

QM A1

QM A1

$$\Delta x \Delta p \approx \hbar \Rightarrow p \approx \frac{\hbar}{\Delta x} \Rightarrow K = \frac{p^2}{2m_p} \approx \frac{\hbar^2}{2m_p(\Delta x)^2} = 3.69 \times 10^{-13} \text{ J}$$

$$K = 3.69 \times 10^{-13} \text{ J} * \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = \frac{3.69 \times 10^{-13} \text{ J}}{1.602 \times 10^{-19} \text{ J}} \approx 2.3 \text{ MeV}, \text{ binding energy must be of the same order}$$

QM A2

QM A2

a. $\left[\exp(\hat{A}) \right]^\dagger = \left[\hat{I} + \hat{A} + \frac{1}{2!} \hat{A}^2 + \frac{1}{3!} \hat{A}^3 + \dots \right]^\dagger = \left[\hat{I}^\dagger + \hat{A}^\dagger + \frac{1}{2!} (\hat{A}^2)^\dagger + \frac{1}{3!} (\hat{A}^3)^\dagger + \dots \right] =$
 $= \left[\hat{I} + \hat{A} + \frac{1}{2!} \hat{A}^2 + \frac{1}{3!} \hat{A}^3 + \dots \right] = \exp(\hat{A}) \quad \text{Hermitian}$

b. $\left(i[\hat{Q}, \hat{Q}^\dagger] \right)^\dagger = [\hat{Q}, \hat{Q}^\dagger]^\dagger i^\dagger = -i(\hat{Q}\hat{Q}^\dagger - \hat{Q}^\dagger\hat{Q})^\dagger = -i(\hat{Q}^{\dagger\dagger}\hat{Q}^\dagger - \hat{Q}^\dagger\hat{Q}^{\dagger\dagger}) = -i(\hat{Q}\hat{Q}^\dagger - \hat{Q}^\dagger\hat{Q}) = -i[\hat{Q}, \hat{Q}^\dagger]$
not Hermitian

c. $[\hat{x}^n, \hat{p}] \psi = \hat{x}^n \hat{p} \psi - \hat{p} \hat{x}^n \psi = x^n \frac{\hbar}{i} \frac{d}{dx} \psi - \frac{\hbar}{i} \frac{d}{dx} (x^n \psi) = \cancel{x^n} \frac{\hbar}{i} \cancel{\frac{d}{dx}} \psi - \frac{\hbar}{i} n x^{n-1} \psi - \cancel{x^n} \cancel{\frac{\hbar}{i} \frac{d}{dx}} \psi =$
 $= i\hbar n x^{n-1} \psi = (i\hbar n \hat{x}^{n-1}) \psi$

Quantum Solutions

QM A3

Easy

1. (a)

$$\psi(x, 0) = C \sin \frac{2\pi x}{a} \cos \frac{\pi x}{a} = \frac{C}{2} \left[\sin \frac{\pi x}{a} + \sin \frac{3\pi x}{a} \right]$$

$$\psi(x, t) = \frac{C}{2} \left[\sin \frac{\pi x}{a} e^{-iE_1 t/\hbar} + \sin \frac{3\pi x}{a} e^{-iE_3 t/\hbar} \right]$$

(b) $\langle E \rangle = \frac{1}{2} (E_1 + E_3)$ since ψ_1 and ψ_3 have equal amplitudes

$$(c) P_1 = \frac{1}{2}, P_3 = \frac{1}{2} \quad P_n = 0, n \neq 1, 3$$

QM A4

(a)

$$\langle S_x^2 + S_y^2 \rangle = \langle S^2 - S_z^2 \rangle = \hbar^2 [2 - 1] = \hbar^2$$

$$\text{since } \langle S_x^2 \rangle = \langle S_y^2 \rangle, \quad \langle S_x^2 \rangle = \frac{\hbar^2}{2}$$

$$(b) \left\{ \begin{array}{l} \text{outcomes } +\hbar, 0, -\hbar \\ \langle S_x^2 \rangle = \hbar^2 P(S_x=+\hbar) + 0 \cdot P(S_x=0) + \hbar^2 P(S_x=-\hbar) \end{array} \right.$$

$$\frac{1}{2} = P(S_x=+\hbar) + P(S_x=-\hbar)$$

$$P(S_x=+\hbar) = P(S_x=-\hbar) = \frac{1}{4}$$

$$P(S_x=0) = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$$

Quantum answers

QM B1

Hard 1.

