UNL Department of Physics and Astronomy

Preliminary Examination – Day 1

August, 2011

This test covers the topics of Mechanics (Topic 1) and Thermodynamics and Statistical Mechanics (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work 2 problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Mechanics Group A. Answer only 2 Group A questions.

A1. The trajectory of a body of mass m, measured in a frame A, is given by

$$\boldsymbol{r} = (v_o t + x_o sin\omega t)\hat{\boldsymbol{x}} + (\frac{1}{2}gt^2 + y_o cos\omega t)\hat{\boldsymbol{y}} \ ,$$

where v_0, g, ω, x_0, and y_0 are constants. The force applied to the body is known to be

$$\boldsymbol{F} = \frac{1}{2}(x_o m\omega^2 sin\omega t)\hat{\boldsymbol{x}} + \left(mg + \frac{1}{2}y_o m\omega^2 cos\omega t\right)\hat{\boldsymbol{y}}.$$

Is the frame A inertial? Explain.

A2. A mass m is attached to a spring and is oscillating as x = Acos(ωt + ϕ). A very short impulse is to be applied to the mass to stop the motion. What is the magnitude of the impulse? When must it be applied?

A3. Consider two cylindrical bodies (A and B) of equal mass and dimensions with moments of inertia about the cylindrical axis of I_A and I_B, respectively. For both bodies, the center of mass coincides with the geometric center. Each is released from rest and allowed to roll down an inclined ramp. Assuming the center of mass of each body is initially at the same location, which body will have the greater center of mass velocity at the bottom of the ramp. Why?

A4. A particle of mass m moves in a potential $V(x) = \beta x^4 - \alpha x^2$, where α and β are constants and are both greater than zero. Find the equilibria and determine their stability. Find the frequency of small oscillations of a mass about the stable equilibria. What are the dimensions of α and β ?

Mechanics Group B. Answer only 2 Group B questions.

B1. A satellite is in a circular orbit of radius r_1 about the center of the Earth. A short thrust of the satellite's engine is used to reduce the velocity by a factor s without changing its direction. This puts the satellite into an elliptical orbit having radius r_1 at apogee (farthest distance) and radius r_2 at perigee (closest distance). Find the ratio r_1/r_2 as a function of s.

B2. For the Lagrangian

$$L = \frac{m}{2}[\dot{q}_1^2(1 + sin^2 q_2) + \dot{q}_2^2] - \frac{k}{3} q_2^3$$

where k and m are constants:

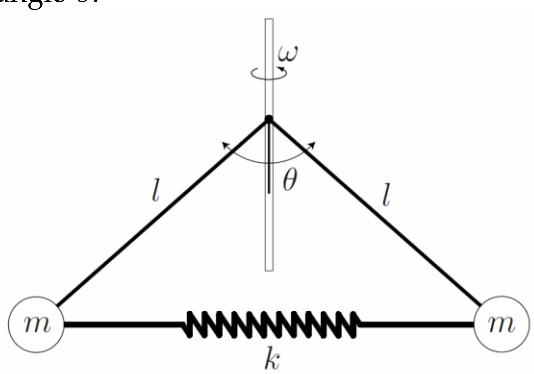
(a) Find the equations of motion. (Don't try to solve the equations.)

(b) Are there any ignorable (i.e. cyclic) coordinates? If so, what are the corresponding conserved momenta?

(c) Find the energy of the system. Is it conserved? Why?

B3. A block of mass m is released from the top of a fixed wedge. The face of the wedge is inclined at an angle θ to the horizontal. Initially, the coefficient of friction between block and the wedge is μ. As the surface of the block heats up, μ decreases linearly with the energy absorbed. Find the coefficient of friction as a function of the distance traveled by the block.

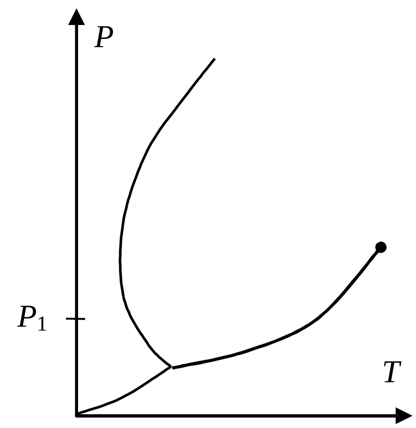
B4. A pair of masses is attached to a pivot on a vertical shaft by massless rods of length ℓ and can swing in the plane as shown. The shaft is rotating with a constant angular frequency ω. The masses are also connected to each other by a spring as shown. Find the equilibrium value of the angle θ.



Thermodynamics and Statistics Group A. Answer only 2 Group A questions.

