UNL Department of Physics and Astronomy

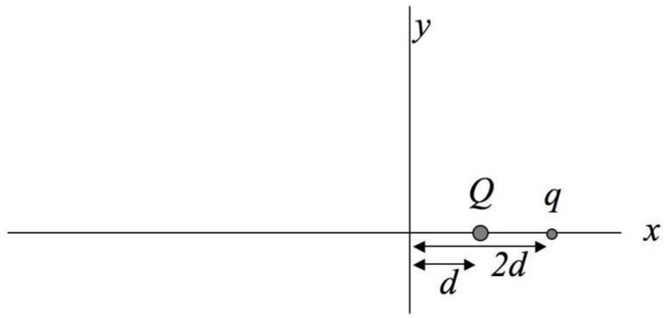
Preliminary Examination – Day 2

August, 2011

This test covers the topics of Electricity and Magnetism (Topic 1) and Quantum Mechanics (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work 2 problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Electricity and Magnetism Group A. Answer only 2 Group A questions.

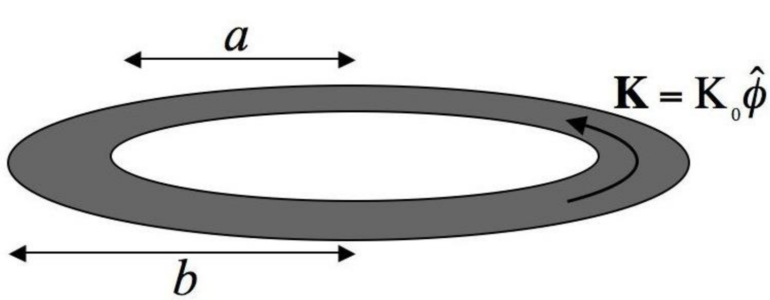
A1. A positive charge q is placed on the x-axis at $x = 2d$ and a second charge $Q = -2q$ at $x = d$. How does the net force acting on the positive charge compare to the case where an infinite, grounded plane conductor is placed in the yz-plane? Is it in the same direction? Larger, smaller, or just the same? Find the ratio of the two forces.



A2. Consider a region of space containing an E field. A Linear Isotropic Homogeneous dielectric is then brought into this region of space and the presence of the dielectric modifies the E field. Does the electrostatic energy U_E increase, decrease or stay the same? Why?

A3. A charged particle with zero initial velocity enters a region containing both an E and B field. The E field is given by $\vec{E} = E_0\hat{i}$ and the particle moves in a trajectory that exactly follows the E field. What is the direction of the B field and what is the sign of the particle's charge? (In this problem $\hat{i},\hat{j},\hat{k}$ are unit vectors along the x,y, and z direction respectively.)

A4. What is the magnetic dipole moment of a flat circular disc with inner radius a, outer radius b, and surface current density $\mathbf{K}=K_0\hat{\varphi}$**?** Hint: the dipole moment of a infinitesimally thin ring of radius r carrying a current I is $\pi r^2 I$.



Electricity and Magnetism Group B. Answer only 2 Group B questions.

B1. Consider an infinitely long solid cylinder of radius R along the z axis. A current flows up the wire with a volume current density given by $\vec{J} = J_0 \hat{z}$.

(a) Draw a figure indicating coordinate axes, the cylinder and the direction of current flow. Explain clearly the limitations that the symmetry of this structure imposes on the B field as well as the limitations resulting from the Biot-Savart Law.

(b) Using Ampere's law calculate the B field vector both inside and outside the cylinder. Draw and describe the Amperian loop to be used in each case.

B2. Consider two dipoles $\mathbf{m}_1$ and $\mathbf{m}_2$ a distance d apart oriented so that both are perpendicular to d as shown. $\mathbf{m}_1$ is fixed and $\mathbf{m}_2$ is free to move in any fashion.

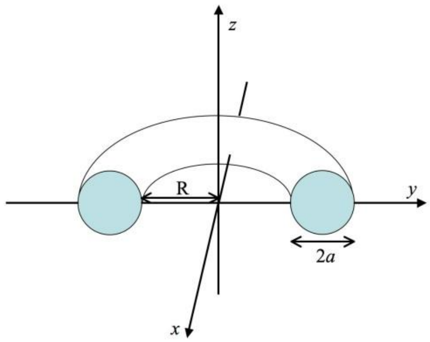
(a) Re-draw the diagram above and indicate an appropriate choice of coordinate system.

