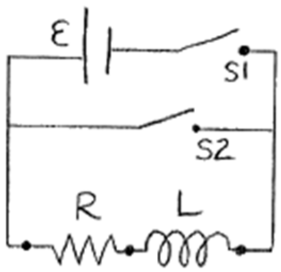
E&M A1

Consider the following bizarre circuit shown below. Initially, S1 is closed, and remains closed until a steady-state situation is reached. The battery potential is 5V, L = 4 mH, and R = 2Ω. What is the energy stored in the inductor? Now, S1 is opened, and, simultaneously, S2 is closed. How much power is being dissipated in the resistor 5 msec after S2 is closed?



$$I_{initial} = \frac{\varepsilon}{R} = \frac{5}{2} V/\Omega = \underline{2.5\,A}$$

$$U = \frac{1}{2} L I^2 = \frac{25}{8} \times 4 \times 10^{-3}\ J = \boxed{12.5\ mJ}$$

$$I = I_0 e^{-(R/L)t}$$

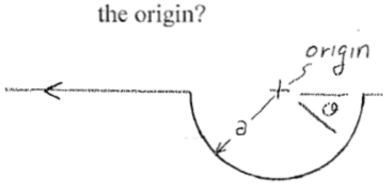
@ 5 ms

$$= 2.5A * \exp\left(-\frac{2}{4\times10^{-3}} \times 5\times10^{-3}\right) = \boxed{205\ mA}$$

$$P = i^2 R = \boxed{84\ mW}$$

E&M A2

Consider a long current-carrying wire extending to infinity along the +. and –x axes. Near the origin, it is given a semi-circular deformation of radius a as shown. The wire carries a current I from right to left. What is the magnitude and direction of the B-field at the origin?



- No contribution @ origin from straight segments.
- all $d\vec{B}$s into page from semi-circular segment

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

$$|\vec{B}(0)| = \frac{\mu_0}{4\pi} I \frac{1}{a^2} \int_0^{180°} d\vec{\ell} = \frac{\mu_0 I}{4\pi a^2} \int_0^{\pi} a\,d\theta = \frac{\mu_0 I \pi a}{4\pi a^2} \Rightarrow \boxed{\frac{\mu_0 I}{4a}(-\hat{z}) = \vec{B}}$$

E&M A3

Consider a Gaussian cylinder of radius 2.5cm, concentric with the conductors.

$$\frac{1}{\epsilon_0} \lambda(2cm)\, l = -2\pi r_0\, l\, E_0$$

$$\frac{1}{\epsilon_0} \lambda(2cm) = -2\pi \times 1.5\times10^{-2} m \times 7 N/C$$

$$8.98\times10^{9}\ \lambda(2cm) = -0.75\times10^{-2}\times 7$$

$$\lambda(2cm) = \frac{-7.5\times10^{-3}\times 7}{8.98\times 10^{9}} = -5.85\times10^{-12}\ C/m$$

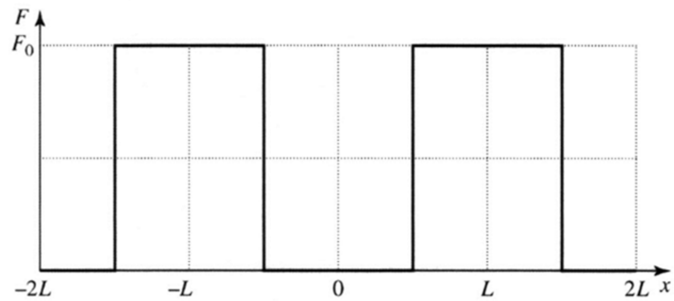
$$15 = X - 5.85$$

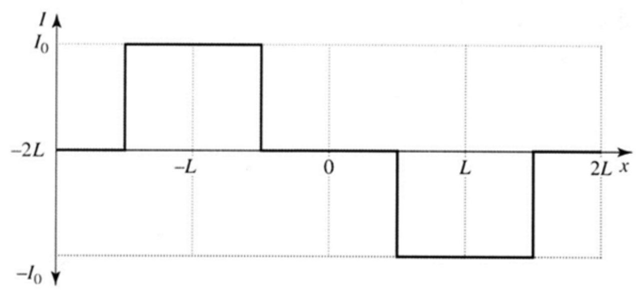
$$\boxed{\lambda(2.5cm) = +20.85\ pC/m}$$

E&M A4

Where $\varepsilon = vBL = IR \Rightarrow I_0 = \frac{vBL}{R}$ and $F_0 = ILB = \frac{vB^2L^2}{R}$.

For the loop pulled through the region of magnetic field,





Where $\varepsilon = vBL = IR \Rightarrow I_0 = \frac{vBL}{R}$ and $F_0 = ILB = \frac{vB^2L^2}{R}$.

