- A1.
- 1. Find the angle θ at which the rope will settle. Express your answers in terms of the given variables M, g, and a as needed.

(1)

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Solution: Applying Newton's second law to mass M,

$$\sum F_y = 0 = T \cos \theta - Mg$$
$$\sum F_y = Ma = T \sin \theta$$

Solving for θ yields

$$\tan \theta = \frac{a}{g}$$

+5

+10

2. What will the tension T of the rope be once it settles into this angle? Express your answers in terms of the given variables M, g, and a as needed.

Solution: Taking the result for θ obtained in the previous step and substituting into the equations of motion yields

$$f(0) T = M\sqrt{g^2 + a^2}$$
From (1) $T = Mg = Mg \sec 0 = Mg [f + fan 0 = Mg [f + a/g^2]$
A2.
$$= M \cdot a^2 + g$$
Solution: Applying Newton's second law to masses 1 and 2 yields:

where we have used that the masses have a common angular acceleration, $\alpha = \omega^2$, to express the left-hand side in terms of the angular rotation ω . It follows that

$$T_{B} = m_{2}\omega^{2}d$$

$$T_{A} = (2m_{2} + m_{1})\omega^{2}d$$

$$T_{B} - \overline{I_{A}} = -m_{1}\omega^{2}d$$

$$T_{B} - \overline{I_{A}} = -m_{1}\omega^{2}d - 2m_{2}\omega^{2}d$$

$$T_{B} - \overline{I_{B}} = -2m_{2}\omega^{2}d$$

$$T_{B} = 0 - \overline{I_{A}} = -m_{1}\omega^{2}d - 2m_{2}\omega^{2}d$$

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A3. Solution:

$$L = T - U$$
$$= \frac{1}{2}m\dot{x}^2 - U(x)$$
$$= \frac{1}{2}m\dot{x}^2 - \frac{k}{x}e^{-t/\tau},$$

where the potential U(x) is determined from F(x,t),

$$\int U(x) = -\int F dx = \frac{k}{x} e^{-t/\tau}$$

The Hamiltonian is $H = p_x \dot{x} - L$ with canonical momentum given by

$$+ \int p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

Substituting and simplifying yields

$$H = p_x \dot{x} - L$$

$$= \frac{p_x^2}{2m} + \frac{k}{x} e^{-t/\tau}$$

$$H = nust be expressed$$

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The Hamiltonian is the total energy. $\frac{\partial L}{\partial t} \neq 0$, thus energy is not conserved. $\neq 5$

Ereliminary Exam problems [1] May By Maral Ngoko classical Pb #1: Easy - Mechanics Adnder the critical circumstance that the astronaut just starts to slide, one has: $f_s = m a_h$ where $f_s = \mu_s N = \mu_s mg$ is the static friction $a_n = \frac{v^2}{R} = R w^2$ is the centripetal acceleration IR DW +10 $\implies mRW^2 = \mu_s mg$ $R = \frac{M_s g}{w^2}$ where g on that planet has to be determined. $vy = v_{oy} + a_y t \implies a_y t_{og} = \frac{v_y - v_{oy}}{t}$ +10 $= q = |ay| = 3.0 \text{ m/s}^2 = \frac{-30 \text{ m/s}}{20.5}$ Nog 1 : Vy $R = \frac{0.4 \times 125}{2^2} = 0.30 m = 30 m + 5$ R is independent on m.