(b) What is the torque exerted by dipole $\mathbf{m}_1$ on dipole $\mathbf{m}_2$? Describe the resultant motion of $\mathbf{m}_2$ under the action of this torque.

(c) What is the force exerted by dipole $\mathbf{m}_1$ on dipole $\mathbf{m}_2$? Describe the resultant motion of $\mathbf{m}_2$ under the action of this force.

Electricity and Magnetism Group B (continued). Answer only 2 Group B questions.

B3. A cylinder of length L and radius a has a uniform magnetization M_0 directed along its axis. The cylinder is then bent into a torus of inner radius R, without changing its magnetization. The center of the torus is at the origin and the central plane of the torus lies in the x-y plane. A cut away view is shown below.



(a) Find the magnitude and direction of the bound volume current density, $\mathbf{J}_b$. Indicate the direction on the diagram (which you should redraw), on both the right and left cross sectional views.

(b) Find the magnitude and direction of the bound surface current density, $\mathbf{K}_b$. Indicate the direction on the diagram, on both the right and left cross sectional views.

(c) The **B** field everywhere inside the "hole" of the torus is zero. Using the appropriate boundary conditions, calculate the magnitude and direction of the **B** field at a point P just within the torus and on the y axis i.e. at a coordinate point (0, R+δ, 0) where δ is very small. Note: There are two tangential directions-be careful which one you choose.

B4. The electric field in a region **r > R** is given by $E_r = \dfrac{2A\cos\theta}{r^3}$, $E_\theta = \dfrac{A\sin\theta}{r^3}$ and $E_\phi = 0$, where these refer to the components of the E field in a spherical coordinate system and where A is a positive constant.

(a) Draw a figure of a sphere of radius r = R, including appropriate axes, and the *direction and magnitude* of the E field at three points at the surface of the sphere, i.e. with r=R: at the North pole, at the South pole, and at the Equator.

(b) Calculate the charge density in the region r > R. Show all work.

(c) From your results in (ii) and from the E field given in the problem, what can you say about the volume charge density? Where do the charges exist? Is the charge density isotropic? Justify your answers.

Quantum Mechanics Group A. Answer only 2 Group A questions.

A1. What is the kinetic energy of an electron that has the same momentum as an 80-keV photon?

A2. A particle with mass m is described by the normalized wavefunction

$$\psi(x) = \begin{cases} C\cos(\tfrac{1}{2}\pi x/a) & \text{for } -a < x < a \\ 0 & \text{elsewhere} \end{cases},$$

Where a and C are positive real-valued constants. If we measure the particle's momentum, what is the probability that we find it moving to the left? Explain your answer.

A3. This problem refers to motion in one dimension along the x-axis. Is $[x^2, p_x^2]$ Hermitian? Explain.

A4. A three-dimensional Hilbert space $\mathcal{H}_3$ is spanned by the complete set of orthonormal kets: $|u_1\rangle, |u_2\rangle$, and $|u_3\rangle$. At some point in time, the quantum system is in the state

$$|\psi\rangle = -\tfrac{1}{2} e^{\frac{1}{3}\pi i}\sqrt{3}\,|u_1\rangle + \tfrac{1}{2}|u_3\rangle .$$

(a) Show that $|\psi\rangle$ is normalized.

At the same point in time, we do a measurement of an observable S with the associated operator S. We find the non-degenerate value $S = s$, which corresponds to the eigenket

$$|s\rangle = -\tfrac{5}{13} i\,|u_2\rangle - \tfrac{12}{13}|u_3\rangle .$$

(b) What was the probability to find this value s?

Quantum Mechanics Group B. Answer only 2 Group B questions.

B1. A free particle of mass m, moving in a one-dimensional space, has the following normalized wavefunction in momentum space (momentum probability amplitude):

$$\langle p|\psi\rangle = \varphi(p) = \begin{cases} K & \text{for } |p| \le \dfrac{h}{2a} \\ 0 & \text{for all other } p \end{cases} \qquad (K \text{ is a real-valued constant})$$

The eigenkets $|p\rangle$ of the momentum operator p satisfy completeness: $\int_{-\infty}^{\infty} dp\,|p\rangle\langle p| = 1$.

(a) Find the constant *K*, including units.