E&M B1

$$\Phi = q\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right)$$

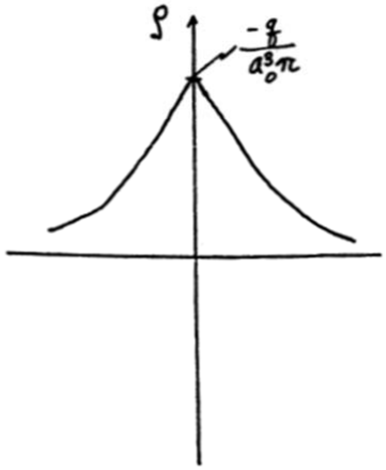
(a) When r = 0, Φ has a singularity at r = 0.

$$r \to 0,\quad \nabla^2 \Phi = \nabla^2 (q/r) = -4\pi q\,\delta(r)$$

It is interpreted as a point charges situates at r = 0 with a magnitude of $4\pi q$.

(b) r ≠ 0 $\quad \nabla^2 \Phi = -4\pi \rho(r)$

By shperical coordinates



$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right)$$

$$= -\frac{q}{r^2}\frac{\partial}{\partial r}\left[e^{-\alpha r}\left(\alpha r + 1 + \frac{\alpha^2 r^2}{2}\right)\right] = \frac{\alpha^3 q e^{-\alpha r}}{2}$$

$$\rho = -\frac{q}{a_0^3 \pi} e^{-\frac{2r}{a_0}}$$

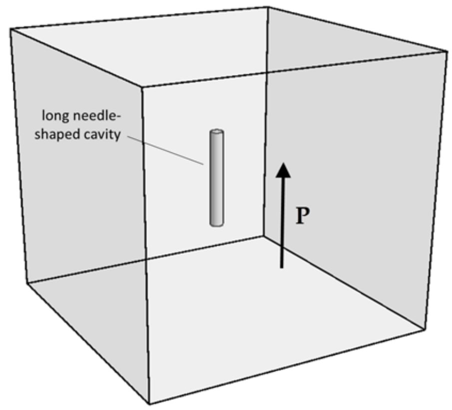
The charge distribution is continuous except at r = 0. With a maximum value $-\frac{q}{a^3\pi}$, the charge density decay exponentially. The total orbital electronic charge Q

$$Q = \int_V \rho(r)\,dv = -\frac{4q}{a_0^3}\int_0^\infty r^2 e^{-\frac{2r}{a_0}}\,dr = -\frac{q}{2}\Gamma(3)$$

$$= q$$

i.e. there is a charge equal and opposite to the total electronic charge of the atom at r= 0, which is the nucleus.

E&M B2



B2 Suppose the electric field inside a large piece of isotropic dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \varepsilon_0 \mathbf{E}_0 + \mathbf{P}$. Now a long, thin, needle-shaped cavity is hollowed out of the material. This cavity runs parallel to $\mathbf{P}$. We assume the polarization is "frozen in", so it doesn't change when the cavity is made. We also assume the cavity is small enough that $\mathbf{P}$, $\mathbf{E}_0$, and $\mathbf{D}_0$ are essentially uniform in the solid.

a. Find the electric field vector $\mathbf{E}$ at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$.

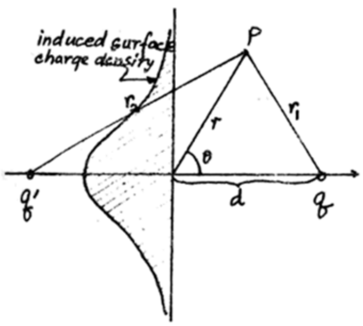
b. Find the electric displacement vector $\mathbf{D}$ at the center of the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$.

Part a. The boundary condition for the *E* field is $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$, but here $\sigma = \sigma_{\text{b}} = \mathbf{P}\cdot\hat{\mathbf{n}} = 0$, so $\mathbf{E}_{\text{in needle}} = \mathbf{E}_{\text{solid}} = \mathbf{E}_0$.

Part b. We have no polarization in the cavity, so $\mathbf{P}_{\text{in needle}} = \mathbf{0}$.

Hence, $\mathbf{D}_{\text{in needle}} = \varepsilon_0 \mathbf{E}_{\text{in needle}} + \mathbf{P}_{\text{in needle}} = \varepsilon_0 \mathbf{E}_0 = \mathbf{D}_0 - \mathbf{P}$

E&M B3



The potential at point P (Φ_P)

$$\Phi_P = \frac{q}{|\vec{r}_1|} - \frac{q}{|\vec{r}_2|} \quad \ldots\ldots \quad (1)$$

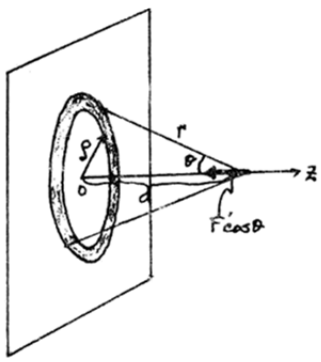
where $r_1 = \sqrt{r^2+d^2-2rd\cos\theta}$; $r_2 = \sqrt{r^2+d^2+2rd\cos\theta}$

$$\hat{n}\cdot(\vec{E}_2 - \vec{E}_1) = 4\pi\sigma$$

$$\hat{n}\cdot(\nabla\Phi) = -4\pi\sigma$$

Use spherical polar co-ordinates with the z axis passing through q and q'. Since Φ is not a function of φ, we obtain

$$\sigma = -\frac{1}{4\pi}\,\hat{n}\cdot\left(\hat{e}_r\frac{\partial\Phi}{\partial r} + \frac{\hat{e}_\theta}{r}\frac{\partial\Phi}{\partial\theta}\right) \quad (2)$$



