UNL - Department of Physics and Astronomy

Preliminary Examination - Day 2
Friday, May 11, 2018

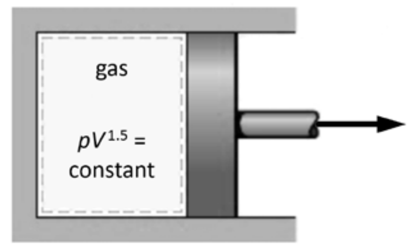
This test covers the topics of *Thermodynamics and Statistical Mechanics* (Topic 1) and *Quantum Mechanics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

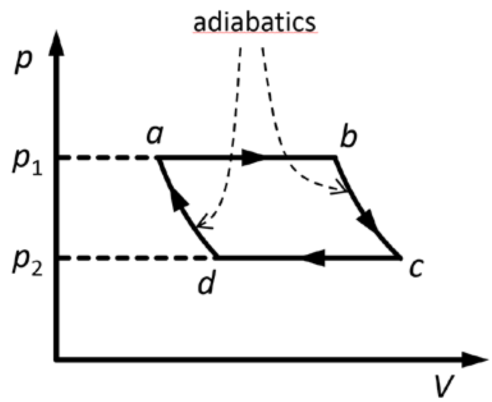
WRITE

Thermodynamics and Statistical Mechanics Group A - Answer only two Group A questions

A1 A mass of 0.4 kg of a certain gas is contained within a piston-cylinder assembly. The gas undergoes a process for which the pressure-volume relationship is $pV^{1.5} = \text{constant}$. The initial pressure is 300 kPa, the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. The change in specific internal energy of the gas in the process is $u_2 - u_1 = -55$ kJ/kg. Determine the net heat transfer for the process.



A2 Consider an engine working in a reversible cycle and using an ideal gas with constant heat capacity c_p as the working substance. The cycle consists of two processes at constant pressure, joined by two adiabatics. Which temperature of T_a, T_b, T_c, T_d is the highest and which is the lowest? Justify your answer.



A3 An automobile engine whose thermal efficiency is $\varepsilon = 22.0\%$ operates at 95.0 cycles per second and does work at the rate of 120 hp. How much heat does the engine absorb per cycle? (1 hp = 1 horsepower = 746 watts.)

A4 A mixture of 1.78 kg of water and 262 g of ice at 0°C is, in a reversible process, brought to a final equilibrium state where the water/ice ratio (by mass) is 1:1 at 0°C. Calculate the entropy change of the system during this process.

<u>Thermodynamics and Statistical Mechanics Group B</u> <u>- Answer only two Group B questions</u>

B1 Given a round table, we randomly place three legs under the table. What is probability that the table will fall?

B2 Two fluids, F_1 and F_2, of fixed volumes and constant heat capacities C_1 and C_2, are initially at temperatures T_1 and T_2 ($T_1 > T_2$), respectively. They are adiabatically insulated from each other. A quasistatically acting Carnot engine uses F_1 as heat source and F_2 as heat sink, and acts between the systems until they reach a common temperature T_0. Find this T_0, and also find the total work done by the Carnot engine.

B3 Consider the Joule-Thomson expansion in which a gas is allowed to flow slowly through a porous plug between two containers, which are otherwise isolated from each other and from their surroundings. The enthalpy is defined as $H = U + pV$. The temperature change of such an expansion is measured by the Joule-Thomson coefficient $j \equiv \left(\frac{\partial T}{\partial p}\right)_H$.

a. Show that $dH = TdS + Vdp$.

b. Subsequently, show that $j = \frac{V}{C_p}\left(T\alpha_p - 1\right)$ where the coefficient of volume expansion at constant pressure is defined as $\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$.

c. Finally, compute the coefficient *j* for an ideal gas.

B4 Consider a system of *N* particles with only three possible energy levels: 0, ε, and 2ε. The system occupies a fixed volume *V* and is in thermal equilibrium with a reservoir at temperature *T*. Ignore interactions between particles, and assume that Boltzmann statistics applies.

a. What is the partition function for a single particle in the system?

b. What is the average energy per particle?

c. For each of the three energy levels, what is probability that it is occupied in the high-temperature limit, $k_B T \gg \varepsilon$?

d. What is the average energy per particle for $k_B T \gg \varepsilon$?

e. Find the heat capacity of the system, C_V, and give the limits for $k_B T \ll \varepsilon$ and for $k_B T \gg \varepsilon$.

