UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
August 14, 2014

This test covers the topics of *Quantum Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Quantum Mechanics Group A - Answer only two Group A questions

A1 A plane monochromatic electromagnetic wave has intensity 5 W/cm^2 and wavelength 500 nm. Find the number of photons per unit volume.

A2 The angular momentum quantum number of the electron in a hydrogen atom is $\ell = 6$. What is its lowest possible energy (in eV)?

A3 An electron is moving in a rectangular potential well of depth 20 eV. Suppose the electron energy is 5 eV relative to the bottom of the well. How far into the exterior region beyond the well does the electron wavefunction penetrate? (You should give an estimate).

A4 Muonium is an atom consisting of a proton and a muon, a negatively-charged particle whose mass is 207 times that of the electron. Find the ground-state energy of muonium .

Quantum Mechanics Group B - Answer only two Group B questions

B1 A 200-keV photon collides with an electron at rest.

a. What should be the direction of the scattered photon in order to obtain the maximum kinetic energy for the recoil electron?
b. Calculate this kinetic energy.

B2 Consider a particle of mass m moving in the one-dimensional potential $V(x) = ax^4$, $a > 0$. Using the uncertainty principle, estimate the energy of the ground state.

B3 Consider a particle of mass m in an infinite square well such that

$$V(x) = \begin{cases} 0 & \text{for } -\tfrac{1}{2}a < x < \tfrac{1}{2}a \\ \infty & \text{elsewhere} \end{cases}$$

The particle is initially described by the superposition of the ground and first excited states of the well:

$$\Psi(x, t=0) = C\,[\,\psi_1(x) + \psi_2(x)\,]\,.$$

a. Find C assuming that ψ_1 and ψ_2 are normalized.
b. Find $\Psi(x,t)$ and the average energy of the particle for an arbitrary t.
c. Show that the average particle position $\langle x \rangle$ oscillates with time as $\langle x \rangle = A\cos(\omega t)$, where $A = \int x\,\psi_1^*(x)\,\psi_2(x)\,dx$ (you don't have to do this integral).
d. Find the frequency ω, and the time T for the particle to oscillate back and forth in the well once.
e. Calculate the same time classically assuming that the energy equals the average, $\tfrac{1}{2}(E_1 + E_2)$. Find the ratio of the quantum time to the classical time.

Note: The energy levels in the well are $E_n = n^2 \dfrac{\pi^2 \hbar^2}{2ma^2}$.

B4 The radial part of the wavefunction for the hydrogen atom in the 3*d* state is given by

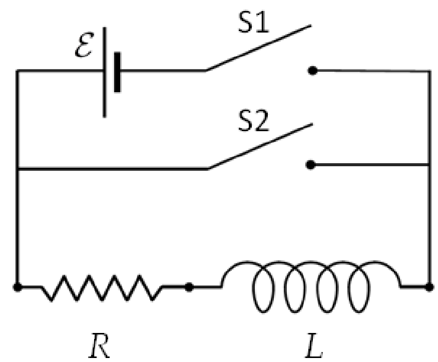
$$R_{3d}(r) = Ar^2 e^{-r/3a_0},$$

where A is a constant and a_0 is the Bohr radius. In parts *a.* and *b.* the answers should be expressed in terms of the Bohr radius (don't assume A to be known). In this 3*d* state:

a. Calculate the average value of r.
b. Calculate the most probable value of r.
c. Calculate the orbital angular momentum in units of $\hbar$.
d. Calculate the energy of this state in eV.

<u>Electrodynamics Group A</u> <u>- Answer only two Group A questions</u>

A1 Consider the circuit in the adjacent diagram. Initially, switch 1 (S1) is closed, and remains closed until a steady-state current in the circuit is reached. The battery potential is 5 V, $L = 4$ mH, and $R = 2\ \Omega$.

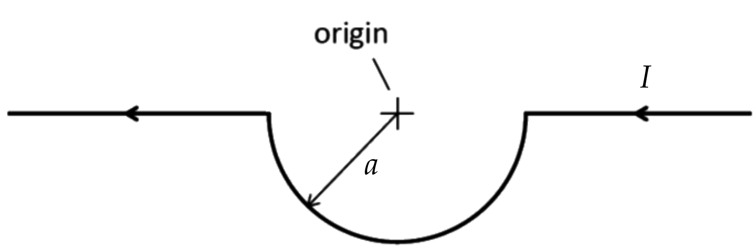


a. What is the energy stored in the inductor?

