UNL Department of Physics and Astronomy

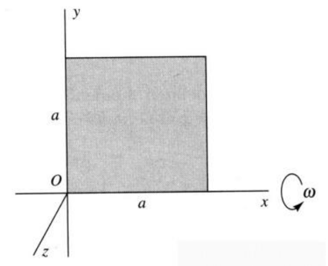
Preliminary Examination – Day 1

9 August, 2012

This test covers the topics of Mechanics (Topic 1) and Thermodynamics and Statistical Mechanics (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work 2 problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Mechanics Group A. Answer only 2 Group A questions.

A1. Find the moment of inertia of a uniform square lamina (thin sheet) of side a and mass m about (a) the x-axis, (b) the y-axis, and (c) the z-axis.



A2. A particle of mass m and angular momentum L moves in a central force field given by
$f(r)=-kr$.

(a) If the particle is in a circular orbit, what is its radius?

(b) If the particle instead has energy $E = 2L(k/m)^{1/2}$, what are the maximum and minimum radii it reaches?

$V_{\mathrm{eff}}(r) = V(r) + L^2/(2mr^2)$

A3. Find the force law for a central-force field that allows a particle to move in a logarithmic spiral orbit $r = ke^{\alpha\theta}$, where k and α are constants.

Note: the equation of the orbit

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2}F\left(\frac{1}{u}\right)$$

$$u = \frac{1}{r}, \quad L = mr^2\dot{\theta} = const$$

A4. Given the two-dimensional potential energy function

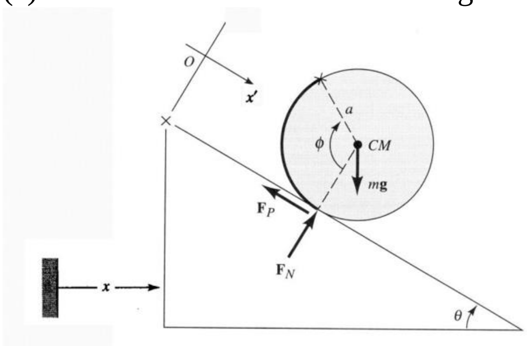
$$V(\mathbf{r}) = V_0 - \tfrac{1}{2}k\delta^2 e^{-r^2/\delta^2}$$

Where $\boldsymbol{r} = \boldsymbol{i}\,x + \boldsymbol{j}\,y$, and V_o, k, and δ are constants, find the force function.

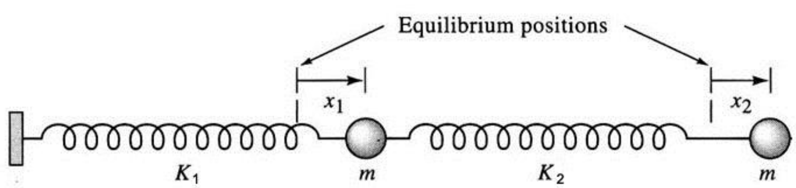
Mechanics Group B. Answer only 2 Group B questions.

B1. A ball of mass m rolls down a movable wedge of mass M. The angle of the wedge is θ, and it is free to slide on a smooth horizontal surface. The contact between the ball and the wedge is perfectly rough.

(a) How many degrees of freedom does this system have?
(b) Choose a suitable set of generalized coordinates and find the Lagrangian for this system.
(c) Find the acceleration of the wedge along the horizontal surface.

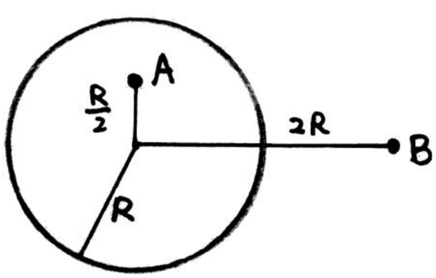


B2. In the system of coupled oscillators shown in the figure, the balls have the same mass m and $K_1 = (3/2)K_2$.
(a) Find the frequencies for the two normal modes.
(b) Find and describe the motion for each of the normal modes.



B3. The force acting on a particle of mass m is given by $F = kvx$ in which k is a positive constant and v is velocity along x-axis. The particle passes through the origin with speed v_o at t = 0. Find x as a function of t.

B4. A planet of radius R has a non-uniform density ρ = cr, where c is a constant. Calculate the magnitude of the gravitational field at points A and B.



Thermodynamics and Statistics Group A. Answer only 2 Group A questions.

A1. 2 kg of water with approximately constant heat capacity

$$C_P^M = \frac{1}{M}\frac{Q}{\Delta T} = 4.2\ kJ/(kg\ K)$$

is initially at temperature T_i =283.15 K. The water is brought into contact with a heat reservoir of constant temperature T_R=363.15 K.

a) What is the change in entropy of the entire system of water and reservoir?

b) Is the process reversible or irreversible? Explain in a brief statement.