(b) Find the wavefunction $\psi(x)$ in position space.

B2. A point particle with mass m moves in one dimension (coordinate x) in the potential

$$V(x) = \tfrac{1}{2} m \omega_0^2 x^2,$$

where ω_0 is a constant. The normalized stationary states are given by $\varphi_0(x), \varphi_1(x), \varphi_2(x), \ldots$ for increasing eigenenergies $E_0, E_1, E_2, \ldots$, respectively.

At time $t = 0$ the system is described by the normalized wave function

$$\psi(x, t=0) = \tfrac{1}{2}\sqrt{3}\, i\, \varphi_0(x) - K\varphi_2(x),$$

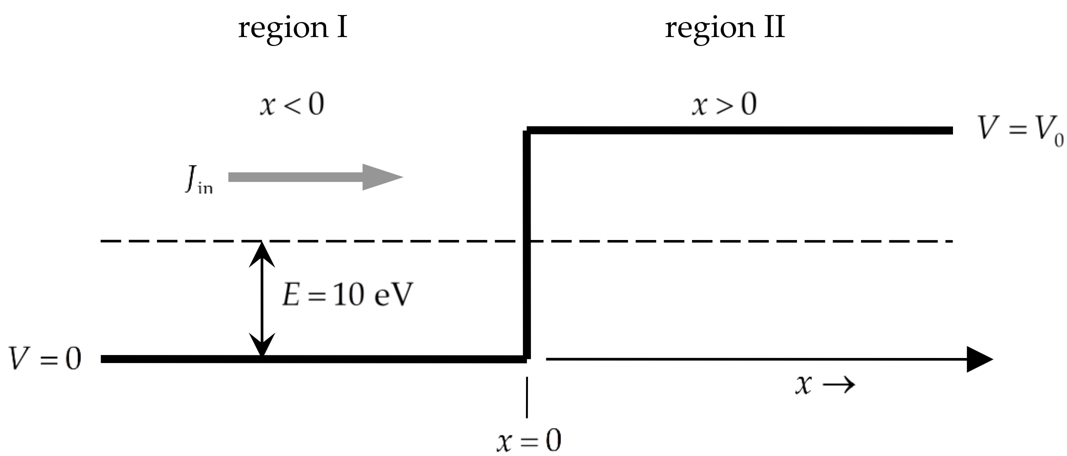
where K is a positive real-valued constant.

(a) Give the Hamiltonian for this system.

(b) Determine the value of K.

(c) If, at $t = 0$, we measure the energy, what is the probability we find the value E_0?

(d) Calculate the expectation value of the energy at $t = 0$.



B3. A beam of electrons (see above) with kinetic energy $E = 10$ eV impinges on a step with height $V_0 > E$. The incoming probability current is $J_{\text{in}} = 4.0\times10^5\ \text{s}^{-1}$ (left to right). We describe the wavefunction of the incoming electrons as $\varphi_{\text{in}}(x) = Ae^{ikx}$.

(a) Determine the quantities A and k, making sure to include units in your answers.

(b) Demonstrate that the wavefunction $\varphi_{\text{II}} = Be^{-\kappa x}$ is the solution of the time-independent Schrödinger equation in region II. Give an expression for κ.

(c) What is the probability current J_{II} in region II?

(d) What is the reflection coefficient R of this potential step?

At the position $x = d = 1.4$ Å (in region II) the absolute value of the *amplitude* of the evanescent wave has dropped to 1% of what it is at the location of the step ($x = 0$).

(e) What is the height V_0 of the step? Give your answer in eV.

B4. The wavefunction of a particle moving in one dimension (x) is given, at some time, by

$$\psi(x)=\begin{cases} K\,e^{ip_0x/\hbar}\sqrt{b^2-x^2} & \text{for } -b\le x\le b \\ 0 & \text{elsewhere} \end{cases}$$

in which p_0 and the normalization constant K are positive real numbers.

(a) Calculate the normalization constant K. What are the units of K (SI) ?

(b) For the given wavefunction, you measure the particle's position. Calculate the probability that you find it between $x=-\frac{1}{2}b$ and $x=0$.

(c) Give a symmetry argument to explain why $\langle x\rangle=0$ (no calculation).

(d) Calculate $\langle x^2\rangle$.

(e) Calculate Δx.

