UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
Thursday, May 10, 2018

This test covers the topics of *Classical Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Classical Mechanics Group A - Answer only two Group A questions

A1 A neutron star consists of neutrons bound together by gravitational attraction and has an extremely high density ρ. What is the maximum angular frequency of rotation if a mass is not to fly off the surface? Give your answer in terms of the density of the star and some constants. You may assume a constant density and a spherical star.

A2 In a homonuclear noble gas dimer (Ne_2, Ar_2, etc.), the repulsive/attractive balance between the two atoms is well represented by the Lennard-Jones potential, given by $V(r) = 4\varepsilon\left[(\sigma/r)^{12} - (\sigma/r)^{6}\right]$.

a. Find the equilibrium position.
b. When m is the mass of each atom in the dimer, find the frequency ω of small oscillations about the equilibrium position.

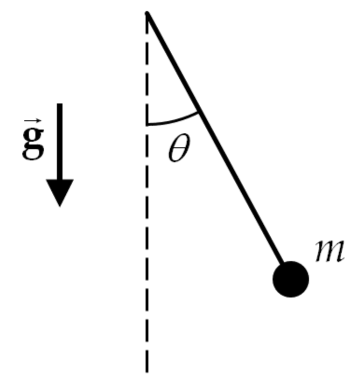
A3 A point particle of mass m traces out a complicated path given by $x = at^3$, $y = bt$, and $z = ct^2$.

a. Find the angular momentum **L** as a function of time
b. Find the torque **N** as a function of time.

A4 Consider two particles of equal mass m. One is stationary and the other moves with velocity **v**, striking the stationary particle. After the collision the particles both move, with momenta $\mathbf{p}_{1F}$ and $\mathbf{p}_{2F}$. Assuming a perfectly elastic collision, what is the angle between particles' velocity vectors after the collision?

Classical Mechanics Group B - Answer only two Group B questions

B1 A pendulum consists of a rod of zero mass and a small object of mass m attached to it at the end. The angle the rod makes with the vertical is called θ, as shown.



a. Write the Lagrangian of the system.
b. Write the Hamiltonian of the system.
c. What is the generalized momentum p_θ?
d. Write the equation of motion.
e. If, at time $t = 0$, we have $\theta = \theta_0$ and $\dot{\theta} = 0$, what is the maximum speed at some later time?

B2 A small object of mass m is projected into the air with initial velocity whose Cartesian components are $(v_{0x}, v_{0y}, 0)$. Assuming the linear air resistance, $\mathbf{F} = -c\mathbf{v}$, and the constant acceleration of gravity g

(a) Find the object's velocity as a function of time.

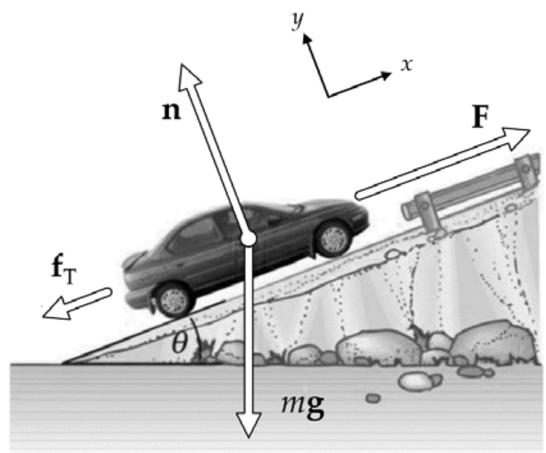
(b) Find the object's position (x, y) as a function of time. Assume the initial position at the origin of the coordinate system.

(c) Investigate the behavior of velocity and the position in the limit $t \gg m/c$.

(d) How much energy has been dissipated into the air by the time $t \gg m/c$? What is the rate of the energy dissipation in this limit?

B3 A box of mass m is pushed against a spring of negligible mass and force constant k, compressing it a distance x. The box is then released and travels up a ramp that is at an angle α above the horizontal. The coefficient of kinetic friction between the box and the ramp is μ_k, where $\mu_k < 1$. The box is still moving up the ramp after traveling some fixed distance $s > |x|$ along the ramp. Calculate the angle α for which the speed of the box after traveling this distance s is a minimum.

