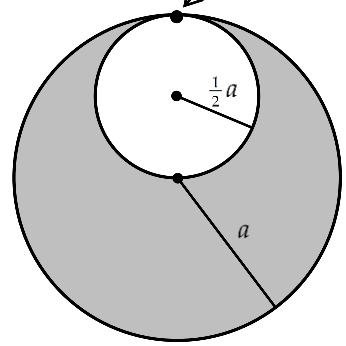
UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
August 15, 2013

This test covers the topics of *Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Mechanics Group A - Answer only two Group A questions



A1 (See adjacent figure.) The thin disk of radius a has a cavity of radius $a/2$ centered at a point $a/2$ from the center of the disk and hangs at a point at top of the disk. Find the period of the physical pendulum. The mass density of the disk is σ (mass/unit area).

A2 A particle (mass m) is falling vertically under the influence of gravity, and a frictional force $|\mathbf{F}| = kv$ (v is speed, k is a positive constant) is present.

a. Obtain the equation of motion (for the vertical coordinate only).

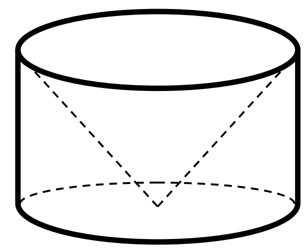
b. Integrate the equation to obtain the velocity as a function of time; find the terminal velocity.

A3 Certain neutron stars (extremely dense stars) are believed to be rotating at about 1 revolution per second. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?

A4 A railroad freight car of mass 3.18×10^4 kg collides with a stationary car. They couple together, and 27.0% of the initial kinetic energy is transferred to thermal energy, sound, vibrations, and so on. Find the mass of the car that was initially stationary.

<u>Mechanics Group B</u> <u>- Answer only two Group B questions</u>

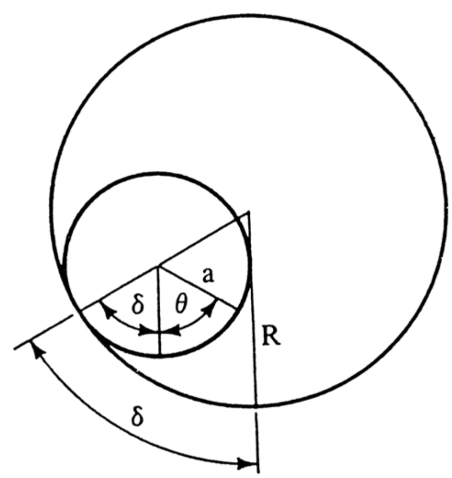
B1 A solid cylinder of radius 4 m, height 5 m, and volume density 2500 kg/m^3 has a conical section removed from its middle. The removed cone has a base radius of 4 m and a height of 5 m; its axis of symmetry is the same as that of the initial cylinder.



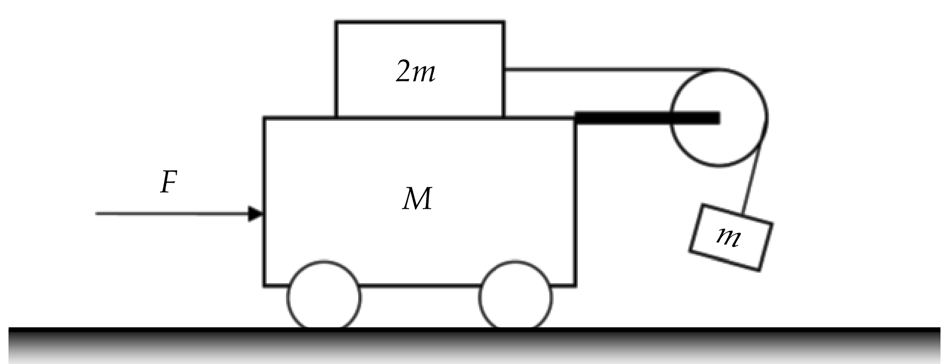
What is the moment of inertia of the remaining solid about its axis of cylindrical symmetry?

B2 A solid homogeneous cylinder of radius a rolls without slipping on the inside of a stationary larger cylinder of radius R.

a. Write down the Lagrangian function for this system.
b. Find the equation of motion.
c. Find the period of small oscillations about the stable equilibrium position.