Now, S1 is opened at the same time S2 is closed.

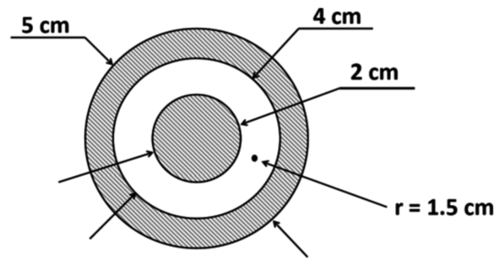
b. How much power is being dissipated in the resistor 5 ms after S2 is closed?

A2 A long, current-carrying wire extends to infinity along the $+x$ and $-x$ axes. Near the origin, it is given a semi-circular deformation of radius a as shown. The wire carries a current I that goes from the right to the left.



What is the magnitude and direction of the B-field at the origin?

A3 Two infinitely long, concentric, cylindrical conductors have a cross section as shown. A free charge of +15 pC/m exists on the outer cylinder. The radial electric field at $r = 1.5$ cm (the point indicated in the diagram) points outward and has a magnitude of 7 N/C. What is the magnitude and sign of the areal charge density residing on the outer surface of the outer cylinder at $r = 2.5$ cm?



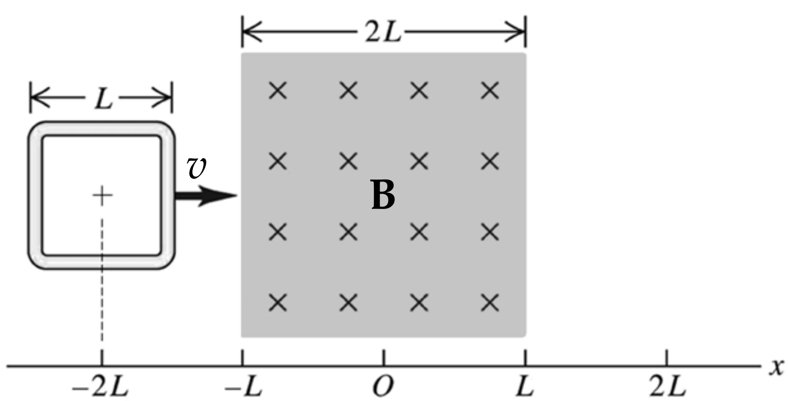
A4 A square loop of wire with resistance R is moved at constant speed v across a uniform magnetic field confined to a square region whose sides are twice the length of those of the square loop. The x coordinate of the loop is given by the value of x at its center.

a. What is the largest force $|F|_{\max}$ acting on the loop?

b. Graph the external force F needed to move the loop at constant speed as a function of its coordinate x from $x = -2L$ to $x = +2L$. Take positive force to be to the right.

c. What is the largest induced current $|I|_{\max}$ in the loop?

d. Graph the induced current I in the loop as a function of its x coordinate. Take counterclockwise currents to be positive.



Electrodynamics Group B - Answer only two Group B questions

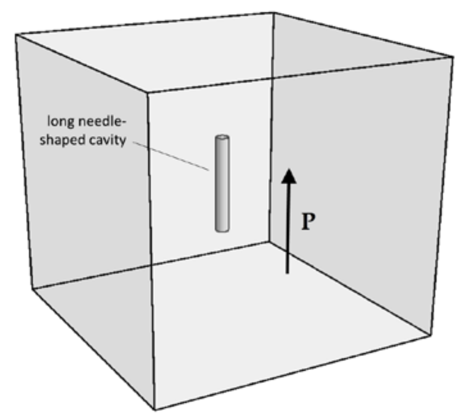
B1 The time-average potential of a neutral hydrogen atom is given by

$$\Phi(r) = q\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right)$$

where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius.

Find the charge distribution (both continuous and discrete) which will give this potential and interpret your result physically.

