Magnitude of

If taking the cross product, set it up as **,** L to R

|  |  |
| --- | --- |
|  | E.g. for a cone are **not** constant. For a cylinder, *r* is constant. |

**Curves and line integrals**

**Note: R** is the vector that traces out the curve. For example, if the curve is a semicircle in the quadrant I and II, then where 0θ**.** And in this case, τ =θ.

**Parameterization:** The goal is to solve for your curve or surface in terms of a variable that suits your preferred coordinate system. If your curve is an intersection of two surfaces, then solve for the intersection just like you would a system of equations (Gauss elimination, etc.), but before you solve, chose a parameterization that makes sense for the surfaces in question. I.e. if you have a plane intersecting a cylinder, chose θ as the parameter (from 0 to 2π), make the appropriate substitutions for the cylinder and then solve for x,y, and z in terms of the new parameter.

Where **v** is a force vector field and **R** is the position vector to some reference point.

Note 1: Vector field:

Example: We have the equation for a paraboloid: First, get all the variables to one side, so: This is now The gradient of *g* is now the normal to the surface. This is a “level surface”. If we want to find the normal to a “level curve”, then we set *x,y,*or *z* to a constant and the take the gradient, e.g.: This is now our

Level Curve: The function parameters that yield a specified “z”.

**Trig Identities:**

|  |  |
| --- | --- |
|  | We can derive all the others from these two and  |

Length of any one side of a triangle cannot exceed the sum of the lengths of the other two sides.



**Change of Variables Example:**

**Geometry:**

If *a* and *b* are 1, then it is a circular cylinder.



**Parameterization of curves, surfaces, and volumes:**

Computationally, we can express these in terms of the components *x,y,z* of **R:**

Notes: Find x,y, and z in terms of the two parameters u and v. z will be in terms of u and v, e.g. . Then take the appropriate derivatives. The limits of integration are over the new parameters u and v.

Special cases of the above:

When we integrate Case 2, the limits of integration are defined by the region in the xy plane under the surface.

Note: is the function, e.g. if our equation is , then . And is .

 We only use this equation if we can write the surface as *z=f(x,y)*.

Notes: Fix j, Find the M’s (little determinants), carry out the summation.

**Properties of determinants:**

1. If a row or column is modified by adding times another row, then det**A** does not change.
2. If any two rows are changed then det**A**=-det**B**.
3. If **A** is triangular, then det**A** is the product of the diagonal.
4. If a row or column is 0 then the det is 0.
5. If a row or a column is a linear combo of other rows or columns then the det = 0
6. If any two rows are equal, det=0.
7. If det , then **A**x=**c** has a unique solution.
8. If det=0 then **A** is singular.
9. If then system is underdetermined.
10. If then system is overdetermined.
11. Underdetermined and 2 unknowns🡪 2 parameter family of solutions and solutions lie in a plane. 1 parameter family and solutions like on a line, etc.
12. Inconsistent system: 0= -15 for a solution (for example).
13. Scaling a row or column scales the determinant.

**A=**

Recall that the cofactor is: . Remember to take the **Transpose** of **A**jk to get the adj**A**

Another way to find Augment **A|I**, then use elementary ops to get

If **A** is invertible, then **AB**=**AC** implies that **B**=**C**, **BA**=**CA** implies that **B**=**C**, **AB**=0 implies that **B**=0.

Theorem:

**Cramer’s Rule**: If **Ax**=**c** where A is invertible, then each component of **x** may be computed as the ratio of two determinants; the denominator is det **A**, and the numerator is det **A** but with the *i*th column replaced by **c**.

Notes: Decompose **A** into **LU** (e.g. start with *u11* = *a11, then u11 l21=a21,* etc.), then solve **Ly=c** for **y**, then solve **Ux**=**y** for **x**.

**Ill conditioned**: Small changes in matrix elements yield large changes in det, inverse, etc. To test if then the matrix is ill conditioned.

**Vector Spaces:**

 This means we have a vector with “*n”* dimesions.

Subspace: If a subset T of a vector space is itself a vector space (with the same definitions as S for vector addition u+v, scalar multiplication au, zero vector 0, and negative vector –u), then T is a subspace of S.

**Dot product, norm, and angle for n-space**

We can “weight” each component:

Orthonormal: a set of vectors where each vector is normalized and each vector in one set is orthogonal to every other vector in the other set. This can be described as the

Norms:

, if we are using weights.

If vector space consists of functions, say, then the inner product is: **,** where a and b are the bounds of the function. Note that the norm can be found for a vector

**Span**: The set of all linear combinations of the vectors in a vector space is called the span. E.g. for a vector space , the span is and is denoted as The span is a subspace of S. The span has to include the origin.



The above example shows a few linear combinations of the vectors *u* and *v*; the span of this vector space would include **all** the linear combinations (a plane).

* To discover if two spans are equal, say **,** write the equation and solve for in terms of **w.** Compare the **w** vectors. E.g. (for a ):

. In this case we have a non-square matrix, so reduce using elementary operations so that the last row of **A** is zero (keeping track of the operations on **w**). This gives us an equation only in terms of **w**, e.g.