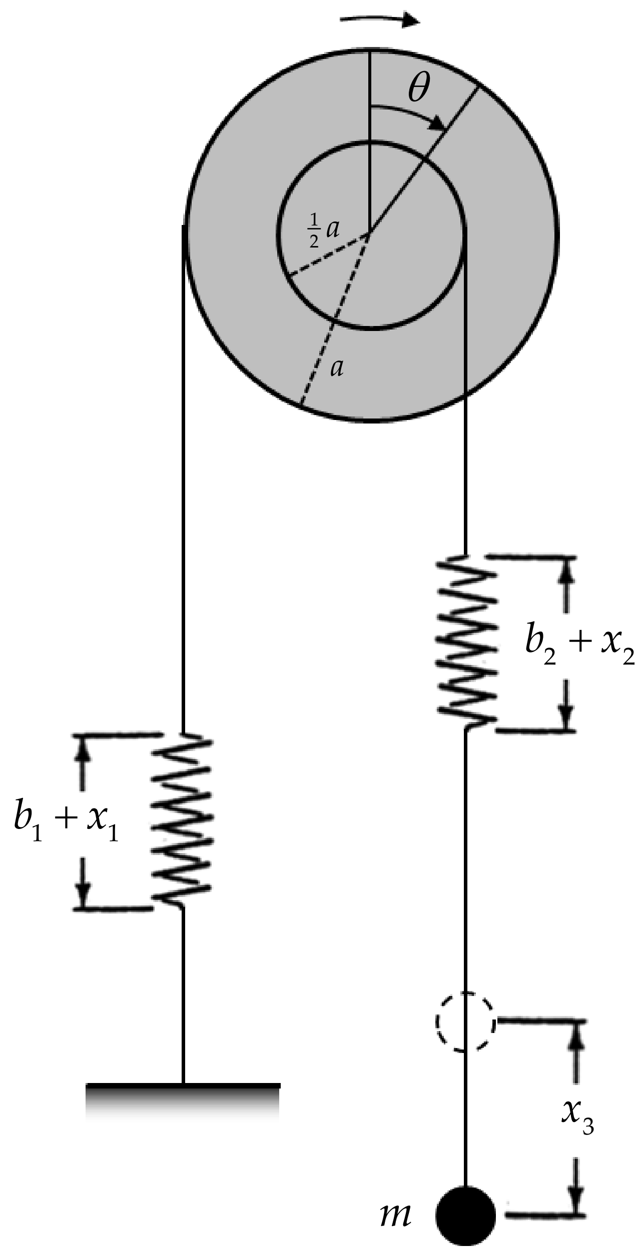


B3 A cart of mass M rolls without friction on a flat surface under the action of a force F. The cart supports a massless, frictionless pulley over which passes a light cord attached at either end to masses m and $2m$. The coefficient of static friction between the top of the cart and the upper mass $2m$ is 0.2. What is the minimum force F that must be applied to the cart in order that the upper mass $2m$ not slip on the top of the cart? $M = 10$ kg; $m = 1$ kg.



B4 A uniform disk of radius a and mass M rotates about a fixed axis. A massless cord is fixed to a point on the outside circumference and leads to a massless spring (spring constant k_1) which is in turn fastened to a fixed point. At a radius $a/2$, another cord is fastened to a spring (spring constant k_2) which connects to a mass m.

Set up Lagrange's equations for the disk and the weight. (Do not solve.)



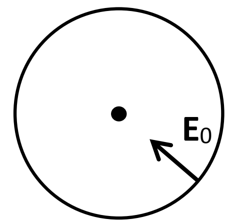
The quantities b_1 and b_2 are the original lengths of the left and right springs when the whole system is at equilibrium and the disk is not rotating.

In this equilibrium state, the quantities x_1, x_2, x_3, and θ all equal zero.

Electrodynamics Group A - Answer only two Group A questions

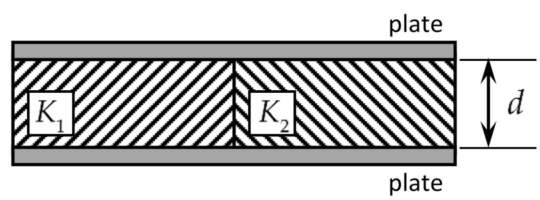
A1 In a certain region of space, the free current density $\mathbf{J}_{\text{free}}$ is given by $\mathbf{J}_{\text{free}} = J_0(y^2 + z^2)\hat{\mathbf{x}}$ where $J_0 = 500\ \text{A/m}^4$. What is the *B*-field vector at $(x,y,z) = (1,1,1)\ \text{m}$?