$$\Psi(\vec{r}) = \frac{1}{\sqrt{2}} [\Psi_{n=0}(\vec{r}) - \Psi_{n=1}(\vec{r})]$$

- (a) no for $n=1$ $\Psi_{1,00}$ and $\Psi_{2,10}$ correspond to different energies
yes for $n=2$ $\Psi_{2,00}$ and $\Psi_{2,10}$ have the same energy

(b)

$$\langle H \rangle = \frac{1}{2} \langle \Psi_{n=0} - \Psi_{n=1} | H | \Psi_{n=0} - \Psi_{n=1} \rangle = \\ = \frac{1}{2} [\langle \Psi_{n=0} | H | \Psi_{n=0} \rangle + \langle \Psi_{n=1} | H | \Psi_{n=1} \rangle]$$

for $n=1$ we get

$$\frac{1}{2} (E_1 + \frac{E_1}{4}) = \frac{5}{8} E_1$$

for $n=2$ $\langle H \rangle = \frac{E_2}{4}$

(c) $\langle \vec{L}^2 \rangle = \frac{1}{2} [\langle \Psi_{n=0} | \vec{L}^2 | \Psi_{n=0} \rangle + \langle \Psi_{n=1} | \vec{L}^2 | \Psi_{n=1} \rangle]$
 $= \frac{1}{2} [0 + \hbar^2 \cdot 1 \cdot 2] = \hbar^2$

(d)

$$\langle \vec{r} \rangle = \frac{1}{2} \int \vec{r} (\Psi_{n=0} - \Psi_{n=1})^2 d\vec{r} =$$

$$= - \int \vec{r} \Psi_{n=0} \Psi_{n=1} d\vec{r} \quad (\text{Two other integrals disappear because of parity})$$

in the remaining integral $\langle x \rangle = \langle y \rangle = 0$ because of ϕ integration. The only nonzero component

is

$$\langle z \rangle = - \int z \Psi_{n=0} \Psi_{n=1} d\vec{r} = 3a_0$$

QM B2

Hand 2.

$$(a) \quad S_x = \frac{\hbar}{2} \sigma_x \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad S_x = \frac{\hbar}{2} \lambda$$

$$\lambda = \pm a$$

$$a = \lambda b \rightarrow \lambda = \pm 1$$

$$\text{for } \lambda = +1 \quad S_x = \frac{\hbar}{2} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (\text{normalized})$$

$$\text{for } \lambda = -1 \quad S_x = -\frac{\hbar}{2} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$(b) \quad \langle \sigma_z \rangle = \frac{1}{2} (1+1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2} (1+1) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

$$\langle \sigma_y \rangle = \frac{1}{2} (1+1) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} (1+1) \begin{pmatrix} -i \\ i \end{pmatrix} = 0$$

$$\langle \sigma_z^2 \rangle = \frac{1}{2} (1+1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \quad \langle S_z^2 \rangle = \frac{\hbar^2}{4}$$

$$\langle \sigma_y^2 \rangle = \frac{1}{2} (1+1) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2} (1+1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \quad \langle S_y^2 \rangle = \frac{\hbar^2}{4}$$

$$(c) \quad \langle S_x^2 \rangle + \langle S_y^2 \rangle + \langle S_z^2 \rangle = \frac{3}{4} \hbar^2$$

also follows from $\vec{S}^2 = \hbar^2 S(S+1) \Big|_{S=\frac{1}{2}} = \frac{3}{4} \hbar^2$

$$(d) \quad H = -\mu_x B = -\gamma S_x B$$

eigenstates of S_x are eigenstates of H

energy eigenvalues $\mp \frac{\hbar}{2} \gamma B$

QM B3

$$L_+ |\ell, m\rangle = \hbar \sqrt{(\ell+m+1)(\ell-m)} |\ell, m+1\rangle$$

$$L_- |\ell, m\rangle = \hbar \sqrt{(\ell+m)(\ell-m+1)} |\ell, m-1\rangle$$

ANSWERS

Part a.