A4

Solution: The position of the stone as a function of time is

$$s(t) = h - \frac{1}{2}gt^2,$$

while the position of the rocket can be determined starting from $a_y(t)$:

The maximum height the rocket attains occurs at time T when $v_y(T) = 0$.

$$\begin{array}{rcl}
 & v_y(T) &= & 0 = T \left(A - \frac{1}{2} BT \right) \\
 & T &= & 0, \ \frac{2A}{B}
\end{array}$$

We want the latter (non-zero) time. The initial height h can be solved by equating s(t) = r(t) at time t = T.

$$r(T) = r_{\max} = \frac{2A^3}{3B^2}$$

$$s(T) = h - \frac{1}{2}g\left(\frac{2A}{B}\right)^2$$

$$h = \frac{2A^3}{3B^2} + \frac{1}{2}g\left(\frac{2A}{B}\right)^2$$

$$= \frac{2A^2}{B^2}\left(g + \frac{A}{3}\right)$$

+6

Solution: For simple projectile motion under constant acceleration and ignoring air resistance, the equations of motions can be solved yielding

$$+5 \qquad \qquad x(t) = v_0 \cos \theta t \\ y(t) = v_0 \sin \theta t - \frac{1}{2}gt^2,$$

where v_0 is the initial speed of the projectile, θ is the angle above the horizontal at which the projectile is fired, and g is the constant acceleration due to gravity near the surface of the Earth. The distance of the projectile from the launch point is

$$d^2 = x^2(t) + y^2(t)$$
$$= t^2 \left[v_0^2 - v_0 \sin \theta g t + \left(\frac{gt}{2}\right)^2 \right]$$

The distance from the origin increases at least until the projectile reaches its maximum height. Thus, $\frac{d^2}{dt^2}(d^2) > 0$ shortly after t = 0 and the distance increases until $\frac{d^2}{dt^2}(d^2)$ becomes negative. So, let's find the turning point, i.e. when $\frac{d^2}{dt^2}(d^2) = 0$.

$$\frac{d^2}{dt^2}(d^2) = 2v_0^2 t - 3v_0 \sin\theta g t^2 + g^2 t^3 = 0$$

= $t \left(2v_0^2 - 3v_0 \sin\theta g t + g^2 t^2\right)$
 $t = 0, \frac{3v_0 \sin\theta}{2g} \pm \frac{v_0}{2g} \sqrt{9\sin^2\theta - 8}$

The first solution is trivial so let's focus on the others. These solutions are real only when the term under the square root is non-negative, i.e.

$$9\sin^2\theta - 8 \ge 0$$

$$\sin\theta \ge \frac{2\sqrt{2}}{3}$$

Imaginary time is unphysical and thus for

$$\oint \int \theta < \sin^{-1} \frac{2\sqrt{2}}{3}$$

there is no turning point and $\frac{d^2}{dt^2}(d^2) > 0$.

B2.

B3.

(b)
(a) for rolling sphere 2nd Neuton tau

$$\frac{mv^{1}}{R+a} = mg\cos\theta - N$$

$$\frac{mv^{1}}{R+a} = mg\cos\theta - N$$

$$\frac{mv^{1}}{L+a} = mg\cos\theta - N$$

$$\frac{mv^{1}}{L+a} = mg\cos\theta - (i)$$
(conservation of Calryy (no slipping))

$$mg(R+a)(4-\cos\theta) = \frac{mv^{1}}{L} + \frac{Tw^{1}}{2}$$

$$w = \frac{w}{a} (nc slipping) \quad J = \frac{2}{5} mat$$

$$mg(R+a)(4-cos\theta) = \frac{m}{L} (v^{1} + \frac{2}{5} v^{1})$$

$$drom(1) \quad \frac{3v^{1}}{2va} = g\cos\theta$$

$$g(t+cos\theta) = g\frac{d}{2} cos\theta + \frac{7}{5}$$

$$(c)\theta = \frac{1}{1+\frac{7}{10}} = \frac{10}{7} \quad \theta = 54.0^{\circ}$$

$$(c) \quad L = T - V = \frac{m}{L} (v^{1} + \frac{2}{5} v^{1}) - mg(R+a)(t+cos\theta)$$

$$L = \frac{7}{10} mR^{1}\theta^{1} - my(R+a)(t-cos\theta)$$

+8

(3) Small oscillations Oscil = Durid 20 (rad) $= \left[\dot{\theta} + \frac{2}{3} \left(\frac{g}{R-r} \right) \theta = 0 \right] \begin{array}{c} \text{equation} \\ \text{characteristic } g \\ \text$ simple harmonic with frangular frequency $W^2 = \frac{2}{3} \frac{g}{R-r}$ and thus period : $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{3(R-r)}{2g}}$ +5 $T = \pi \sqrt{\frac{6(R-r)^{7}}{g}}$ Also accept approximating the Lagrangian for small I and then Finding E-L equations