A2 A capacitor comprises a long thin straight wire and a long thin, cylindrical shell of radius *R*. The electric field outside the shell is zero everywhere; the field immediately inside the cylinder has magnitude E_0 and points toward the wire.



Calculate the line charge density, λ, on the wire.

A3 An ideal parallel-plate capacitor (plate area *A*) has the gap between its plates filled with two slabs of different dielectrics, with dielectric constants K_1 and K_2 (see figure). The thickness of the slabs equals the plate separation d, and each slab fills half of the volume between the plates.

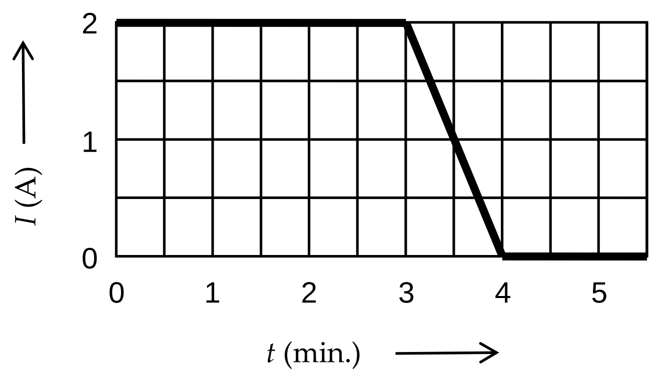


Calculate the capacitance of this capacitor.

A4 An ohmic resistor is immersed in 200 g of water at room temperature (20°C). At time $t = 0$ the resistor is connected to a DC battery with $V_B = 120\ \text{V}$. The graph shows the current as a function of time. Initially, the current is constant, but after 3 min. the battery starts to die, causing the current to gradually drop to zero. Assume the resistor's resistance is temperature-independent, and that the water is thermally insulated from the environment.

a. Calculate the resistance *R* of the resistor.
b. Find the final temperature of the water.

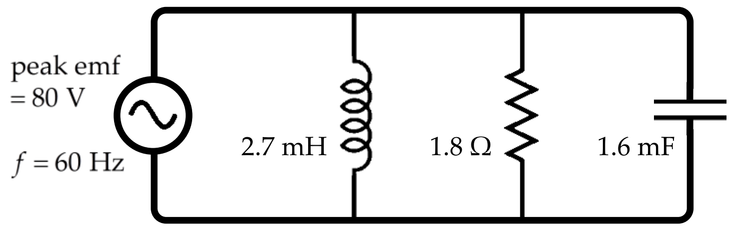
Info: the specific heat of water is $c_w = 4.18\ \text{J g}^{-1}\ \text{K}^{-1}$.



Electrodynamics Group B - Answer only two Group B questions

B1 A solid cylinder of radius R and infinite length, with its axis of symmetry being the z-axis, has a polarization field $\mathbf{P} = P_0(1 - r/R)\hat{\mathbf{z}}$. (Here r is the perpendicular distance from the z-axis.) In addition, it has a uniform surface charge density on its surface of $+\sigma$.
What is the electric field everywhere?

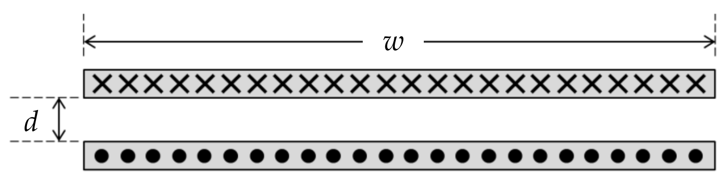
B2 We consider the adjacent circuit. It is driven by a (sinusoidal) AC generator with 60 cycles/sec. ($f = 60$ Hz) and peak voltage amplitude $\mathcal{E} = 80$ V. We consider the circuit's *steady-state* behavior.



Calculate …

a. … the current through the inductor as a function of time, $I_L(t)$
b. … the current through the resistor as a function of time, $I_R(t)$
c. … the current through the capacitor as a function of time, $I_C(t)$
d. … the amplitude of total current through the generator , I
e. … the circuit's impedance Z
f. … the phase angle φ by which the total current $I(t)$ lags the generator's emf

B3 Equal but opposite currents $+I$ and $-I$ flow in two long parallel strips as shown in the figure. The currents are uniformly distributed over the strips. The width of each strip is w, and the distance between them is d ($w \gg d$).



a. Find the magnetic field between the strips. (Neglect edge effects).
b. Calculate the magnetic field energy per unit length.
c. What is the self-inductance per unit length?

