxed{S(T,V) = \left(\frac{\alpha}{2}\right) T^2 V^2}$$

Helmholtz free energy:

$$F = U - TS = \frac{\alpha}{3} T^3 V^2 - T\frac{\alpha}{2} T^2 V^2 = \underline{-\frac{\alpha}{6} T^3 V^2}$$

Thermodynamics and Statistical Mechanics 2

Thursday, May 04, 2017 12:10 AM

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Solution.

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$$\left(\frac{\partial E}{\partial T}\right)_V = C_v, \quad \begin{cases} dE = T\,dS - p\,dV \\ dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \\ \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \end{cases} \Rightarrow \left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{(\partial E/\partial V)_T}{(\partial E/\partial T)_V} = -\frac{T\left(\frac{\partial P}{\partial T}\right)_V - P}{C_v}$$

Thermodynamics and Statistical Mechanics 3

Thursday, May 04, 2017 12:10 AM

(b) $dE = T\,dS - p\,dV$

$\Downarrow$

$$0 = T\left(\frac{\partial S}{\partial V}\right)_E - p \implies \left(\frac{\partial S}{\partial V}\right)_E = \frac{p}{T}$$

(c) For van der Waals gas:

$$p = \frac{\nu R T}{V - \nu b} - \frac{\nu^2 a}{V^2}$$

$$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{\nu^2 a}{V^2 \cdot C_V}$$

$$T_2 - T_1 = \int_{V_1}^{V_2} \left(\frac{\partial T}{\partial V}\right)_E dV = -\frac{a\nu^2}{C_V} \int_{V_1}^{V_2} \frac{dV}{V^2} = \frac{a\nu^2}{C_V}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

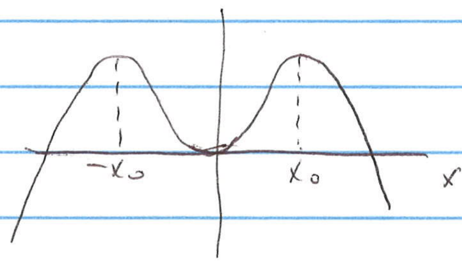
CM A1

Mechanics easy — Ilya F.

1. $F(x) = -Ax + Bx^3$

(a) $V(x) = -\int F(x)dx = \frac{1}{2}Ax^2 - \frac{1}{4}Bx^4$ (take const = 0)

$x_0 = \sqrt{\frac{A}{B}}$



(b) $E = \frac{mv_0^2}{2} = \frac{mv^2}{2} + V(x)$ (conservation of energy)

$$v(x) = \left[v_0^2 + \frac{2}{m}\left(\frac{1}{2}Ax^2 - \frac{1}{4}Bx^4\right)\right]^{1/2}$$

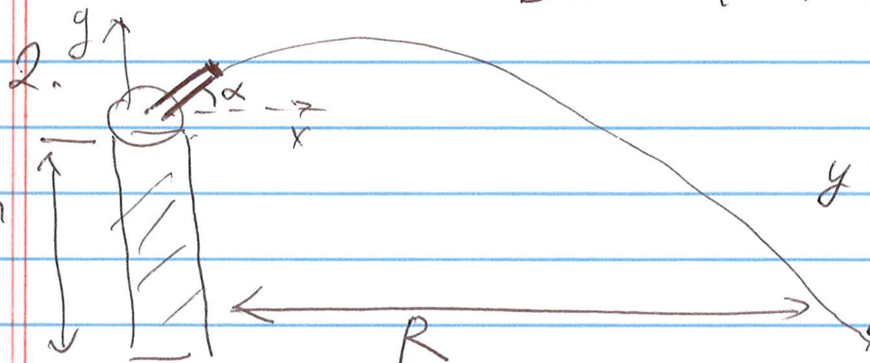
(c) $E < V(x_0)$

$$\frac{mv_0^2}{2} < \frac{1}{2}A\frac{A}{B} - \frac{1}{4}B\left(\frac{A}{B}\right)^2 = \frac{1}{4}\frac{A^2}{B}$$

$$v_0^2 < \frac{1}{2m}\frac{A^2}{B} \qquad |v_0| < \frac{A}{(2mB)^{1/2}}$$

CM A2

2.