A1. A 600 g copper ball has a temperature of 700°C when it is placed in 3.00 kg of water at a temperature of 20°C. Calculate the temperature (in °C) of the system when equilibrium has been reached? Assume the system is thermally insulated. Data: $C_{water} = 4.18 \text{ Jg}^{-1} \text{ K}^{-1}$, $C_{Cu} = 0.39 \text{ Jg}^{-1} \text{ K}^{-1}$

Answer

Heat entering water = heat leaving copper \sum	6	po;nt3
$m_{\rm w}C_{\rm w}(T_{\rm f}-T_{\rm w}) = m_{\rm Cu}C_{\rm cu}(T_{\rm Cu}-T_{\rm f}) \implies m_{\rm w}C_{\rm w}T_{\rm f} - m_{\rm w}C_{\rm w}T_{\rm w} = m_{\rm Cu}C_{\rm cu}T_{\rm cu} - m_{\rm cu}C_{\rm cu}T_{\rm f} \implies $	6	points
$T_{\rm f} = \frac{m_{\rm w}C_{\rm w}T_{\rm w} + m_{\rm Cu}C_{\rm cu}T_{\rm Cu}}{m_{\rm w}C_{\rm w} + m_{\rm Cu}C_{\rm cu}} = 305 \text{ K} = 32.5^{\circ}\text{C}$	13	, points

A2. One mole of diatomic ideal gas ($C_V = 2.5 nR$) performs a transformation from an initial state for which temperature and volume are, 290 K and 30000 ml to a final state in which temperature and volume are 310 K and 16000 ml. The transformation is represented on the (V, P) diagram by a straight line. Find the work performed and the heat absorbed by the system.

Solution:

A straight line in (P,V) diagram: $P = P1 + \frac{(V-V1)\Delta P}{\Delta V}$, $\mathcal{F}_{0} = \mathcal{F}_{0} =$ $\frac{\Delta U = CV\Delta T = 416J}{Q = \Delta U + W = -1275} \int 12 points$

A3. Consider a gas with the following speed probability distribution f(v) = Awhen $0 < v < v_0$, = 0 otherwise. Find (a) average speed, (b) rms speed in the 1 dimensional, 2 dimensional, and 3 dimensional cases.

Solution:

1D:
$$\langle v \rangle = \frac{\int f(v)vdv}{\int f(v)dv} = v_0/2, v_{rms}^2 = \frac{\int f(v)v^2dv}{\int f(v)dv} = v_0^2/3$$
 for the points
2D: $\langle v \rangle = \frac{\int f(v)vd^2v}{\int f(v)d^2v} = \frac{2}{3}v_0, v_{rms}^2 = \frac{\int f(v)v^2d^2v}{\int f(v)d^2v} = v_0^2/2$ for the points
3D: $\langle v \rangle = \frac{\int f(v)vd^3v}{\int f(v)d^3v} = \frac{3}{4}v_0, v_{rms}^2 = \frac{\int f(v)v^2d^3v}{\int f(v)d^3v} = v_0^23/5$ for the point of all coefficients are correct

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A4. An ideal gas (γ = 1.4) expand in an adiabatic process to 10 times of its original volume. If the initial temperature is 0 °C, and the initial pressure 1 atm find the final temperature.