B4 Consider a car of mass m that is accelerating up a hill, as shown in the figure. The road makes an angle θ with the horizontal, as shown. An automotive engineer has measured the magnitude of the total resistive force to be $f_T = 218 + 0.70v^2$, where f_T is in N and v is the speed in m/s.

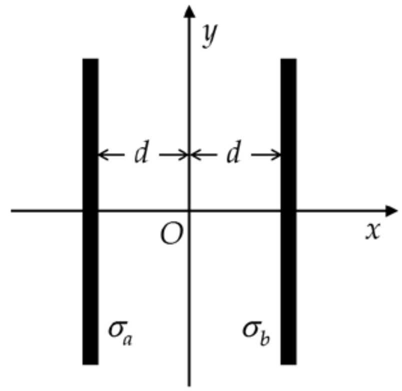


a. Determine the power the engine must deliver to the wheels as a function of speed and acceleration.

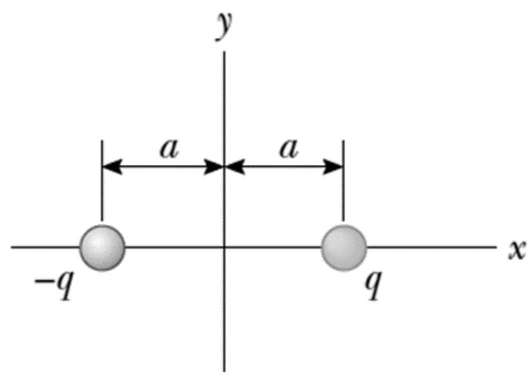
b. The mass of the car is 1450 kg, and the hill has $\theta = 10°$. When the car accelerates with a constant acceleration $a = 1.0\ \text{m/s}^2$ from speed 0 m/s to 27 m/s, what is the output power as function of time?

Electrodynamics Group A - Answer only two Group A questions

A1 Two thin plates of infinite area and made of insulating material are on either side of the origin and a distance d away from it. They carry uniformly-distributed surface change densities σ_a and σ_b as shown in the figure. Find the electric potential difference between the two plates.



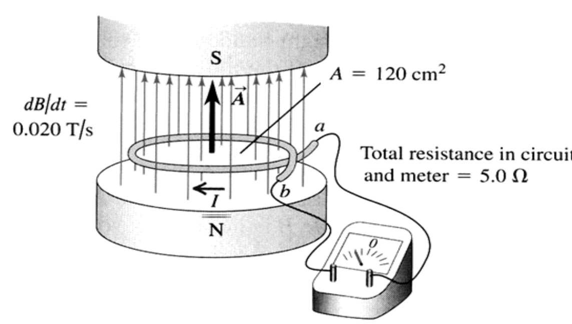
A2 An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2a, as shown in the figure. The dipole is along the x axis and is centered at the origin.



a. Calculate the potential V and the electric field E_x on the x axis anywhere between the two charges (so for $-a < x < a$)

b. Give expressions for V and E_x on the x-axis very far from the dipole ($x \gg a$).

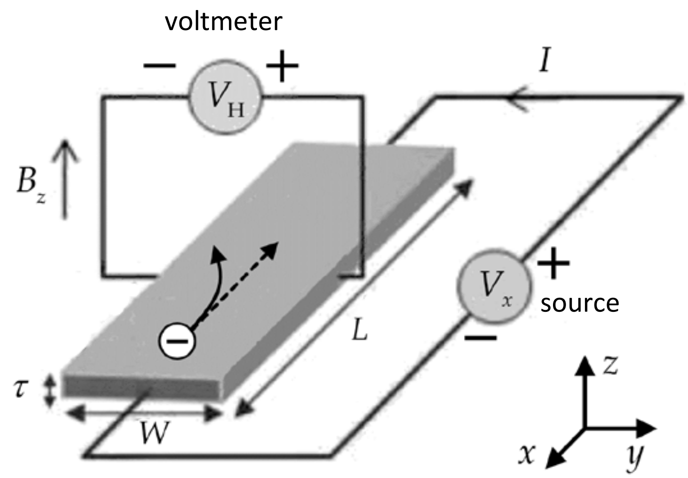
A3 The magnetic field between the poles of the electromagnet in the figure is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm². The total resistance of the circuit, including the meter, is 5 Ω.



a. Find the induced emf and the induced current in the circuit.

b. If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and the induced current?