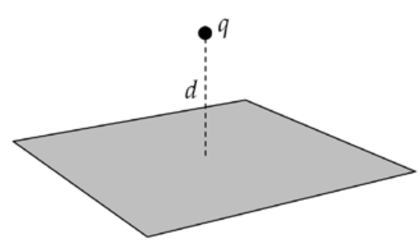
B2 Suppose the electric field inside a large piece of isotropic dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \varepsilon_0\mathbf{E}_0 + \mathbf{P}$. Now a long, thin, needle-shaped cavity is hollowed out of the material. This cavity runs parallel to $\mathbf{P}$. We assume the polarization is "frozen in", so it doesn't change when the cavity is made. We also assume the cavity is small enough that $\mathbf{P}$, $\mathbf{E}_0$, and $\mathbf{D}_0$ are essentially uniform in the solid.



a. Find the electric field vector $\mathbf{E}$ at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$.

b. Find the electric displacement vector $\mathbf{D}$ at the center of the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$.

B3 A point charge q is held stationary at a distance d away from an infinite plane conductor held at zero potential.



Find …

a. … the surface charge density induced on the plane, and plot it;

b. … the force between the plane and the charge.

B4 Consider a sphere of radius R with a constant uniform magnetization $\mathbf{M}$. The magnetic field inside the sphere is given by $\mathbf{B} = \frac{2}{3}\mu_0\mathbf{M}$.

a. Calculate the surface current density at any point on the surface.
b. Calculate the tangential component of the B field just outside the sphere.
c. Calculate the normal component of the B field just outside the sphere.

Physical Constants

speed of light .... $c = 2.998 \times 10^{8}$ m/s

Planck's constant .... $h = 6.626 \times 10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055 \times 10^{-34}$ J·s

Boltzmann constant ...... $k_{\mathrm{B}} = 1.381 \times 10^{-23}$ J/K

elementary charge .... $e = 1.602 \times 10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257 \times 10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

Avogrado constant .... $N_{\mathrm{A}} = 6.022 \times 10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}$ m/F

electron mass .... $m_{\mathrm{el}} = 9.109 \times 10^{-31}$ kg

electron rest energy...... 511.0 keV

Compton wavelength .. $h / m_{\mathrm{el}} c = 2.426$ pm

proton mass .... $m_{\mathrm{p}} = 1.673 \times 10^{-27}\,\mathrm{kg} = 1836 m_{\mathrm{el}}$

1 bohr............................ $a_0 = \hbar^2 / k e^2 m_{\mathrm{el}} = 0.5292$ Å

1 hartree (= 2 rydberg) ... $E_{\mathrm{h}} = \hbar^2 / m_{\mathrm{el}} a_0{}^2 = 27.21$ eV

gravitational constant ... $G = 6.674 \times 10^{-11}$ m^3/ kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

ELECTROSTATICS

$$\oiint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\mathrm{enc}}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\boldsymbol{\ell} = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E} \cdot d\boldsymbol{\ell} = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \tfrac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2 / 2C$

Multipole expansion:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho(\mathbf{r}')d\tau' + \frac{1}{r^2}\int r' \cos(\theta')\rho(\mathbf{r}')d\tau' + \frac{1}{r^3}\int (r')^2 \left\{\tfrac{3}{2}\cos(\theta') - \tfrac{1}{2}\right\} \rho(\mathbf{r}')d\tau' + \dots\right]$$

where the first term is the monopole term, the second is the dipole term, the third is the quadru-pole term … ; $\mathbf{r}$ and $\mathbf{r}'$ are field point and source point and θ' is the angle between them.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \nabla \cdot \mathbf{D} = \rho_{\mathrm{f}} \qquad \rho_{\mathrm{b}} = -\nabla \cdot \mathbf{P} \qquad \sigma_{\mathrm{b}} = \mathbf{P} \cdot \hat{\mathbf{n}}$$

The above are true for *all* dielectrics. Confining ourselves to linear, isotropic, and homogeneous (LIH) dielectrics, we also have:

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{P} = \chi_{\mathrm{e}} \varepsilon_0 \mathbf{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_{\mathrm{e}}) \qquad \kappa_{\mathrm{e}} = \varepsilon / \varepsilon_0 \qquad \chi_{\mathrm{e}} = \kappa_{\mathrm{e}} - 1$$

$$C(\text{dielectric}) = \kappa_{\mathrm{e}} C(\text{vacuum})$$

Boundary condition: $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$

MAGNETOSTATICS

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$ Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\ell$

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\int \dfrac{Id\ell\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

For surface currents: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\int \dfrac{\mathbf{K}\times\hat{\mathbf{R}}}{R^2}da$

For straight wire segment: $B = \dfrac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]$ where s is the perpendicular distance from wire.