Quantum Mechanics Group A - Answer only two Group A questions

A1 A monochromatic point source of light radiates 25 W at a wavelength 500 nm. A metal plate is placed 0.1 m from the source. The work function of the metal is 2.1 eV.

(a) What is the photon current density when they strike the metal?

(b) What is the maximum kinetic energy of the emitted electrons?

A2 An operator B is called anti-Hermitian when $B^\dagger = -B$. Show that the expectation values of anti-Hermitian operators are purely imaginary.

A3 For a set of square-integrable function on the interval $(-\infty, \infty)$ prove that

(a) the operator $\frac{d}{dx}$ is antihermitian;

(b) the expectation value of the kinetic energy of a particle with mass m is

$$\langle T \rangle = \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx.$$

A4 Let the kets $|\varphi_i\rangle$ $(i = 1,2,3,\ldots)$ be an orthonormal basis of some Hilbert space $\mathcal{H}$. We consider the operator $P = \Sigma_i |\varphi_i\rangle\langle\varphi_i|$.

a. Show that P is idempotent, i.e. $P^2 = P$.
b. Is P Hermitian?

Quantum Mechanics Group B - Answer only two Group B questions

B1 Consider a charged oscillator, of positive charge q and mass m, which is subject to an oscillating electric field $E_0 \cos(\omega t)$. The particle's Hamiltonian is $\hat{H} = \dfrac{\hat{P}^2}{2m} + \dfrac{1}{2}k\hat{X}^2 + qE_0\hat{X}\cos(\omega t)$.

a. Calculate $d\langle\hat{X}\rangle/dt$, $d\langle\hat{P}\rangle/dt$, and $d\langle\hat{H}\rangle/dt$.

b. Find $\langle\hat{X}\rangle(t)$, the expectation value of $\hat{X}$ as a function of time.

B2 At time $t = 0$, a hydrogen atom is in the superposition state

$$\psi(\mathbf{r}, 0) = \frac{1}{\sqrt{2}}[\psi_{100}(\mathbf{r}) + \psi_{210}(\mathbf{r})]$$

where $\psi_{nlm}(\mathbf{r})$ are stationary eigenstates.

(a) Find the expectation value of the Cartesian coordinate z at $t = 0$.

(b) Find the expectation value of $z(t)$ at $t > 0$.

(c) Write the Heisenberg equation of motion for $z(t)$, and find the expectation value of $p_z(t)$ from it.

B3 (a) Using the properties of the lowering and raising angular momentum operators,

$$\langle jm|J_\pm|j'm'\rangle = [(j' \mp m')(j' \pm m' + 1)]^{1/2}\hbar\delta_{jj'}\delta_{m,m'\pm1}$$

$$J_x = \frac{1}{2}(J_+ + J_-),$$

find the matrix representation for the Cartesian component J_x of the spin operator $\mathbf{J}$ of a particle with spin $j = 1$.

(b) Find the eigenvalues and eigenstates of J_x.

(c) Suppose the particle is in the eigenstate of J_x with the eigenvalue $\hbar$, and J_z is measured. What are possible outcomes of this measurement, and what are the corresponding probabilities?

(d) Answer the question (c), if the particle is initially in the eigenstate of J_x with the eigenvalue 0.

B4 In a one-dimensional scattering experiment, a steady, uniform beam of protons with kinetic energy $E = 400$ eV and density $\rho_i = 225\ \text{m}^{-1}$ is sent to a potential step of height $V_0 = 600$ eV, so that the potential energy $V(x)$ is

$$V = 0 \ \text{ for } \ x < 0, \qquad V = V_0 \ \text{ for } \ x > 0.$$

(a) Calculate the amplitude A of the incident wave.

(b) Write down the wavefunction $\psi(x)$ for $x < 0$ and $x > 0$. All parameters (coefficients) should be expressed in terms of given quantities (ρ_i, E and V_0).

(c) Find the total proton density ρ as a function of x. Simplify it as much as you can.