A4 A current I flows through a rectangular bar (width W and length L) of conducting material along the x direction in the presence of a uniform magnetic field B_z pointing in the z direction, as shown in the figure.

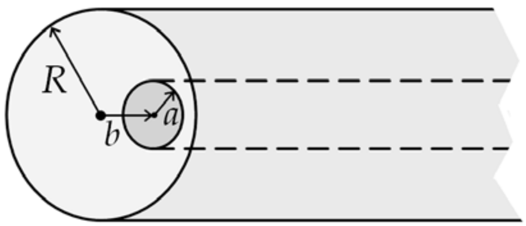


a. If the moving charges are positive holes with speed v_x (as is the case, for example, in certain semiconductors) instead of electrons (which the figure shows), in which direction are these holes deflected by the magnetic field?

This deflection results in an accumulation of charge on the sides of the bar, which in turn produces an electric force counteracting the magnetic one. Equilibrium occurs when the two cancel.

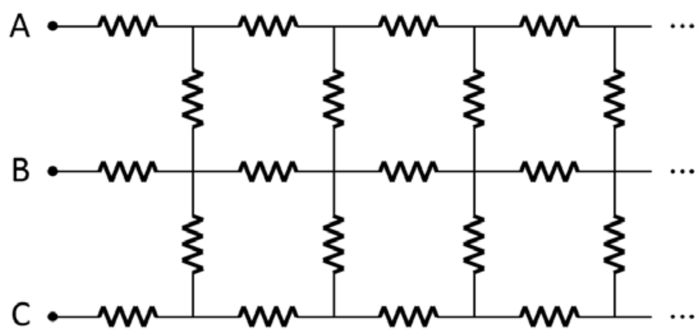
b. Find the resulting potential difference (the Hall voltage, V_H) between the two sides of the bar.

Electrodynamics Group B - Answer only two Group B questions

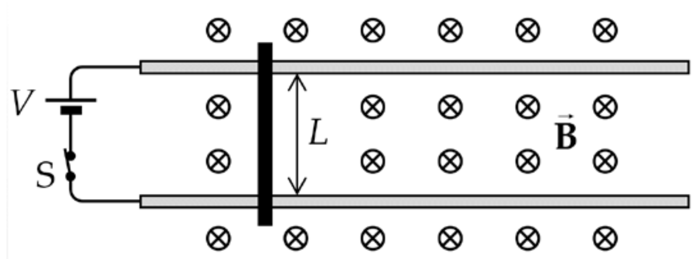


B1 A very long, solid, insulating cylinder with radius R has uniform volume charge density ρ. We then drill a cylindrical hole in the original cylinder along its entire length, as shown in the figure. The hole has radius a and its axis is at a distance b from the center of the original cylinder. The geometry is such that $a < b < R$ and $a < R - b$. Find the magnitude of the electric field in the hole.

B2 Consider the infinite network of identical resistors shown in the figure. Each resistor has resistance r. What is the resistance between the points A and C?

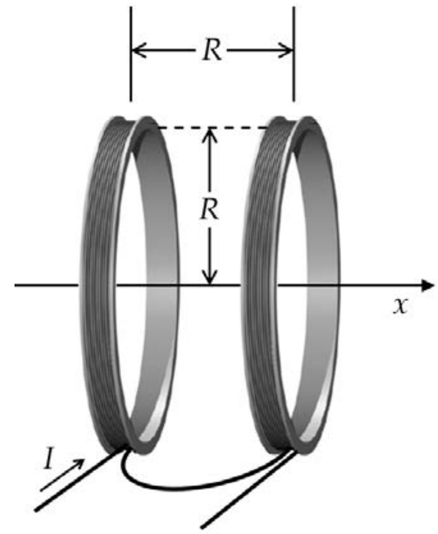


B3 A rod of length L and mass m sits at rest on two frictionless conducting rails. A constant magnetic field **B** is applied as shown in the figure. The resistance of the rod is R. At time $t = 0$ a switch S is closed, connecting a battery with potential V to the rails. The rod begins to move.



a. Find the terminal speed of the rod (after S has been closed for a very long time).
b. Find the speed of the rod as a function of time.
c. Find the current in the loop as a function of time.