A1. A die is thrown N times, and the ***sum*** of all N shown numbers is recorded. Find the expectation value and the standard deviation for this sum.

A2. The figure shows the phase diagram for a one-component substance. Put labels on the graph to indicate the areas of the solid (S), liquid (L), and gas (G) phase. If the solid phase is in equilibrium with the liquid phase at pressure P_1, will the solid float or sink? Is there a pressure at which the solid will neither float nor sink? Explain your reasoning.



A3. Consider a gas characterized by an unknown equation of state P=P(V,T). Express $\left(\frac{\partial P}{\partial T}\right)_V$ in terms of $\left(\frac{\partial P}{\partial V}\right)_T$ and $\left(\frac{\partial V}{\partial T}\right)_P$.

A4. Write down briefly but precisely

A)The Kelvin statement of the second law.

B) The Clausius statement of the second law.

C) The entropy statement of the second law.

Thermodynamics and Statistics Group B. Answer only 2 Group B questions.

B1. Two copper blocks (implying constant heat capacity) at temperature T_h and T_c, where $T_c < T_h$, are allowed to equilibrate by means of a reversible heat engine operating between the two blocks.

a) What is the final temperature of both blocks (when the heat engine stops producing work) in terms of T_h and T_c?

b) What is the total change in entropy of the two blocks?

B2. A reversible heat engine extracts heat Q_h from a reservoir at temperature T_h and heat $Q_m = aQ_h$, with $0 < a < 1$, from a reservoir at temperature $T_m < T_h$ while rejecting heat Q_c to a reservoir at temperature $T_c < T_m$.

Derive an expression for the efficiency of this three-reservoir heat engine in terms of a and the three reservoir temperatures T_h, T_m, and T_c. The efficiency is given by the produced work divided by the heat extracted from the two hotter reservoirs. Check your result by showing that the efficiency reduces to the Carnot efficiency expression in the limits $a \rightarrow 0$ and $T_m \rightarrow T_c$.

B3. 500 grams of ice cubes at 0°C are placed in 1 liter of water at 20°C. The system then comes to equilibrium with no heat exchange with the surroundings.

(a) Does the ice melt completely? If yes, find the temperature of the water in equilibrium. If not, find how much ice remains in equilibrium.

(b) Calculate the total change of entropy for the whole system.

B4. A certain substance in a vessel undergoes the following process:

(a) It expands isothermally while receiving 1000 J of heat from the thermostat at 1000 K.
(b) It is insulated and then expanded somewhat.
(c) It is compressed isothermally while releasing 250 J to the second thermostat at 500 K.
(d) It is insulated and then expanded somewhat.
(e) It is compressed isothermally while releasing some heat to the third thermostat at 300 K.
(f) It is insulated and then compressed in such a way that it ends up in the initial state.

Given these restrictions, what is the maximum amount of work that can be done by the system in such a process?

EQUATIONS THAT MAY BE HELPFUL

Elliptic Orbits

$$r = \frac{m\ell^2}{k} \frac{1}{1 + \epsilon \cos(\theta - \theta_0)}$$

$$\frac{r_2}{r_1} = \frac{1 - \epsilon}{1 + \epsilon}$$

$$\epsilon = \sqrt{1 + \frac{2Em\ell^2}{k^2}}$$

$$r_1 + r_2 = -\frac{k}{E}$$

Rigid Body motion

Body	Axis	I
Sphere; radius r	Any axis	$\frac{2}{5} m r^2$
Cube; side a	Any axis	$\frac{1}{6} m a^2$
Rod; radius a, length l	Along the cylindrical axis	$\frac{1}{2} m a^2$
	Through center, perpendicular to the cylindrical axis	$\frac{1}{4} m a^2 + \frac{1}{12} m l^2$

$$T = \frac{1}{2} I \omega^2$$

$$L = I \omega$$

$$\boldsymbol{a}' = \boldsymbol{a} - 2\boldsymbol{\omega} \times \boldsymbol{v}' - \dot{\boldsymbol{\omega}} \times \boldsymbol{r}' - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}')$$

General efficiency η of a heat engine producing work $|W|$ while taking in heat $|Q_h|$ is $\eta = \frac{|W|}{|Q_h|}$.

For a Carnot cycle operating as a heat engine between reservoirs at T_h and at T_c the efficiency becomes $\eta_C = \frac{T_h - T_c}{T_h}$.

Clausius' theorem: $\sum_{i=1}^{N} \frac{Q_i}{T_i} \leq 0$ which become $\sum_{i=1}^{N} \frac{Q_i}{T_i} = 0$ for a reversible cyclic process of N steps.

$\frac{dP}{dT} = \lambda/(T\Delta V)$; specific heat of water: 4186 J/(kg*K); Latent heat of ice melting: 334 J/g