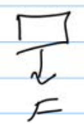
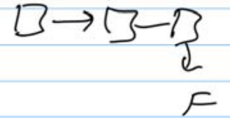
Lecture 10 - Evolution

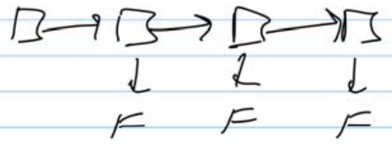
Life History

Semelparous Monocarpic Annual





Iteroparous Polycarpic Perennial



Annual

$$N_A(t+1) = B_A N_A \rightarrow W_A = B_A$$

Cole's paradox

Annual if:

$$B_A > B_p + \underset{\text{Self}}{\underset{\uparrow}{1}}$$

Perennial

$$N_p(t+1) = \underset{\swarrow \text{ Offspring produced}}{B_p N_p} + N_p$$

$$\rightarrow W_p = B_p + 1$$

$$\text{Annual: } W_A = \overset{\text{juvenile survival}}{s_j} \underset{\uparrow \text{ offspring production of annual}}{B_A}$$

$$\text{Perennial } W_p = s_j B_p + \underset{\uparrow \text{ survival of adult to next year}}{s_a}$$

Annual if

$$W_A > W_p \qquad s_j B_A > s_j B_p + s_a$$

$$B_A > B_p + \frac{s_a}{s_j} \rightarrow \frac{\text{adult survival}}{\text{juvenile survival}}$$

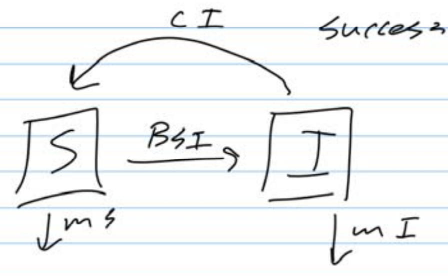
if $s_a = 0.8$

$s_j = 0.008$ $\frac{s_a}{s_j} = 100$

Disease Evolution

Goal: Conventional Wisdom

successful parasites evolve to be harmless



$$\frac{dS}{dt} = \overset{\text{host birth rate}}{b}(S+I) - \beta SI + cI - mS$$

$$\frac{dI}{dt} = \beta SI - cI - mI - vI$$

virulence

↳ increased mortality due to infection

When will disease increase?

$$\frac{dI}{dt} > 0$$

$$\beta SI - (c+m+v)I > 0$$

$$\beta S - (c+m+v) > 0$$

$$\beta S > c+m+v$$

$$\frac{\beta S}{c+m+v} > 1$$

largest possible $S = N$

$$R_0 = \frac{N\beta}{c+m+v}$$

In short maximize R_0 ⤳ fitness of disease

↳ * when singly infected.

Which of the R_0 parameters are under at least partial control of the disease organism?

β transmission rate

c clearance

~~m natural mortality~~

v virulence

$\beta \to \infty$	$c \to 0$	$v \to 0$	What would maximize R_0?

Trade-offs

transmission and virulence → Anderson and May (1982)

clearance and transmission

$B(v)$

transmission ↗ B ; ↖ virulence v

$$\frac{B_{max}\, V}{B_h + V}$$

$$R_0 = \frac{N\, B(v)}{v+m+c}$$

$$\begin{tikzcd} & \text{Disease Reproduction} \arrow[dl] \arrow[d] \\ B & v \end{tikzcd}$$

$$R_0 = \frac{B_{max}\, V}{B_h + V} \times \frac{N}{v+m+c}$$ ↩ solve for v^*

$$v^* = \sqrt{B_h\,(c+m)}$$

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - \frac{aNP}{1+haN}$$ → a → partially under control of the prey

$\frac{dN}{dt}\frac{1}{N}$ → per capita growth rate

i.e. a ~~average~~ fitness

$$\frac{dN}{dt}\frac{1}{N} = r\left(1-\frac{N}{K}\right) - \frac{aP}{1+haN}$$

$$\frac{d\left(\frac{dN}{dt}\frac{1}{N}\right)}{da}$$ → selection gradient

Additive Genetic Variance

$$\underbrace{\left.\frac{d\left(\frac{-aP}{1+haN}\right)}{da}\right|_{a=a^*}}_{\text{selection gradient}} \times V = \frac{da}{dt}$$

$$S \rightarrow I$$

$$\frac{dI}{dt} = \begin{array}{l}\text{fitness}\\ \text{of diseases}\end{array} = \underset{\downarrow}{B(v)}\, S I - (c + m + v) I$$

$$B(v) = \frac{B_{max}\, V}{B_n + V}$$

$$\frac{d\left(\frac{dI}{dt}\right)}{dv}$$

$$\frac{dS}{dt} =$$

$$\frac{dI}{dt} =$$

$$\frac{dv}{dt} = V \cdot g$$