(d) Show that the *total* current density $J(x) = 0$ everywhere.

(e) Explain why J is independent of the position x in spite of the fact that ρ does depend on x.

Physical Constants

speed of light	$c = 2.998 \times 10^{8}$ m/s	electrostatic constant	$k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}$ m/F
Planck's constant	$h = 6.626 \times 10^{-34}$ J·s	electron mass	$m_{\mathrm{el}} = 9.109 \times 10^{-31}$ kg
Planck's constant / 2π	$\hbar = 1.055 \times 10^{-34}$ J·s	electron rest energy	511.0 keV
Boltzmann constant	$k_{\mathrm{B}} = 1.381 \times 10^{-23}$ J/K	Compton wavelength	$\lambda_{\mathrm{C}} = h/m_{\mathrm{el}}c = 2.426$ pm
elementary charge	$e = 1.602 \times 10^{-19}$ C	proton mass	$m_{\mathrm{p}} = 1.673 \times 10^{-27}\,\mathrm{kg} = 1836 m_{\mathrm{el}}$
electric permittivity	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m	1 bohr	$a_0 = \hbar^2 / ke^2 m_{\mathrm{el}} = 0.5292$ Å
magnetic permeability	$\mu_0 = 1.257 \times 10^{-6}$ H/m	1 hartree (= 2 rydberg)	$E_{\mathrm{h}} = \hbar^2 / m_{\mathrm{el}} a_0^{\,2} = 27.21$ eV
molar gas constant	$R = 8.314$ J / mol·K	gravitational constant	$G = 6.674 \times 10^{-11}$ m^3/ kg s^2
Avogadro constant	$N_{\mathrm{A}} = 6.022 \times 10^{23}$ mol^{-1}	hc	$hc = 1240$ eV·nm

Equations That May Be Helpful

TRIGONOMETRY

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

THERMODYNAMICS

Partition function = $Z = \Sigma_i e^{-\beta E_i}$

Average energy = $\langle E \rangle = -\dfrac{\partial}{\partial\beta}\left(\ln Z\right)$

Heat capacity = $\qquad C_V = N\dfrac{d\langle E\rangle}{dT}$

Clausius' theorem: $\sum_{i=1}^{N}\dfrac{Q_i}{T_i}\le 0$, which becomes $\sum_{i=1}^{N}\dfrac{Q_i}{T_i}=0$ for a reversible cyclic process of N steps.

$$\frac{dp}{dT}=\frac{\lambda}{T\Delta V}$$

For adiabatic processes in an ideal gas with constant heat capacity, $pV^{\gamma}=\text{const.}$

$$dU = TdS - pdV$$

$$H = U + pV \qquad F = U - TS \qquad G = F + pV \qquad \Omega = F - \mu N$$

$$C_V = \left(\frac{\delta Q}{dT}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad C_p = \left(\frac{\delta Q}{dT}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p \qquad TdS = C_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \qquad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

Triple product: $\left(\dfrac{\partial X}{\partial Y}\right)_Z\cdot\left(\dfrac{\partial Y}{\partial Z}\right)_X\cdot\left(\dfrac{\partial Z}{\partial X}\right)_Y = -1$

Maxwell's relations:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V, \quad \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p, \quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V, \quad -\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p$$

Data for water

specific heat	$C = 4186$ J/(kg·K)
heat of fusion	$L_F = 334$ kJ/kg
heat of vaporization	$L_V = 2256$ kJ/kg

QUANTUM MECHANICS

Ground-state wavefunction of the hydrogen atom: $\psi(\mathbf{r}) = \dfrac{e^{-r/a_0}}{\pi^{1/2}a_0^{3/2}}$, where $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{me^2}$ is the Bohr radius, using $m\approx m_{\text{el}}$, in which m_{el} is the electron mass.

$$\psi_{nlm}(\mathbf{r}) = R_{nl}(r)Y_{lm}(\hat{\mathbf{r}}) \qquad E_n = -\frac{1}{n^2}\frac{mk^2e^4}{2\hbar^2}$$

$$R_{10}(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$$

$$R_{21}(r) = \frac{1}{3^{1/2}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$$