B4 Two identical, flat, circular coils of wire each have n turns and a radius of R. The coils are arranged as a set of Helmholtz coils (see figure), parallel and with a separation of R. If each coil carries a current of I, determine the magnitude of the magnetic field at a point that is on the common symmetry axis of the coils and halfway between them.



Physical Constants

speed of light .... $c = 2.998\times10^{8}$ m/s

Planck's constant .... $h = 6.626\times10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055\times10^{-34}$ J·s

Boltzmann constant ...... $k_B = 1.381\times10^{-23}$ J/K

elementary charge .... $e = 1.602\times10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854\times10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257\times10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

Avogadro constant .... $N_A = 6.022\times10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988\times10^{9}$ m/F

electron mass .... $m_{el} = 9.109\times10^{-31}$ kg

electron rest energy...... 511.0 keV

Compton wavelength .. $\lambda_C = h/m_{el}c = 2.426$ pm

proton mass .... $m_p = 1.673\times10^{-27}\,\text{kg} = 1836 m_{el}$

1 bohr .... $a_0 = \hbar^2 / ke^2 m_{el} = 0.5292$ Å

1 hartree (= 2 rydberg) ... $E_h = \hbar^2/m_{el}a_0^{\,2} = 27.21$ eV

gravitational constant ... $G = 6.674\times10^{-11}$ m^3/ kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

ELECTROSTATICS

$$\oiint_S \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{encl}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \tfrac{1}{2}\varepsilon_0\int_V E^2 d\tau = Q^2/2C$

Relative permittivity: $\varepsilon_r = 1 + \chi_e$

Bound charges

$\rho_b = -\nabla \cdot \mathbf{P}$
$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$

Capacitance in vacuum

Parallel-plate: $C = \varepsilon_0 \dfrac{A}{d}$

Spherical: $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

Cylindrical: $C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$ (for a length L)

MAGNETOSTATICS

Relative permeability: $\mu_r = 1 + \chi_m$

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

Current densities: $I = \int \mathbf{J} \cdot d\mathbf{A}$, $I = \int \mathbf{K} \cdot d\boldsymbol{\ell}$

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi} \int \dfrac{I d\boldsymbol{\ell} \times \hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

Infinitely long solenoid: B-field inside is $B = \mu_0 n I$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{\text{encl}}$

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I \int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi} \dfrac{1}{r^3} \left[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m} \right]$

Bound currents

$J_b = \nabla \times \mathbf{M}$
$K_b = \mathbf{M} \times \hat{\mathbf{n}}$

Maxwell's Equations in vacuum

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1. $\nabla \cdot \mathbf{D} = \rho_{\mathrm{f}}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{f}} + \dfrac{\partial \mathbf{D}}{\partial t}$ Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\dfrac{d\Phi_B}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \tfrac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \tfrac{1}{2} L I^2 = \tfrac{1}{2} \oint \mathbf{A} \cdot \mathbf{I}\, d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2} ……………	$\dfrac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2} ……………	$\dfrac{1}{2a}$
$x^2e^{-ax^2}$ ……………	$\dfrac{\sqrt{\pi}}{4a^{3/2}}$
$x^3e^{-ax^2}$ ……………	$\dfrac{1}{2a^2}$
$x^4e^{-ax^2}$ ……………	$\dfrac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5e^{-ax^2}$ ……………	$\dfrac{1}{a^3}$
$x^6e^{-ax^2}$ ……………	$\dfrac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+bx^2}dx = \pi / 2b^{1/2}$$
$$\int_0^\infty x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$
$$\int (x^2+b^2)^{-1/2}dx = \ln\left(x+\sqrt{x^2+b^2}\right)$$
$$\int (x^2+b^2)^{-1}dx = \frac{1}{b}\arctan(x/b)$$
$$\int (x^2+b^2)^{-3/2}dx = \frac{x}{b^2\sqrt{x^2+b^2}}$$
$$\int (x^2+b^2)^{-2}dx = \frac{\frac{bx}{x^2+b^2}+\arctan(x/b)}{2b^3}$$
$$\int \frac{x\,dx}{x^2+b^2} = \frac{1}{2}\ln\left(x^2+b^2\right)$$
$$\int \frac{dx}{x(x^2+b^2)} = \frac{1}{2b^2}\ln\left(\frac{x^2}{x^2+b^2}\right)$$
$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$
$$= -\frac{1}{ab}\operatorname{artanh}\left(\frac{ax}{b}\right)$$