$$\langle \psi | \psi \rangle = \sum_{\ell,m} |c_{\ell,m}|^2 = \left| \frac{1}{\sqrt{7}} \right|^2 + |A|^2 + \left| \sqrt{\frac{2}{7}} \right|^2 = |A|^2 + \frac{3}{7} = 1 \Rightarrow |A|^2 = \sqrt{\frac{4}{7}} = \frac{2}{\sqrt{7}}$$

Part b.

$$|\psi\rangle = c_{-1} |1, -1\rangle + c_0 |1, 0\rangle + c_1 |1, 1\rangle$$

$$\langle \psi | = c_{-1} \langle 1, -1 | + c_0 \langle 1, 0 | + c_1 \langle 1, 1 | \quad (\text{because } \forall i : c_i \in \mathbb{R})$$

We have

$$L_+ = L_x + iL_y \quad L_x = \frac{1}{2}(L_+ + L_-)$$

$$L_- = L_x - iL_y \quad L_y = -\frac{1}{2}i(L_+ - L_-)$$

$$\text{We first calculate } \langle \psi | L_+ | \psi \rangle = \langle \psi | L_x | \psi \rangle + i \langle \psi | L_y | \psi \rangle ;$$

$$\text{we note that } L_+ |\psi\rangle = c_{-1} \hbar \sqrt{2} |1, 0\rangle + c_0 \hbar \sqrt{2} |1, 1\rangle$$

$$\langle L_+ \rangle = \left\{ c_{-1} \langle 1, -1 | + c_0 \langle 1, 0 | + c_1 \langle 1, 1 | \right\} \left\{ c_{-1} \hbar \sqrt{2} |1, 0\rangle + c_0 \hbar \sqrt{2} |1, 1\rangle \right\} = c_{-1} c_0 \hbar \sqrt{2} + c_0 c_1 \hbar \sqrt{2} =$$

$$= \frac{2}{7} \hbar \sqrt{2} + \frac{2\sqrt{2}}{7} \hbar \sqrt{2} = \frac{2\sqrt{2} + 4}{7} \hbar$$

$$\langle L_- \rangle = \langle \psi | L_- \psi \rangle = \langle (L_-)^\dagger \psi | \psi \rangle = \langle L_+ \psi | \psi \rangle = \langle \psi | L_+ \psi \rangle^* = \langle L_+ \rangle^* = \langle L_+ \rangle \quad \text{because } \langle L_+ \rangle \in \mathbb{R}$$

So now

$$L_x = \frac{1}{2}(L_+ + L_-) \Rightarrow \langle L_x \rangle = \frac{1}{2}(\langle L_+ \rangle + \langle L_- \rangle) = \frac{1}{2}(\langle L_+ \rangle + \langle L_+ \rangle) = \langle L_+ \rangle = \frac{2\sqrt{2} + 4}{7} \hbar$$

$$L_y = \frac{1}{2i}(L_+ - L_-) \Rightarrow \langle L_y \rangle = \frac{1}{2i}(\langle L_+ \rangle - \langle L_- \rangle) = \frac{1}{2i}(\langle L_+ \rangle - \langle L_+ \rangle) = 0$$

For the z-component and the total angular momentum we simply have

$$\langle L_z \rangle = \sum_{\ell,m} |c_{\ell,m}|^2 m \hbar = |c_{1,-1}|^2 (-1) \hbar + |c_{1,0}|^2 (0) \hbar + |c_{1,+1}|^2 (+1) \hbar = \frac{1}{7} (-\hbar) + \frac{2}{7} \hbar = \frac{1}{7} \hbar$$

$$\langle L^2 \rangle = \sum_{\ell,m} |c_{\ell,m}|^2 \ell(\ell+1) \hbar^2 = |c_{1,-1}|^2 2\hbar^2 + |c_{1,0}|^2 2\hbar^2 + |c_{1,+1}|^2 2\hbar^2 = 2\hbar^2$$

Part c

This probability is $|\langle 1,1 | \psi \rangle|^2 = \left| \langle 1,1 | \sqrt{\frac{2}{7}} | 1,1 \rangle \right|^2 = \frac{2}{7}$