Substitute $\theta = \pi/2$ and $r = \rho$ i.e. on the conducting plane where ρ is the radius on the plane with the origin as the center. Eq. (2) becomes:

$$\sigma = -\frac{qd}{2\pi(\rho^2+d^2)^{3/2}} \quad (3)$$

By Coulomb 's Law of force between its image.

$$\vec{F} = \frac{qq'}{(2d)^2}\hat{k} = -\frac{q^2}{(2d)^2}\hat{k}$$

E&M B4

B4 Consider a sphere of radius R with a constant uniform magnetization $\mathbf{M}$. The magnetic field inside the sphere is given by $\mathbf{B} = \frac{2}{3}\mu_0\mathbf{M}$.

a. Calculate the surface current density at any point on the surface.
b. Calculate the tangential component of the B field just outside the sphere.
c. Calculate the normal component of the B field just outside the sphere.

SOLUTION

We'll need

$$\hat{\mathbf{x}} = \sin\theta\cos\varphi\hat{\mathbf{r}} + \cos\theta cos\varphi\hat{\boldsymbol{\theta}} - \sin\varphi\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{y}} = \sin\theta\sin\varphi\hat{\mathbf{r}} + \cos\theta\sin\varphi\hat{\boldsymbol{\theta}} + \cos\varphi\hat{\boldsymbol{\varphi}}$$

$$\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$$

We also have $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = (\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}})\times\hat{\mathbf{r}} = -\sin\theta\hat{\boldsymbol{\theta}}\times\hat{\mathbf{r}} = -\sin\theta(-\hat{\boldsymbol{\phi}}) = \sin\theta\hat{\boldsymbol{\phi}}$

Define these quantities:

$\mathbf{B}^{(i)}{}_{\perp} =$ inside field perpendicular

$\mathbf{B}^{(i)}{}_{\parallel} =$ inside field parallel

$\mathbf{B}^{(o)}{}_{\perp} =$ outside field perpendicular

$\mathbf{B}^{(o)}{}_{\parallel} =$ outside field parallel

$$\mathbf{B}^{(i)} = \tfrac{2}{3}\mu_0\mathbf{M} = \tfrac{2}{3}\mu_0 M\hat{\mathbf{z}} = \tfrac{2}{3}\mu_0 M(\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}) = \underbrace{\tfrac{2}{3}\mu_0 M\cos\theta\hat{\mathbf{r}}}_{=\mathbf{B}^{(i)}{}_{\perp}} \underbrace{-\tfrac{2}{3}\mu_0 M\sin\theta\hat{\boldsymbol{\theta}}}_{=\mathbf{B}^{(i)}{}_{\parallel}}$$

$$\mathbf{K}_{\text{b}} = \mathbf{K}_{\text{b},\parallel} + \mathbf{K}_{\text{b},\perp} = \mathbf{M}\times\hat{\mathbf{n}} = M\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \underbrace{M\sin\theta\hat{\boldsymbol{\phi}}}_{\text{NOTE: this is }\parallel\text{!}}$$

so $\mathbf{K}_{\text{b},\parallel} = M\sin\theta\hat{\boldsymbol{\phi}}$ and $\mathbf{K}_{\text{b},\perp} = \mathbf{0}$

$$\frac{\mathbf{B}^{(o)} - \mathbf{B}^{(i)}}{\mu_0} = \underbrace{\mathbf{K} \times \hat{\mathbf{n}}}_{\text{is } \parallel} \qquad \text{so} \quad \mathbf{B}_{\perp}^{(o)} = \mathbf{B}_{\perp}^{(i)} = \tfrac{2}{3}\mu_0 M \cos\theta \hat{\mathbf{r}}$$

$$\frac{\mathbf{B}_{\parallel}^{(o)} - \mathbf{B}_{\parallel}^{(i)}}{\mu_0} = \mathbf{K}_{\parallel} \times \hat{\mathbf{n}} = M \sin\theta \hat{\boldsymbol{\phi}} \times \hat{\mathbf{r}} = M \sin\theta \hat{\boldsymbol{\theta}} \quad \Rightarrow$$

$$\Rightarrow \quad \mathbf{B}_{\parallel}^{(o)} = \mathbf{B}_{\parallel}^{(i)} + \mu_0 M \sin\theta \hat{\boldsymbol{\theta}} = -\tfrac{2}{3}\mu_0 M \sin\theta \hat{\boldsymbol{\theta}} + \mu_0 M \sin\theta \hat{\boldsymbol{\theta}} = \tfrac{1}{3}\mu_0 M \sin\theta \hat{\boldsymbol{\theta}}$$