B1. An ideal gas is expanded adiabatically from (p_1, V_1) to (p_2, V_2) . It is then compressed isobarically to (p_2, V_1) . Finally, the pressure is increased to p_1 at constant volume V_1 . Show that the efficiency of the cycle is

$$\eta = 1 - \gamma \frac{V_2 / V_1 - 1}{p_1 / p_2 - 1}$$



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Because AB is adiabatic and an ideal gas has the equations pV = nkT and $C_p = C_v + R$, we get

$$\int_{AB} p dV = -\int_{AB} C_v dT = -C_v (T_2 - T_1)$$

= $\frac{1}{1 - \gamma} (p_2 V_2 - p_1 V_1) .$

During the CA part of the cycle the gas absorbs heat

$$Q = \int_{CA} T dS = \int_{CA} C_v dT = C_v (T_1 - T_2)$$

= $\frac{1}{1 - \gamma} V_1 (p_2 - p_1)$.

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Hence, the efficiency of the engine is

$$\eta = \frac{W}{Q} = 1 - \gamma \frac{\frac{V_2}{V_1} - 1}{\frac{p_1}{p_2} - 1} \ .$$

B2. The entropy of an ideal gas is $S = n/2 [a + 5R \ln(U/n) + 2R \ln(V/n)]$, where *n* is the mole number, *R* is the universal gas constant, *U* is internal energy, *V* is volume, and *a* is a constant.

- (a) Calculate the constant pressure heat capacity (C_P) and the constant volume heat capacity (C_V).
- (b) Rewrite entropy in (T,V), (T,P), and (P,V) representation.

Solution:

$$C_{V} = T \left(\frac{\partial S}{\partial T}\right)_{V} = T \frac{n}{2} \frac{5R}{U} \left(\frac{\partial U}{\partial T}\right)_{V} = \frac{5}{2} \frac{nRT}{U} C_{V} \quad 5 \text{ G point } \text{So}, C_{V} = \frac{5}{2} nR$$

$$C_{P} = C_{V} + nR = \frac{7}{2} nR$$

$$\mathcal{F} \text{ point } \text{So}, T = \frac{5}{2} nR$$

$$S(T, V) = nc_V \ln \frac{T}{T_o} + nR \ln \frac{V}{V_o} + ns_o.$$

$$Y \text{ point}$$

$$S(T, P) = nc_P \ln \frac{T}{T_o} - nR \ln \frac{P}{P_o} + ns_o.$$

$$Y \text{ point}$$

$$S(P, V) = nc_V \ln \frac{P}{P_o} + nc_P \ln \frac{V}{V_o} + ns_o.$$

$$Y \text{ point}$$

Also accept without P_0 , V_0 , and T_0 .

B3. Consider the adiabatic free expansion of an ideal gas (from volume V to 2V).

- What's the work and heat in the process? (a)
- How does the temperature change? (b)
- Show that this process is irreversible. (Hint: calculate the entropy (c)

difference)

(d) answer (b) if the gas is nonideal.

(a) In this process, there is no work and heat. The internal energy does not change, or the temperature does not change.
(b) Suppose the initial state of the gas can be described as P,V,T, the final state of gas can be described as P/2,2V,T.
(c) So, one can use an isothermal process (reversible).
(c) So, one can use an isothermal process (reversible).

Since dU=0,

$$\Delta S = \frac{1}{T} \int (dU + PdV) = \frac{1}{T} \int PdV = \frac{1}{T} \int (\frac{nRT}{V}) \, dV = nR \ln (2V/V)$$

= nR ln 2 > 0.

Because this process is adiabatic and $\Delta S>0$, it is irreversible.

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6 points (d) If the gas is not ideal, the potential energy of interaction between molecules increases during the expansion, therefore due to conservation of energy the kinetic energy decreases, therefore the temperature slightly decreases

B4.

$$\frac{Thormo}{(B_{1})} (a) p_{0}V_{0} = \frac{L_{0}}{2}V_{1} \rightarrow V_{1} = 2V_{0}$$

$$i : \frac{p_{2}}{2}, V_{1}, T_{0}$$

$$i : \frac{p_{2}}{2}, V_{1}, T_{0}$$