$x = v_0 t\cos\alpha$

$y = v_0 t\sin\alpha - \frac{gt^2}{2}$

eliminate t:

$$y = x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$

at $x = R$ $y = -h$

$$-h = R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}$$

$$v_0^2 = \frac{gR^2}{2\cos^2\alpha\,(R\tan\alpha + h)}$$

Mechanics 2

Tuesday, July 25, 2017 1:00 PM

$$\frac{dx}{dt} = \sqrt{2GM}\sqrt{\frac{1}{x}-\frac{1}{r}} \Rightarrow \int dt = \frac{1}{\sqrt{2GM}}\int_0^r \frac{dx}{\sqrt{\frac{1}{x}-\frac{1}{r}}}$$

Substitute $x = r\cos^2\theta$, $dx = -2r\sin\theta\cos\theta\, d\theta$

$$t = \int dt = -2\sqrt{\frac{r^3}{2GM}}\int_{\pi/2}^{0}\cos^2\theta\, d\theta = -\sqrt{\frac{r^3}{2GM}}\int_{\pi/2}^{0}(1-\cos 2\theta)\,d\theta$$

$$= \frac{\pi}{2\sqrt{2}}\sqrt{\frac{r^3}{GM}}$$

Period of earth's orbit $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$t = \frac{T}{4\sqrt{2}} = \frac{365}{4\sqrt{2}} = 64.53 \text{ days}$$

CM A3

(P3) easy A rocket of weight 5000 kg is launched vertically. The ejected gas has speed 1000 m/s. What is the necessary rate of ejection to

(a) support the weigt of the rocket

(b) give it upward acceleration of 2g.

Solution. (a) Let x kg of gas is ejected per second.

$$x \cdot v_e = M_0 \cdot g$$

$$x = \frac{5000 \cdot 9.8}{1000} = 49 \text{ kg/s}$$

(b) Totally we need $F = M_0 g + M_0 \cdot 2g$

$$x' = \frac{3 \cdot 5000 \cdot 9.8}{1000} = 147 \text{ kg/s}$$

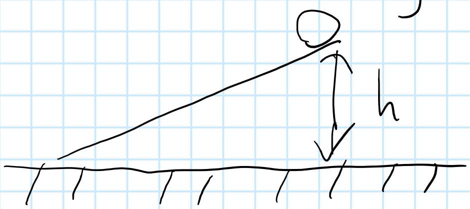
Mechanics 3

Tuesday, July 25, 2017 1:00 PM

CM A4

(P4) easy

A cylinder of mass m and radius R rolls along inclined plane. Find the speed of its center at the bottom if the cylinder is initially launched at hight h. There is no slipping.



Solution. The kinetic energy of the cylinder:

$$K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \frac{v^2}{R^2} = \frac{3}{4} m v^2$$

From the conservation of energy:

$$\frac{3}{4} m v^2 = m g h \quad \Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

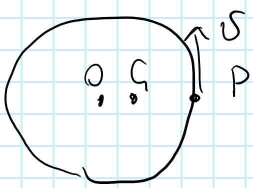
Mechanics 1

CM B1 Tuesday, July 25, 2017 1:00 PM

P1 hard

A circular ring of mass M and radius r lies on a smooth horizontal surface. An insect of mass m sits on it and crawls round the ring with a uniform speed v relative to the ring. Find the angular velocity of the ring.

Solution.



Angular momentum is conserved.

$$m\cdot PG\,(v - PG\cdot\omega) - I_G\cdot\omega = 0$$

G – center of mass.

$$PG = \frac{Mr}{M+m}, \quad OG = \frac{mr}{M+m}$$

$$I_G = Mr^2 + M(OG)^2$$

$$\Downarrow$$

$$\omega = \frac{m\cdot PG\cdot v}{m\,PG^2 + I_G} = \frac{mv}{(M+2m)r}$$

CM B3

P2 hard

Suppose that the earth suddenly stopped orbiting the sun. Find the time it would take for the earth to fall into the sun.

Solution. Acceleration: $\dfrac{dv}{dt} = -\dfrac{GM}{x^2}$ M - the earth mass.

$$v\cdot dv = -\frac{GM\,dx}{x^2}$$

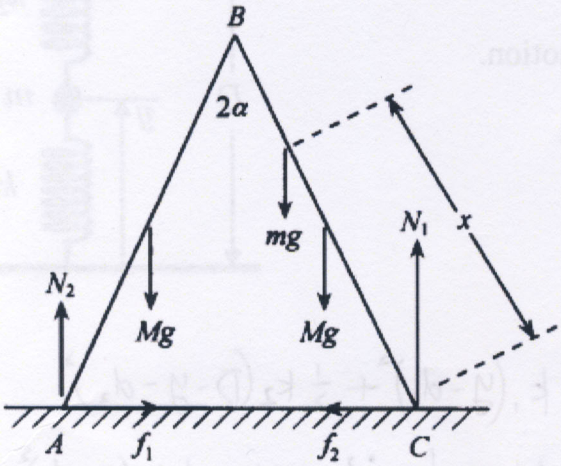
$$\int v\,dv = \frac{v^2}{2} = -GM\int\frac{dx}{x^2} + C$$