Physical Constants

speed of light .... $c = 2.998\times 10^{8}$ m/s

Planck's constant .... $h = 6.626\times 10^{-34}$ J·s

Planck's constant / 2π ... $\hbar = 1.055\times 10^{-34}$ J·s

elementary charge .... $e = 1.602\times 10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854\times 10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257\times 10^{-6}$ H/m

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988\times 10^{9}$ m/F

electron mass .... $m_{\text{el}} = 9.109\times 10^{-31}$ kg

proton mass .... $m_{\text{p}} = 1.673\times 10^{-27}$ kg

1 bohr .... $a_0 = 0.5292$ Å

1 hartree .... $E_{\text{h}} = 27.21$ eV

EQUATIONS THAT MAY BE HELPFUL

Electrostatics:

$$\oiint_A \vec{E}\bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \vec{\nabla}\bullet\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}X\vec{E} = 0 \qquad \vec{E} = -\nabla V$$

$$-\int_{r_1}^{r_2} \vec{E}\bullet d\vec{l} = V(r_2) - V(r_1) \qquad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r}^{\,'})}{|\vec{r}-\vec{r}^{\,'}|}$$

Work done $W = -\int_{\vec{a}}^{\vec{b}} q\vec{E}.d\vec{l} = q[V(\vec{b}) - V(\vec{a})]$

Multipole expansion:

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho(\vec{r}^{\,'})d\tau^{'} + \frac{1}{r^2}\int r^{'}Cos\theta^{'}\rho(\vec{r}^{\,'})d\tau^{'} + \frac{1}{r^3}\int (r^{'})^2(\frac{3}{2}Cos\theta^{'} - \frac{1}{2})\rho(\vec{r}^{\,'})d\tau^{'} + ...\right]$$

where the first term is the monopole term, the second is the dipole term, the third is the quadrupole term..; r and r' are the field point and source point as usual and θ' is the angle between them.

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \vec{\nabla}\bullet\vec{D} = \rho_f \qquad \rho_b = -\vec{\nabla}\bullet\vec{P} \qquad \sigma_b = \vec{P}\bullet\hat{n}$$

The above are true for ALL dielectrics. Confining ourselves to LIH dielectrics, we also have:

$$\vec{D} = \varepsilon\vec{E} \qquad \vec{P} = \chi_e\varepsilon_0\vec{E} \qquad \varepsilon = \varepsilon_0(1+\chi_e) \qquad \kappa_e = \varepsilon/\varepsilon_0 \qquad \chi_e = \kappa_e - 1$$

C(dielectric)=κ_e C (vacuum)

Boundary Conditions:

$$E_{2t} - E_{1t} = 0 \qquad E_{2n} - E_{1n} = \frac{\sigma}{\varepsilon_0}$$

Magnetostatics:

Lorentz Force $\vec{F} = q(\vec{v} \times \vec{B}) + q\vec{E}$ Current densities $\vec{I} = \int \vec{J} \bullet d\vec{A}$ $\vec{I} = \int \vec{K} \bullet \vec{dl}$

Biot Savart Law $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl}' \times \hat{\Re}}{\Re^2}$ where R is the is the vector from the source point r' to the field point r.

This can also be written (for surface currents) as $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \hat{\Re}}{\Re^2} \bullet dA'$

For a straight wire segment $B = \frac{\mu_0 I}{4\pi s}[Sin(\theta_2) - Sin(\theta_1)]$ where s is perpendicular distance from the wire.

For a circular loop of radius R, the B field at a point on the axis is given by

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$

For an infinitely long solenoid, the B field inside is given by B=μ_0NI where N is the number of turns per unit length.

$\vec{\nabla} \bullet \vec{B} = 0$ $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ Ampere's law $\oint \vec{B} \bullet \vec{dl} = \mu_0 I_{enclosed}$

Magnetic vector potential A

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \Rightarrow \vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{r - r'} d\tau'$$

Also for line and surface currents $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I}{r - r'} \vec{dl}'$ $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{r - r'} da'$

From Stokes theorem $\oint \vec{A} \bullet \vec{dl} = \int \vec{B}.\vec{da}$

For a magnetic dipole m , $\vec{A}_{dipole} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$