Infinitely long solenoid: B-field inside is $B = \mu_0 nI$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B}\cdot d\ell = \mu_0 I_{\text{enclosed}}$

Magnetic vector potential A

$\mathbf{B} = \nabla\times\mathbf{A}$

$\nabla^2\mathbf{A} = -\mu_0\mathbf{J} \quad\Rightarrow\quad \mathbf{A} = \dfrac{\mu_0}{4\pi}\int \dfrac{\mathbf{J}}{r-r'}d\tau'$

For line and surface currents $\mathbf{A} = \dfrac{\mu_0}{4\pi}\int \dfrac{I}{r-r'}d\ell$ $\qquad \mathbf{A} = \dfrac{\mu_0}{4\pi}\int \dfrac{\mathbf{K}}{r-r'}da'$

From Stokes' theorem $\oint \mathbf{A}\cdot d\ell = \int \mathbf{B}\cdot d\mathbf{a}$

For a magnetic dipole $\mathbf{m}$, $\mathbf{A}_{\text{dipole}} = \dfrac{\mu_0}{4\pi}\dfrac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$

Magnetic dipoles

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I\int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\dfrac{1}{r^3}\left[3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}\right]$

The dipole-dipole interaction energy is $U_{\text{DD}} = \dfrac{\mu_0}{4\pi R^3}\left[(\mathbf{m}_1\cdot\mathbf{m}_2) - 3(\mathbf{m}_1\cdot\hat{\mathbf{R}})(\mathbf{m}_2\cdot\hat{\mathbf{R}})\right]$, where $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$.

Material with magnetization M

produces a magnetic field equivalent to that of (bound) volume and surface current densities

$$\mathbf{J}_{\mathrm{b}} = \nabla \times \mathbf{M} \quad \text{and} \quad \mathbf{K}_{\mathrm{b}} = \mathbf{M} \times \hat{\mathbf{n}}$$

$$\oint \mathbf{H} \cdot d\ell = I_{\text{free, enclosed}} \qquad \mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$$

For linear magnetic material $\mathbf{M} = \chi_{\mathrm{m}}\mathbf{H}$ and $\mathbf{B} = \mu_0(1+\chi_{\mathrm{m}})\mathbf{H}$ or $\mathbf{B} = \mu\mathbf{H}$

Boundary conditions: $\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$

Maxwell's Equations in vacuum

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho_{\mathrm{f}}}{\varepsilon}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu \mathbf{J}_{\mathrm{f}} + \varepsilon \mu \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E} \cdot d\ell = -\dfrac{d\Phi}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \frac{1}{2} L I^2 = \frac{1}{2} \oint \mathbf{A} \cdot \mathbf{I}\, d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl : $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian : $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

Curl :
$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

Curl : $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$

Laplacian : $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2}	$\frac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2}	$\frac{1}{2a}$
$x^2e^{-ax^2}$	$\frac{\sqrt{\pi}}{4a^{3/2}}$
$x^3e^{-ax^2}$	$\frac{1}{2a^2}$
$x^4e^{-ax^2}$	$\frac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5e^{-ax^2}$	$\frac{1}{a^3}$
$x^6e^{-ax^2}$	$\frac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+ay^2}\,dy = \pi / 2a^{1/2}$$

$$\int_0^\infty y^n e^{-ay}\,dy = \frac{n!}{a^{n+1}}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right), \qquad \left|\tan^{-1}\left(\frac{x}{a}\right)\right| < \frac{\pi}{2}$$

$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2)$$

$$\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$

$$= -\frac{1}{ab}\coth^{-1}\left(\frac{ax}{b}\right)$$

$$= -\frac{1}{ab}\tanh^{-1}\left(\frac{ax}{b}\right), \qquad a^2x^2 < b^2$$