Part d

Applying the raising operator twice leaves us with just an $m = +1$ term:

$$L_+ L_+ |\psi\rangle = \frac{1}{\sqrt{7}} L_+ L_+ |1,-1\rangle = \frac{1}{\sqrt{7}} \hbar \sqrt{2} L_+ |1,0\rangle = \frac{1}{\sqrt{7}} \hbar \sqrt{2} \hbar \sqrt{2} |1,1\rangle = \frac{2}{\sqrt{7}} \hbar^2 |1,1\rangle$$

$$\text{This has } \langle 1,m | L_+ L_+ | \psi \rangle = \frac{2}{\sqrt{7}} \hbar^2 \langle 1,m | 1,1 \rangle = \frac{2}{\sqrt{7}} \hbar^2 \delta_{1m}$$

Similarly, applying the lowering operator twice gives just an $m = -1$ term:

$$L_- L_- |\psi\rangle = \sqrt{\frac{2}{7}} L_- L_- |1,1\rangle = \sqrt{\frac{2}{7}} \hbar \sqrt{2} L_- |1,0\rangle = \sqrt{\frac{2}{7}} \hbar \sqrt{2} \hbar \sqrt{2} |1,-1\rangle = \frac{2\sqrt{2}}{\sqrt{7}} \hbar^2 |1,-1\rangle$$

$$\text{so } \langle 1,m | L_- L_- | \psi \rangle = \frac{2\sqrt{2}}{\sqrt{7}} \hbar^2 \langle 1,m | 1,-1 \rangle = \frac{2\sqrt{2}}{\sqrt{7}} \hbar^2 \delta_{m,-1}$$

QM B4

ANSWERS

Part a

$$\begin{aligned}\langle \psi | \psi \rangle &= CC^* \sum_{n'=0}^{\infty} \frac{(\alpha^*)^{n'}}{\sqrt{n'!}} \sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}} \langle n' | n \rangle = |C|^2 \sum_{n'=0}^{\infty} \frac{(\alpha^*)^{n'}}{\sqrt{n'!}} \sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}} \delta_{n,n'} = |C|^2 \sum_{n=0}^{\infty} \frac{(\alpha^* \alpha)^n}{n!} = \\ &= |C|^2 \sum_{n=0}^{\infty} \frac{|\alpha|^n}{n!} = |C|^2 e^{|\alpha|^2} = 1 \quad \Rightarrow \quad C = e^{-\frac{1}{2}|\alpha|^2}\end{aligned}$$

Part b

Yes, it is an eigenstate, with eigenvalue α :

$$\hat{a}|\psi\rangle = C \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \hat{a}|n\rangle = C \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \sqrt{n}|n-1\rangle = \alpha C \sum_{n=1}^{\infty} \frac{\alpha^{n-1}}{\sqrt{(n-1)!}} |n-1\rangle = \alpha |\psi\rangle$$

Part c

For some general state $|\chi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$ we have: $\hat{a}^\dagger |\chi\rangle = \hat{a}^\dagger \sum_{n=0}^{\infty} c_n |n\rangle = \sum_{n=0}^{\infty} c_n \sqrt{n+1} |n+1\rangle$. The

component with the lowest value of n is no longer there after the creation operator acted, so the state that results from the action of the creation operator can never be proportional to the original state.

Part d.

$$\text{We use } \hat{a} + \hat{a}^\dagger = \frac{\beta}{\sqrt{2}} \left(\hat{x} + \frac{i}{m\omega_0} \hat{p} \right) + \frac{\beta}{\sqrt{2}} \left(\hat{x} - \frac{i}{m\omega_0} \hat{p} \right) = \sqrt{2}\beta \hat{x} \quad \Rightarrow \quad \hat{x} = (\frac{1}{2}\sqrt{2}/\beta)(\hat{a} + \hat{a}^\dagger)$$

$$\langle \psi | \hat{a} + \hat{a}^\dagger | \psi \rangle = \langle \psi | \hat{a} | \psi \rangle + \langle \psi | \hat{a}^\dagger | \psi \rangle = \langle \psi | \hat{a} \psi \rangle + \langle \hat{a} \psi | \psi \rangle = 2 \operatorname{Re}(\langle \psi | \hat{a} \psi \rangle) = 2 \operatorname{Re}(\alpha \langle \psi | \psi \rangle) = 2 \operatorname{Re}(\alpha)$$

$$\text{So } \langle \hat{x} \rangle = \sqrt{2} \operatorname{Re}(\alpha) / \beta$$