**Bases:**

|  |  |
| --- | --- |
| **e1****e2****u** | * . A set of **e**’s is a basis for **u** iff it is LI and span S.
* Orthogonal basis are preferred. Given orthog. basis vectors:**,** suppose we wish to expand a given **u** in terms of these, then

 **,**  |

**Orthogonalization process:** Given *k* LI vectors we can get *k* ON vectors by:

**Dimensions:** The dimension of a vector space is the greatest number of LI vectors in that vector space. If a vector space contains only a zero vector, the dimension is 0. Dimension relates to bases: The number of basis vectors equals the dimension (because the bases are LI). The dim of a space will be no greater than *n* (.

**Linear Independence:** A set of vectors is LD if at least one of them can be expressed as a linear combination of the ***others***. Example: are LD (

A set of vectors is LD iff there exist scalars, not all zero, such that To solve for the scalars: 1) Set up a system of equations: **.** Where **A** is the matrix of the vectors. 2) Use elementary row operations to reduce to “row echelon form” or as close to it. The non-zero rows are LI.

* A set containing the zero vector is LD.
* Every orthogonal set of nonzero vectors is LI.

**Best Approximations:**

If **u** is any vector with

Where we are given **u** and an orthonormal basis set **.** The more basis vectors we use, the closer our approximation will be (up to the number of bases equal to the dimension of **u.)** The error of our approximation is:

Where:

**Row-echelon form:**

1) In each row not made up entirely of zeros, the first nonzero element is a 1.

2) In any two consecutive rows not made up entirely of zeros, the leading 1 in the lower row is to the right of the leading 1 in the upper row.

3) If a column contains a leading 1, every other element in that column is a zero.

4) All rows made up entirely of zeros are grouped together at the bottom of the matrix.

**Rank:**

1. A matrix (maybe not square) is of rank *r* if it contains at least one *r* *r* submatrix with nonzero determinant but no square submatrix larger than *r* *r* with nonzero determinant. You can swap rows and columns to find these submatrices. The zero matrix is of rank 0.
2. Elementary row operations do not alter the rank.
3. # of LI row vectors = # of LI column vectors = rank.

**Elementary operations:**

1) Operate on the augmented matrix (glue **c** onto **A**).

2) Addition of a multiple of one row to another.

3) Multiplication of a row by a nonzero constant.

4) Interchange of two rows.

**Terminology**

1) Consistent: one or more solutions

2) Unique: only one solution

3) Non-unique: more than one solution

4) Inconsistent: No solutions.

5) If m<n: Consistent or inconsistent. If consistent no unique solution exists (p parameter family, where

6) If m>n: Consistent or inconsistent. Can have unique or non-unique solution. (p parameter family, where

**Eigenvalue Problem**

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If *v* is an eigenvector of A, then *v* lies on the vector A*v.* In other words, if *v* is an eigenvector of A, then A*v* is the same as some constant times A, e.g.

To find eigenvalues, solve the characteristic equation for . To find the eigenspaces, 1) find the eigenvalues , 2) for each , plug back into **,** and solve for **x.** This will give us an eigenvector for each eigenvalue. 3) We should end up with at least one arbitrary solution (0=0) for each eigenvector. This will give us our arbitrary constants that when multiplied by the eigenvector gives us our eigenspace.

**Symmetric Matrices:**

* If **A** is **symmetric**, then all of its eigenvalues are real.
* If an of a **symmetric** matrix **A** is of multiplicity *k*, then the eigenspace corresponding to is of dimension *k.*
* If **A** is **symmetric,** then eigenvectors corresponding to distinct eigenvalues are orthogonal.
* If an matrix **A** is symmetric, then its eigenvectors provide an orthogonal basis for *n-*space.
* If **A** is symmetric, then

Where **e** is an eigenvector of **A.**

**Diagonalization:**

Especially useful when solving a system of differential equations. Given a system **,** our goal is to solve for **x** by 1) Find the **Q** and **D** matrices. **Q** has the eigenvectors for rows, **D** has the eigenvalues in the diagonal (the order of the eigenvalues matches the order of the eigenvectors in **Q**). 2) Write **.** 3) This gives you uncoupled equations, so you can do things like: . 4) Now that you have **,** you can solve for **x,** by **.**

* Every symmetric matrix is diagonalizable.
* If an matrix has *n* distinct eigenvalues, then it is diagonalizable. (It may be diagonalizable anyway—does not read iff).
* If an matrix has eigenvalues, then the corresponding eigenvectors are LI.
* **A** is diagonalizable iff it has *n* LI eigenvectors.

**Quadratic Form/Cononical Form:** A quadratic form is said to be canonical if all mixed terms (such as ) are absent.

Example: reduce to canonical form. 1) Identify the **A** matrix: **,** chose to be equal so that the matrix is symmetrical. 2) Find the eigenvalues. 3) Plug in to get the canonical form: . 4) Find the connection between by where **Q** is the eigenvector matrix from **A.**

**Tensors:**

1st invariant of stress is hydrostatic pressure.

Eigenvalues of stress and strain tensors are principal stresses and strains.

Eigenvectors of stresses and strains are the directions.

**=**3rd Invariant

To find eigenvalues solve for the roots of:

**Fourier Series:**

Even function**:** ,

Odd function**:** ,

Decompose any function into its even and odd parts:

Elementary integral formulas:

Integration by parts:

Questions:

If we have 4 vectors of Rank 3, are they guaranteed to be LD? It seems they are if we have more vectors than the rank…

**Integral Table:**



u substitution for simplifying integrals: 1) Substitute a single variable (*u*) for a hard-to-integrate portion (*x*). 2) Find *du*. 3) Rework limits of integration.