Initially $v=0$, $x=r$ $\Rightarrow$ $C = -\dfrac{GM}{r}$

$$v = \sqrt{2GM}\sqrt{\frac{1}{x} - \frac{1}{r}}$$

CM B2

1. Two identical uniform ladders with length 2a and mass M are connected via a frictionless hinge and put on the ground. The coefficient of friction between the ladders and the ground is μ. And the angle between two ladders is 2α. What coefficient of friction (μ) is required so that a person with mass m can climb to the top from either side safely?



$$\begin{cases} ① & f_1 - f_2 = 0 \\ ② & N_1 + N_2 - 2Mg - mg = 0 \end{cases}$$

$$\begin{cases} ③ & f_1 \cdot 2a \cdot \cos\alpha + Mg \cdot a \cdot \sin\alpha - N_2 \cdot 2a \cdot \sin\alpha = 0 \\ ④ & N_1 \cdot 2a \cdot \sin\alpha - f_2 \cdot 2a \cdot \cos\alpha - Mg \cdot a \cdot \sin\alpha - mg \cdot (2a - x) \cdot \sin\alpha = 0 \end{cases}$$

③ + ④ $\Rightarrow N_1 - N_2 = mg\left(1 - \frac{x}{2a}\right)$ (*)

② $\Rightarrow N_1 + N_2 = (2M + m)g$ (**)

from (*) & (**), easily get $\begin{cases} N_1 = Mg + \left(1 - \frac{x}{4a}\right)mg \\ N_2 = Mg + \frac{x}{4a}mg \end{cases}$

$\because 0 \le x \le 2a \quad \therefore N_1 \ge N_2$

to prevent the ladders from moving, we need $\begin{cases} f_1 \le \mu N_1 \\ f_2 \le \mu N_2 \end{cases}$

since $f_1 = f_2$, $N_1 \ge N_2$, we only need $f_1 = f_2 \le \mu N_2$

from ③, $f_1 = \left(N_2 - \frac{1}{2}Mg\right)\tan\alpha = \left(Mg + \frac{x}{4a}mg - \frac{1}{2}Mg\right)\tan\alpha$

$\therefore \mu \ge \frac{f_1}{N_2} = \left(1 - \frac{M}{2M + \frac{x}{2a}m}\right)\tan\alpha$, for $x \in [0, 2a]$

the max of $1 - \frac{M}{2M + \frac{x}{2a}m}$ is $\frac{M+m}{2M+m}$, so $\mu \ge \frac{M+m}{2M+m} \cdot \tan\alpha$

Mechanics 1

CM B1 Tuesday, July 25, 2017 1:00 PM

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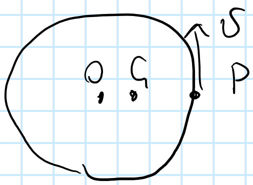

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Mechanics 2

Tuesday, July 25, 2017 1:00 PM

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easy

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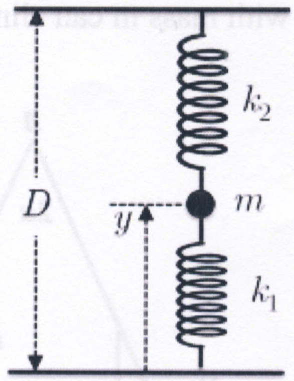
CM B4

2. As shown in the figure, a block with mass m is hung between two springs k_1, k_2 with free length d_1, d_2 respectively ($D>d_1+d_2$). Suppose the mass of each spring is negligible and the block can only move vertically. Gravitational acceleration is g.

(a) Write down the Lagrangian.

(b) Find the equation of motion.

(c) Find the balance point.



(a.) $T = \frac{1}{2} m \dot{y}^2$

$$V = mgy + \frac{1}{2} k_1 (y-d_1)^2 + \frac{1}{2} k_2 (D-y-d_2)^2$$

$$\therefore L(y,\dot{y}) = T - V = \frac{1}{2} m \dot{y}^2 - mgy - \frac{1}{2} k_1 (y-d_1)^2 - \frac{1}{2} k_2 (y+d_2-D)^2$$

(b.) $$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = 0$$

$$m\ddot{y} + mg + k_1(y-d_1) + k_2(y+d_2-D) = 0$$

$$\therefore \ddot{y} + \frac{k_1+k_2}{m} y + mg - k_1 d_1 + k_2(d_2-D) = 0$$

(c.) $\ddot{y} = 0$

$$y = \frac{m}{k_1+k_2} \cdot [k_1 d_1 + k_2 (D-d_2) - mg]$$