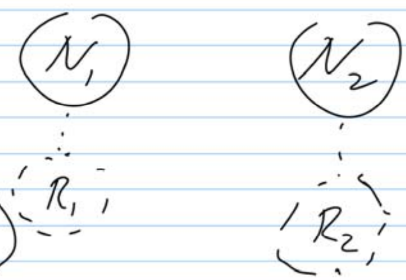
5 lecture - competition

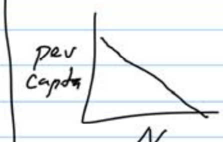
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

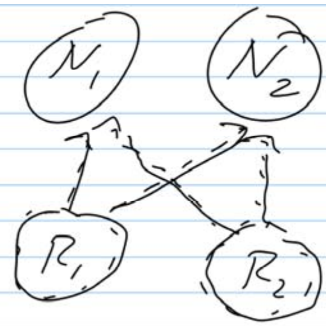
$$\frac{dN_1}{dt} = r_1 N_1\left(1 - \frac{N_1}{K_1}\right) = r_1 N_1\left(\frac{K_1 - N_1}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2\left(1 - \frac{N_2}{K_2}\right) = r_2 N_2\left(\frac{K_2 - N_2}{K_2}\right)$$



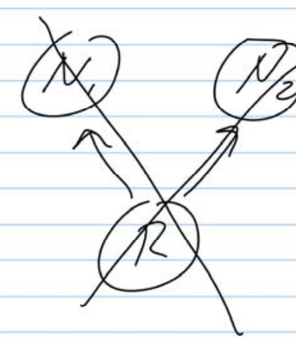
$$\boxed{\begin{aligned} \frac{dN_1}{dt} &= r_1 N_1\left(\frac{K_1 - N_1 - \alpha_{12} N_2}{K_1}\right) \\ \frac{dN_2}{dt} &= r_2 N_2\left(\frac{K_2 - N_2 - \alpha_{21} N_1}{K_2}\right) \end{aligned}}$$

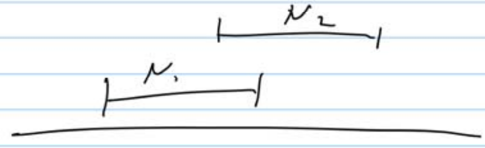




$\left.\begin{matrix}\alpha_{12} \\ \alpha_{21}\end{matrix}\right\}$ competition coefficients

$$\frac{\text{interspecific}}{\text{intraspecific}}$$





resources gradient

Find equilibrium densities

$$\frac{dN_1}{dt} = 0 \qquad \frac{dN_2}{dt} = 0$$

trivial $\rightarrow \boxed{\hat{N}_1 = 0}$

$$0 = r_1 \hat{N}_1\left(\frac{K_1 - \hat{N}_1 - \alpha_{12}\hat{N}_2}{K_1}\right) \rightarrow 0 = K_1 - \hat{N}_1 - \alpha_{12}\hat{N}_2$$

$$\frac{dN_1}{dt} = 0 \Rightarrow \boxed{\begin{aligned} \hat{N}_1 &= K_1 - \alpha_{12}\hat{N}_2 \\ \hat{N}_2 &= K_2 - \alpha_{21}\hat{N}_1 \end{aligned}}, \ \boxed{\hat{N}_2 = 0}$$

$$\frac{dN_2}{dt} = 0 \longrightarrow$$

$$\hat{N}_1 = K_1 - \alpha_{12}(K_2 - \alpha_{21}\hat{N}_1) = K_1 - \alpha_{12}K_2 + \alpha_{12}\alpha_{21}\hat{N}_1$$

$$\hat{N}_1 - \alpha_{12}\alpha_{21}\hat{N}_1 = K_1 - \alpha_{12}K_2$$

$$\hat{N}_1(1 - \alpha_{12}\alpha_{21}) = K_1 - \alpha_{12}K_2$$

do the same for N_2 $\Rightarrow$

$$\hat{N}_1 = \frac{K_1 - \alpha_{12}K_2}{1 - \alpha_{12}\alpha_{21}}$$

equilibria

$$\hat{N}_2 = \frac{K_2 - \alpha_{21}K_1}{1 - \alpha_{12}\alpha_{21}}$$

For what values are $\hat{N}_1$ and $\hat{N}_2$ positive?