A2.

a) Consider the reversible isothermal expansion of an ideal gas in contact with a heat reservoir at temperature T. The volume of the gas doubles during the expansion process. Calculate the entropy change of the gas. The amount of the gas is n moles.
b) Consider now the free expansion of the same amount of ideal gas taking place inside an insulated container with rigid walls after an internal partitioning wall is punctured. Again the volume of the ideal gas doubles. Find the entropy change of the gas.
c) Compare the results obtained in a) and b). Reconcile them with the Second Law of Thermodynamics, which requires that the entropy stay constant in a reversible process and increase in an irreversible one.

A3. Explain why internal energy is a function of state. Show mathematically (with the help of differentials) that exchanged heat is not a function of state. Hint: Use the differential form of the First Law.

A4. What is the minimal amount of electric energy required to operate a heat pump in order to deliver 10 kJ of heat to a house, if the outside temperature is 0°C and the room temperature is 20°C? (Assume that the room temperature does not change appreciably in the process.)

Thermodynamics and Statistics Group B. Answer only 2 Group B questions.

B1. Calculate $\left(\frac{\partial U}{\partial P}\right)_T$ and $\left(\frac{\partial T}{\partial P}\right)_U$ for a gas with the property $U=U(T)$.

B2. a) Derive the differential dH of the enthalpy H(S,P) with the help of a Legendre transformation. Start from the differential dU of the internal energy U(S,V).

b) Calculate $\left(\frac{\partial S}{\partial P}\right)_H$. Express the result in terms of V and T. Hint: rearrange dH.

B3. The Maxwell Boltzmann velocity distribution function reads

$$f_1(\vec{p}) = \frac{1}{\left(2\pi m k_B T\right)^{3/2}} e^{-\frac{p^2}{2mk_BT}} ,$$

where $f_1(\vec{p})d^3p$ is the probability to find a particle in the momentum element d^3p. Use the Maxwell Boltzmann velocity distribution function to calculate the Maxwell Boltzmann speed distribution function $f(v)$ where $f(v)\,dv$ is the probability of finding a particle with its speed in the interval $[v, v+dv]$.

B4. A copper block with heat capacity C_P is initially at temperature T_h. A reversible heat engine is operated between this block and a very large heat reservoir, whose temperature is $T_c<T_h$. In the course of this process, the copper block eventually reaches thermal equilibrium with the heat reservoir.

a) How much work can you extract from the process?

b) What is the total change in entropy of the copper block and the reservoir? Provide an answer in a brief statement without calculation. Calculate explicitly the entropy change of the copper block, the reservoir and the total system.

EQUATIONS THAT MAY BE HELPFUL

General efficiency η of a heat engine producing work $|W|$ while taking in heat $|Q_h|$ is $\eta = \dfrac{|W|}{|Q_h|}$.

For a Carnot cycle operating as a heat engine between reservoirs at T_h and at T_c the efficiency becomes $\eta_C = \dfrac{T_h - T_c}{T_h}$.

Clausius' theorem: $\sum_{i=1}^{N} \frac{Q_i}{T_i} \le 0$ which become $\sum_{i=1}^{N} \frac{Q_i}{T_i} = 0$ for a reversible cyclic process of N steps.

$\frac{dP}{dT} = \lambda/(T\Delta V)$; specific heat of water: 4186 J/(kg*K); Latent heat of ice melting: 334 J/g

$$\vec{a'} = \vec{a} - 2\vec{\omega} \times \vec{v'} - \dot{\vec{\omega}} \times \vec{r'} - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$$

H = E + PV   F = E – TS   G = F + PV   Ω = F – μN

dE = TdS – PdV + μdN   dS = dE/T + PdV/T – μdN/T   dH = TdS + VdP + μdN

dF = -SdT – PdV + μdN   dG = -SdT + VdP + μdN   dΩ = -SdT – PdV – Ndμ

E.1 ALGEBRAIC FUNCTIONS

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \qquad \left|\tan^{-1}\left(\frac{x}{a}\right)\right| < \frac{\pi}{2} \qquad \text{(E.1)}$$

$$\int \frac{x\,dx}{a^2 + x^2} = \frac{1}{2}\ln(a^2 + x^2) \qquad \text{(E.2)}$$

$$\int \frac{dx}{x(a^2 + x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2 + x^2}\right) \qquad \text{(E.3)}$$

$$\int \frac{dx}{a^2x^2 - b^2} = \frac{1}{2ab}\ln\left(\frac{ax - b}{ax + b}\right) \qquad \text{(E.4a)}$$

$$= -\frac{1}{ab}\coth^{-1}\left(\frac{ax}{b}\right) \qquad \text{(E.4b)}$$

$$= -\frac{1}{ab}\tanh^{-1}\left(\frac{ax}{b}\right), \qquad a^2x^2 < b^2 \qquad \text{(E.4c)}$$