B4 A solid sphere of radius R, centered at the origin, is uniformly charged with charge density ρ. Then a small sphere is removed, making a spherical cavity centered at a point **a** within the bigger sphere. What is the electric field vector in the cavity?

Physical Constants

speed of light .... $c = 2.998 \times 10^{8}$ m/s

Planck's constant .... $h = 6.626 \times 10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055 \times 10^{-34}$ J·s

Boltzmann constant ...... $k_B = 1.381 \times 10^{-23}$ J/K

elementary charge .... $e = 1.602 \times 10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257 \times 10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

electrostatic constant .... $k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}$ m/F

electron mass .... $m_{el} = 9.109 \times 10^{-31}$ kg

proton mass .... $m_p = 1.673 \times 10^{-27}$ kg

1 bohr............................. $a_0 = 0.5292$ Å

1 hartree .... $E_h = 27.21$ eV

Avogrado constant ..... $N_A = 6.022 \times 10^{23}$ mol^{-1}

gravitational constant.. $G = 6.674 \times 10^{-11}$ m^3/ kg s^2

EQUATIONS THAT MAY BE HELPFUL

ELECTROSTATICS:

$$\oint\!\!\oint_A \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{enc}}{\varepsilon_0} \qquad \vec{\mathbf{E}} = -\vec{\nabla} V$$

$$-\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\ell} = V(r_2) - V(r_1) \qquad V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0} \frac{q(\vec{\mathbf{r}}')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}$$

work done $W = -\int_{\vec{\mathbf{a}}}^{\vec{\mathbf{b}}} q\vec{\mathbf{E}} \cdot d\vec{\ell} = q\left[V(\vec{\mathbf{b}}) - V(\vec{\mathbf{a}})\right]$

Multipole expansion:

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho(\vec{\mathbf{r}}')d\tau' + \frac{1}{r^2}\int r'\cos(\theta')\rho(\vec{\mathbf{r}}')d\tau' + \frac{1}{r^3}\int (r')^2\left\{\tfrac{3}{2}\cos(\theta') - \tfrac{1}{2}\right\}\rho(\vec{\mathbf{r}}')d\tau' + \ldots\right]$$

where the first term is the monopole term, the second is the dipole term, the third is the quadrupole term … ; $\vec{\mathbf{r}}$ and $\vec{\mathbf{r}}'$ are field point and source point and θ' is the angle between them.

$$\vec{\mathbf{D}} = \varepsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}} \qquad \vec{\nabla} \cdot \vec{\mathbf{D}} = \rho_f \qquad \rho_b = -\vec{\nabla} \cdot \vec{\mathbf{P}} \qquad \sigma_b = \vec{\mathbf{P}} \cdot \hat{\mathbf{n}}$$

The above are true for *all* dielectrics. Confining ourselves to linear, isotropic, and homogeneous (LIH) dielectrics, we also have:

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{P}} = \chi_e\varepsilon_0\vec{\mathbf{E}} \qquad \varepsilon = \varepsilon_0(1 + \chi_e) \qquad \kappa_e = \varepsilon / \varepsilon_0 \qquad \chi_e = \kappa_e - 1$$

$C(\text{dielectric}) = \kappa_e C(\text{vacuum})$

Boundary conditions: $E_{2t} - E_{1t} = 0$, $E_{2n} - E_{1n} = \dfrac{\sigma}{\varepsilon_0}$

MAGNETOSTATICS:

Lorentz Force: $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) + q\vec{\mathbf{E}}$ Current densities: $I = \int \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$, $I = \int \vec{\mathbf{K}} \cdot d\vec{\ell}$

Biot-Savart Law: $\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \dfrac{\mu_0}{4\pi} \int \dfrac{I d\vec{\ell} \times \hat{\mathbf{R}}}{R^2}$ ($\vec{\mathbf{R}}$ is vector from source point to field point $\vec{\mathbf{r}}$) .

For surface currents: $\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \dfrac{\mu_0}{4\pi} \int \dfrac{\vec{\mathbf{K}} \times \hat{\mathbf{R}}}{R^2} da$.

For straight wire segment: $B = \dfrac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]$ where s is perpendicular distance from wire.