$$\frac{K_1 - \alpha_{12}K_2}{1 - \alpha_{12}\alpha_{21}} > 0$$

$K_1 - \alpha_{12}K_2 > 0 \Rightarrow K_1 > \alpha_{12}K_2$

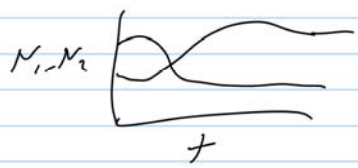
$\frac{K_1}{K_2} > \alpha_{12}$

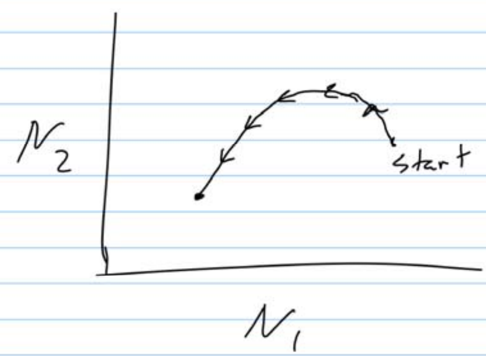
$1 - \alpha_{12}\alpha_{21} > 0$

$1 > \alpha_{12}\alpha_{21}$

(Pause)

Phase Plane



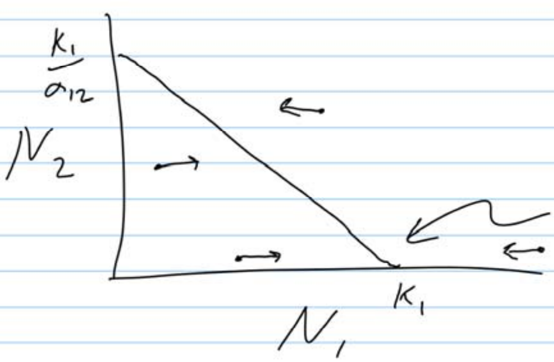


$\frac{dN_1}{dt} = 0$ $\hat{N}_1 = K_1 - \alpha_{12}\hat{N}_2$

$$\alpha_{12}\hat{N}_2 = K_1 - \hat{N}_1$$

$$\hat{N}_2 = \frac{K_1}{\alpha_{12}} - \frac{1}{\alpha_{12}}\hat{N}_1$$

$$y = mx + b$$



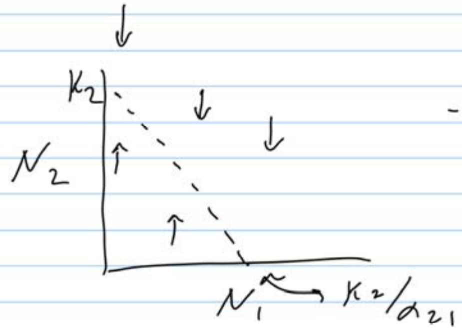
N_1 nullcline

isocline at 0

ZNGI

$\frac{dN_1}{dt} = 0$

$$\frac{dN_2}{dt} = 0$$

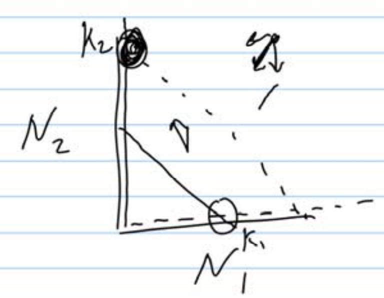


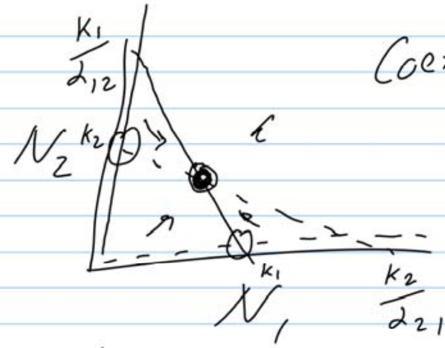
--- $\frac{dN_2}{dt} = 0$

$$\hat{N}_2 = K_2 - \alpha_{21}\hat{N}_1$$

$$0 = K_2 - \alpha_{21}\hat{N}_1$$

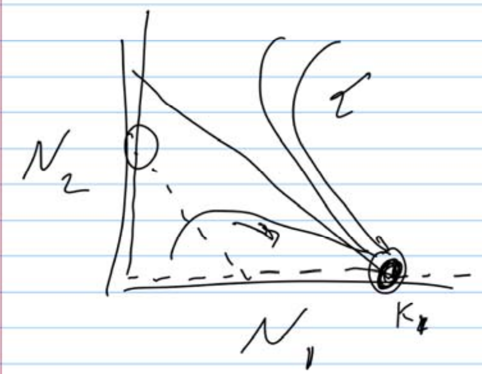
$$\hat{N}_1 = \frac{K_2}{\alpha_{21}}$$

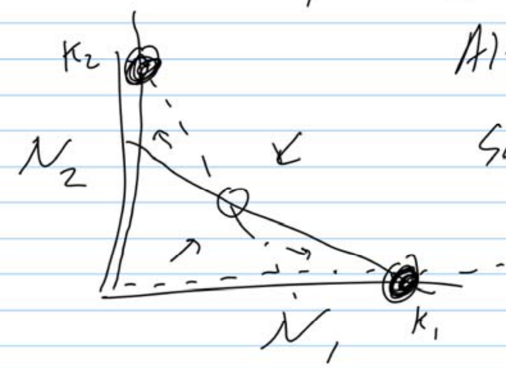




Coexist ---- $\frac{dN_2}{dt} = 0$

— $\frac{dN_1}{dt} = 0$





Alternative Eq.

Saddle

Coexistence of

$$\frac{K_1}{\alpha_{12}} > K_2 \quad \text{and} \quad \frac{K_2}{\alpha_{21}} > K_1$$

$$\frac{K_1}{K_2} > \alpha_{12} \qquad \frac{K_2}{K_1} > \alpha_{21}$$