Particle in one-dimensional, infinitely-deep box with walls at $x = 0$ and $x = a$:
Stationary states $\psi_n = (2/a)^{1/2}\sin(n\pi x/a)$, energy levels $E_n = n^2\dfrac{\pi^2\hbar^2}{2ma^2}$

Angular momentum: $[L_x, L_y] = i\hbar L_z \quad et\ cycl.$

Ladder operators:
$$L_+ |\ell, m\rangle = \hbar\sqrt{(\ell+m+1)(\ell-m)}\,|\ell, m+1\rangle$$
$$L_- |\ell, m\rangle = \hbar\sqrt{(\ell+m)(\ell-m+1)}\,|\ell, m-1\rangle$$

Creation, annihilation operators:

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - i\frac{\hat{p}}{m\omega}\right) \qquad \hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + i\frac{\hat{p}}{m\omega}\right)$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad \hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

Probability current density: $J(x) = \dfrac{\hbar}{2mi}\left(\psi^* \dfrac{\partial\psi}{\partial x} - \psi\dfrac{\partial\psi^*}{\partial x}\right) = \dfrac{\hbar}{m}\mathrm{Im}\left(\psi^*\dfrac{\partial\psi}{\partial x}\right).$

Table Spherical harmonics and their expressions in Cartesian coordinates.

$Y_{lm}(\theta,\varphi)$	$Y_{lm}(x,y,z)$
$Y_{00}(\theta,\varphi)=\frac{1}{\sqrt{4\pi}}$	$Y_{00}(x,y,z)=\frac{1}{\sqrt{4\pi}}$
$Y_{10}(\theta,\varphi)=\sqrt{\frac{3}{4\pi}}\cos\theta$	$Y_{10}(x,y,z)=\sqrt{\frac{3}{4\pi}}\frac{z}{r}$
$Y_{1,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{3}{8\pi}}e^{\pm i\varphi}\sin\theta$	$Y_{1,\pm1}(x,y,z)=\mp\sqrt{\frac{3}{8\pi}}\frac{x\pm iy}{r}$
$Y_{20}(\theta,\varphi)=\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$	$Y_{20}(x,y,z)=\sqrt{\frac{5}{16\pi}}\frac{3z^2-r^2}{r^2}$
$Y_{2,\pm1}(\theta,\varphi)=\mp\sqrt{\frac{15}{8\pi}}e^{\pm i\varphi}\sin\theta\cos\theta$	$Y_{2,\pm1}(x,y,z)=\mp\sqrt{\frac{15}{8\pi}}\frac{(x\pm iy)z}{r^2}$
$Y_{2,\pm2}(\theta,\varphi)=\sqrt{\frac{15}{32\pi}}e^{\pm 2i\varphi}\sin^2\theta$	$Y_{2,\pm2}(x,y,z)=\mp\sqrt{\frac{15}{32\pi}}\frac{x^2-y^2\pm 2ixy}{r^2}$

$H_{\mathrm{mag}}=-\gamma\,\mathbf{S}\cdot\mathbf{B}$

Pauli matrices: $\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$, $\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Compton scattering: $\lambda'-\lambda=\lambda_{\mathrm{C}}(1-\cos\theta)$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

CARTESIAN AND SPHERICAL UNIT VECTORS

$$\hat{\mathbf{x}} = (\sin\theta\cos\phi)\hat{\mathbf{r}} + (\cos\theta\cos\phi)\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{y}} = (\sin\theta\sin\phi)\hat{\mathbf{r}} + (\cos\theta\sin\phi)\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
$e^{-a x^2}$	$\frac{\sqrt{\pi}}{2\sqrt{a}}$
$x e^{-a x^2}$	$\frac{1}{2a}$
$x^2 e^{-a x^2}$	$\frac{\sqrt{\pi}}{4a^{3/2}}$
$x^3 e^{-a x^2}$	$\frac{1}{2a^2}$
$x^4 e^{-a x^2}$	$\frac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5 e^{-a x^2}$	$\frac{1}{a^3}$
$x^6 e^{-a x^2}$	$\frac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+bx^2}\,dx =$$
$$\int_0^\infty x^n e^{-bx}\,dx = \frac{n}{b^n}$$