For circular loop of radius R, the B-field at a point on the axis is $B = \tfrac{1}{2}\mu_0 I \dfrac{R^2}{(R^2 + z^2)^{3/2}}$.

Infinitely long solenoid: B-field inside is $B = \mu_0 n I$ (n is number of turns per unit length).

Ampere's law: $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}}$.

Magnetic vector potential A

$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$

$\nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}} \quad \Rightarrow \quad \vec{\mathbf{A}} = \dfrac{\mu_0}{4\pi} \int \dfrac{\vec{\mathbf{J}}}{r - r'} d\tau'$

For line and surface currents $\vec{\mathbf{A}} = \dfrac{\mu_0}{4\pi} \int \dfrac{I}{r - r'} d\vec{\ell} \qquad \vec{\mathbf{A}} = \dfrac{\mu_0}{4\pi} \int \dfrac{\vec{\mathbf{K}}}{r - r'} da'$

From Stokes' theorem $\oint \vec{\mathbf{A}} \cdot d\vec{\ell} = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$

For a magnetic dipole $\vec{\mathbf{m}}$, $\vec{\mathbf{A}}_{\text{dipole}} = \dfrac{\mu_0}{4\pi} \dfrac{\vec{\mathbf{m}} \times \hat{\mathbf{r}}}{r^2}$

Magnetic dipoles

Magnetic dipole moment of a current distribution is given by $\vec{\mathbf{m}} = I \int d\vec{\mathbf{a}}$.

Force on magnetic dipole $\vec{\mathbf{F}} = \vec{\nabla}(\vec{\mathbf{m}} \cdot \vec{\mathbf{B}})$

Torque on magnetic dipole $\vec{\boldsymbol{\tau}} = \vec{\mathbf{m}} \times \vec{\mathbf{B}}$

B-field of magnetic dipole $\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \dfrac{\mu_0}{4\pi} \dfrac{1}{r^3}\left[3(\vec{\mathbf{m}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \vec{\mathbf{m}}\right]$

Dipole-dipole interaction energy is $U_{\text{DD}} = \frac{\mu_0}{4\pi R^3}\left[(\vec{\mathbf{m}}_1 \cdot \vec{\mathbf{m}}_2) - 3(\vec{\mathbf{m}}_1 \cdot \hat{\mathbf{R}})(\vec{\mathbf{m}}_2 \cdot \hat{\mathbf{R}})\right]$, where $\vec{\mathbf{R}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$

Material with magnetization M

produces a magnetic field equivalent to that of (bound) volume and surface current densities

$\vec{\mathbf{J}}_b = \vec{\nabla} \times \vec{\mathbf{M}}$ and $\vec{\mathbf{K}}_b = \vec{\mathbf{M}} \times \hat{\mathbf{n}}$.

$\oint \vec{\mathbf{H}} \cdot d\vec{\ell} = I_{\text{free, enclosed}}$ $\qquad$ $\vec{\mathbf{H}} = \vec{\mathbf{B}} / \mu_0 - \vec{\mathbf{M}}$

For linear magnetic material $\vec{\mathbf{M}} = \chi_{\text{m}} \vec{\mathbf{H}}$ and $\vec{\mathbf{B}} = \mu_0 (1 + \chi_{\text{m}}) \vec{\mathbf{H}}$ or $\vec{\mathbf{B}} = \mu \vec{\mathbf{H}}$

Boundary conditions: $B_{2\text{n}} - B_{1\text{n}} = 0$ $\qquad$ $B_{2\|} - B_{1\|} = \mu_0 K$

Maxwell's Equations in vacuum:

1. $\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$
3. $\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$ Faraday's Law
4. $\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \varepsilon_0 \mu_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media:

1. $\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho_{\text{f}}}{\varepsilon}$ Gauss' Law
2. $\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$
3. $\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$ Faraday's Law
4. $\vec{\nabla} \times \vec{\mathbf{B}} = \mu \vec{\mathbf{J}}_{\text{f}} + \varepsilon \mu \frac{\partial \vec{\mathbf{E}}}{\partial t}$ Ampere's Law with Maxwell's correction

Alternative way of writing Faraday's Law: $\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d\Phi}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\qquad \Phi = LI$

Energy stored in electric, magnetic field:

$$W = \tfrac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2 / 2C$$

$$W = \tfrac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\oint \vec{\mathbf{A}} \cdot \vec{\mathbf{I}}\, d\ell$$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$