Magnetic dipoles

Magnetic dipole moment of a current distribution is given by **m**=I∫d**a**

Torque on a magnetic dipole in a magnetic field **τ=mXB**

Force on a magnetic dipole $\vec{F} = \nabla(\vec{m} \bullet \vec{B})$

B field of a magnetic dipole $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3}[3(\vec{m} \bullet \hat{r})\hat{r} - \hat{m}]$

Dipole-dipole interaction energy is given by

$$U_{DD} = \frac{\mu_0}{4\pi R^3} \left[(\vec{m}_1 \bullet \vec{m}_2) - 3(\vec{m}_1 \bullet \hat{R})(\vec{m}_2 \bullet \hat{R}) \right]$$ where $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$

Material with magnetization M produces a magnetic field equivalent to that of (bound)volume and surface current densities $\vec{J}_b = \vec{\nabla} X \vec{M}$ and $\vec{K}_b = \vec{M} \times \hat{n}$.

$\oint \vec{H} \bullet d\vec{l} = I_{freeenclosed}$ $\quad\quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

For linear magnetic material

M=χ_m**H** and **B**=$\mu_0(1+\chi_m)$**H** **or B**=μ**H**

Boundary Conditions: $B_{2n}-B_{1n}=0$ $\quad\quad B_{2//}-B_{1//}=\mu_0 K$

Maxwell's Equations in vacuum:

1.$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$ Gauss' Law

2.$\vec{\nabla} \bullet \vec{B} = 0$

3.$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faradays law

4. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in LIH media :

1.$\vec{\nabla} \bullet \vec{E} = \frac{\rho_f}{\varepsilon}$ Gauss' Law

2.$\vec{\nabla} \bullet \vec{B} = 0$

3.$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ Faradays law

4. $\vec{\nabla} \times \vec{B} = \mu \vec{J}_f + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Alternative ways of writing Faraday's Law $\oint \vec{E} \bullet d\vec{l} = -\frac{d\phi}{dt}$

Mutual and self inductance: $\phi_2=M_{21}I_1$ and $M_{21}=M_{12}$

ϕ=LI

Energy stored in a magnetic field: $W = \frac{1}{2\varepsilon_0}\int_V B^2 d\tau = \frac{1}{2}LI^2 = \frac{1}{2}\oint \vec{A} \bullet \vec{I} dl$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

- *Schrödinger Equation.* General and time-independent:

$$
\begin{aligned}
i\hbar\frac{\partial\Psi}{\partial t} &= \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi \\
H\psi &= E\psi \\
\Psi &= \psi e^{-iEt/\hbar}
\end{aligned}
\qquad (1)
$$

- *Formalism and Operator Algebra.* This is probably actually the easiest topic for people who have taken 143a. Some equations of use are:

$$
\begin{aligned}
\langle\alpha|\beta\rangle &= \langle\beta|\alpha\rangle^* \\
\hat{O} &= \hat{O}^\dagger \text{ if } \hat{O} \text{ is Hermitian} \\
\langle\alpha|\hat{O}|\beta\rangle &= \beta\langle\alpha|\beta\rangle \text{ if } |\beta\rangle \text{ is an eigenstate of } \hat{O} \\
&= \int_{-\infty}^{\infty} \mathrm{d}x\, \alpha^*(x)\hat{O}(x)\beta(x) \text{ in 1D} \\
&= A^\dagger \times O \times B \text{ as matrices} \\
[\hat{A},\hat{B}] &= \hat{A}\hat{B} - \hat{B}\hat{A} \\
\therefore\ [\hat{A},\hat{B}] &= -[\hat{B},\hat{A}] \\
[\hat{A}\hat{B},\hat{C}] &= [\hat{A},\hat{C}]\hat{B} + \hat{A}[\hat{B},\hat{C}]
\end{aligned}
\qquad (2)
$$

A Hermitian matrix has real eigenvalues; observables thus must be represented by Hermitian operators.

- *Position and momentum.*

$$
\begin{aligned}
\hat{p} &= -i\hbar\nabla \\
[\hat{x},\hat{p}] &= i\hbar \\
[\hat{f}(x),\hat{p}] &= i\hbar\frac{\mathrm{d}f}{\mathrm{d}x}
\end{aligned}
\qquad (3)
$$

$$
\psi(x) = \langle x|\psi\rangle = \int_{-\infty}^{\infty} dp\,\langle x|p\rangle\langle p|\psi\rangle = \int_{-\infty}^{\infty} dp\,\frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}\,\varphi(p)
$$