EM A1

E & M A1

$$P_{AV} = I_{RMS}^2 R$$

$$I_{RMS} = V_{RMS} / Z$$

$$= V_{RMS} / \sqrt{R^2 + [WL - (1/\omega C)]^2}$$

$$\omega = 120\pi \text{ s}^{-1}$$

$$Z = (90^2 + [120\pi \times 0.45 - \frac{1}{120\pi \times 10^{-4}}]^2)^{\frac{1}{2}} \Omega$$

$$= (8100 + [169.65 - 26.53]^2)^{\frac{1}{2}}$$

$$= 169.1 \Omega$$

$$I_{RMS} = \frac{120V}{169.1\Omega} = 0.71 \text{ A}$$

so $\boxed{P_{AV} = 45.3 \text{ W}}$

EM A2

E&M A2

$$\lambda_{air} = \frac{C}{f} = 12\text{cm}$$

$$L = \frac{\lambda_{beef}}{2} = \frac{\lambda_{air}}{2n}$$

$$5.5\text{cm} = \frac{12\text{cm}}{2n}$$

$$n = 1.09$$

EM A3

$$(a) C_{\text{equiv}} = C_1 + C_2 + C_3 = 60 \mu\text{F}$$

$$Q = CV = (60 \mu\text{F})(120 \text{ V}) = 7200 \mu\text{C}$$

$$(b) \frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{\text{equiv}} = 5.45 \mu\text{F}$$

$$Q = CV = (5.45 \mu\text{F})(120 \text{ V}) = 654 \mu\text{C}$$

EM A4

Part a.

$$\mathbf{E}_{+2} = k \frac{2}{(\frac{1}{2}\sqrt{2})^2} \frac{1}{2} \sqrt{2} (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = k 2 \sqrt{2} (\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

$$\mathbf{E}_{-2} = k 2 \sqrt{2} (\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$$\mathbf{E}_{+1} = -\frac{1}{2} \mathbf{E}_{+2}$$

$$\mathbf{E}_{-1} = -\frac{1}{2} \mathbf{E}_{-2}$$

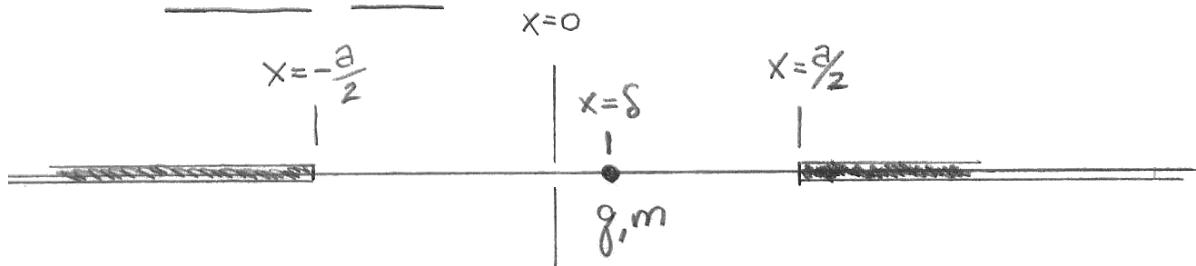
$$\begin{aligned} \mathbf{E}_{\text{tot}} &= \mathbf{E}_{+2} + \mathbf{E}_{-2} + \mathbf{E}_{+1} + \mathbf{E}_{-1} = \mathbf{E}_{+2} + \mathbf{E}_{-2} - \frac{1}{2} \mathbf{E}_{+2} - \frac{1}{2} \mathbf{E}_{-2} = \frac{1}{2} (\mathbf{E}_{+2} + \mathbf{E}_{-2}) = \\ &= \frac{1}{2} \left[k 2 \sqrt{2} (\hat{\mathbf{i}} - \hat{\mathbf{j}}) + k 2 \sqrt{2} (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \right] = k \sqrt{2} \left[(\hat{\mathbf{i}} - \hat{\mathbf{j}}) + (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \right] = k \sqrt{2} \left[2 \hat{\mathbf{i}} \right] = k 2 \sqrt{2} \hat{\mathbf{i}} \end{aligned}$$

Part b.