$$\frac{K_1}{K_2} < \frac{1}{\alpha_{21}}$$

$$\frac{1}{\alpha_{21}} > \frac{K_1}{K_2} > \alpha_{12}$$ for stable coexistence

$\alpha_{12} = \alpha_{21} = 0.9$

$$\frac{1}{0.9} > \frac{K_1}{K_2} > 0.9$$

$$1.1 > \frac{K_1}{K_2} > 0.9$$

$\alpha_{12} = 0.2$

$\alpha_{21} = 0.1$

$$10 > \frac{K_1}{K_2} > 0.2$$

Stability Analysis

Jacobian

$$\frac{dN_1}{dt} = f(N_1, N_2)$$

$$\frac{dN_2}{dt} = g(N_1, N_2)$$

$$= r_1 N_1 \left(\frac{K_1 - N_1 - \alpha_{12} N_2}{K_1} \right)$$

$$\frac{dN_1}{dt} = r_1 N_1 - \frac{r_1}{K_1} N_1^2 - \frac{r_1 \alpha_{12}}{K_1} N_1 N_2$$

$$\begin{pmatrix} \left.\frac{\partial f}{\partial N_1}\right|_{\hat{N}_1, \hat{N}_2} & \frac{\partial f}{\partial N_2} \\ \frac{\partial g}{\partial N_1} & \frac{\partial g}{\partial N_2} \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

$$\frac{\partial \left(\frac{dN_1}{dt} \right)}{\partial N_1} = r_1 - 2 \frac{r_1}{K_1} N_1 - \frac{r_1 \alpha_{12}}{K_1} N_2$$

$$\frac{\partial \left(\frac{dN_1}{dt} \right)}{\partial N_2} = \frac{-r_1 \alpha_{12}}{K_1} N_1$$

for 2 variables, i.e. 2x2 Jacobian

stable if

trace < 0 <u>and</u> determinante > 0

$a_{11} + a_{22} < 0$ $\qquad$ $a_{11} a_{22} - a_{12} a_{21} > 0$

Jacobian

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$