$$V_{\text{tot}} = V_{+2} + V_{-2} + V_{+1} + V_{-1} = (V_{+2} + V_{-2}) + (V_{+1} + V_{-1}) = 0 + 0 = 0 \text{ V}$$

EM B1

E&M B1



$$\begin{aligned}
 F(s) &= \int_{-\infty}^{-a/2} \frac{k g \lambda dx}{(x-s)^2} - \int_{a/2}^{\infty} \frac{k g \lambda dx}{(x-s)^2} \\
 &= k g \lambda \left[\left\{ \frac{-1}{(x-s)} \right\} \Big|_{-\infty}^{-a/2} + \left\{ \frac{1}{(x-s)} \right\} \Big|_{a/2}^{\infty} \right] \\
 &= k g \lambda \left[\frac{1}{\frac{a}{2} + s} - \frac{1}{\frac{a}{2} - s} \right] \\
 &= k g \lambda \left[\frac{1}{1 + \frac{2s}{a}} - \frac{1}{1 - \frac{2s}{a}} \right] \approx k g \lambda (-4s/a) \\
 &= -(4g\lambda/a)s \quad \Rightarrow \quad "k" = \frac{4g\lambda}{a}
 \end{aligned}$$

$$\omega = \sqrt{\frac{4g\lambda}{am}} = 2\pi T$$

$$T = \frac{1}{2\pi} \sqrt{\frac{4g\lambda}{am}} = \boxed{\frac{1}{\pi} \sqrt{\frac{g\lambda}{ma}}}$$

EM B2

EM B2

$$\frac{EMF_A}{EMF_B} = 0.5 = \frac{N_A A_A \Delta B / \Delta t}{N_B A_B \Delta B / \Delta t}$$
$$= \frac{20 \pi (5\text{cm})^2}{10 \pi (X\text{cm})^2}$$

$$\frac{1}{2} = 2 \cdot \frac{25}{X^2}$$

$$X^2 = 100 \text{ cm}^2 \Rightarrow \text{Radius of } B = 10\text{cm}$$

EM B3

a) $V = \frac{\lambda}{2\pi\epsilon_0}(\ln(b/a) - \ln(b/b)) = \frac{\lambda}{2\pi\epsilon_0}\ln(b/a).$

b) $V = \frac{\lambda}{2\pi\epsilon_0}(\ln(b/r) - \ln(b/b)) = \frac{\lambda}{2\pi\epsilon_0}\ln(b/r).$

c) $V = 0.$

$$V_{ab} = V(a) - V(b) = \frac{\lambda}{2\pi\epsilon_0}\ln(b/a).$$

d) Between the cylinders:

$$\begin{aligned} V &= \frac{\lambda}{2\pi\epsilon_0}\ln(b/r) = \frac{V_{ab}}{\ln(b/a)}\ln(b/r) \\ \therefore E &= -\frac{\partial V}{\partial r} = -\frac{V_{ab}}{\ln(b/a)}\frac{\partial}{\partial r}(\ln(b/r)) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}. \end{aligned}$$

e) The potential difference between the two cylinders is identical to that in part (b) (for $r=a$) even if the outer cylinder has no charge.

EM B4

The equation $V_C = IX_C$ allows us to calculate I and then $V = IZ$ gives Z . Solve $Z = \sqrt{R^2 + (X_L - X_C)^2}$ for X_L .

(a) $V_C = IX_C$ so $I = \frac{V_C}{X_C} = \frac{135}{480} = 0.28$ A

(b) $V = IZ$ so $Z = \frac{V}{I} = \frac{120}{0.28} = 429$ Ω

(c) $Z^2 = R^2 + (X_L - X_C)^2$.

$X_L - X_C = \pm \sqrt{Z^2 - R^2}$, so

$$X_L = X_C \pm \sqrt{Z^2 - R^2} = 480 \pm \sqrt{(429)^2 - (80)^2} = 480 \pm 421 = 59 \text{ Ω or } 901 \text{ Ω}$$

(d)

$X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. At resonance, $X_C = X_L$. As the frequency is lowered below the resonance frequency X_C increases and X_L decreases. Therefore, for $\omega < \omega_0$, $X_L < X_C$. So for $X_L = 59 \text{ Ω}$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L = 901 \text{ Ω}$