UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
Thursday, August 13, 2015

This test covers the topics of *Quantum Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Quantum Mechanics Group A - Answer only two Group A questions

A1 A particle is in the second excited state ($n = 3$) in a one-dimensional, inifinitely deep square potential well with walls at $x = 0$ and $x = L$.

Find the probability of the particle being in the region $\frac{1}{3}L < x < \frac{2}{3}L$.

A2 The ground-state wavefunction of a particle of mass m in a 1D potential $V(x) = \frac{1}{2}kx^2$ has the form $\varphi(x) = Ae^{-\alpha x^2}$, where A is the normalization factor and α is some positive real constant. Assume that this is all we know about α.

Use the time-independent Schrödinger equation to determine the energy of the particle in this state in terms of m and k.

A3 A short light pulse of energy $E = 7.5$ J falls in the form of a narrow and almost parallel beam on a flat mirror plate whose reflection coefficient is 0.60. The beam is perpendicular to the mirror plate.

Find the momentum transferred to the plate.

A4 Photoelectrons emitted from a caesium plate illuminated with ultraviolet light of wave-length 200 nm are stopped by a potential of 4.2 V.

What is the work function of caesium (in eV)?

B1 A beam of electrons is "chopped" in length so that the wavefunction of each electron in the beam at $t=0$ is given by

$$\psi(x)=\begin{cases} Ae^{ikx} & \text{for } 0<x<a \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the real-valued normalization constant A.
b. Find the position uncertainty Δx.
c. Find $\varphi(p)$, the electron wavefunction in the momentum space.
d. Find the minimum momentum uncertainty Δp.
e. A student argues that since, for the given wavefunction, $\langle p\rangle=-i\hbar\int\psi^*(x)\frac{d\psi(x)}{dx}dx=\hbar k$, and $\langle p^2\rangle=-\hbar^2\int\psi^*(x)\frac{d^2\psi(x)}{dx^2}dx=\hbar^2k^2$, we have $(\Delta p)^2=\langle p^2\rangle-\langle p\rangle^2=0$. What is wrong in this argument?
f. Find the integral representation of $\psi(x,t)$ for $t>0$ (but don't do the integral).

B2 A harmonic oscillator (mass *m*, frequency ω_0) is initially in the ground state. A uniform field with the potential energy $V(x)=-Fx$ is suddenly applied.

a. Write down the Hamiltonian of the perturbed oscillator.
b. What is the energy spectrum of the perturbed oscillator?
c. What is the probability that the oscillator will remain in the ground state? *Hint: complete the square to write the perturbed Hamiltonian and to do the integral in part c.*

B3 A hydrogen atom is in its ground state. Its wavefunction is $\psi(\mathbf{r})=Ae^{-r/a_0}$.

a. Find the normalization constant *A*;
b. Find the radial probability density and the most probable distance of the electron from the proton;
c. Find $\langle r\rangle$, the expectation value of *r*;
d. Find the classical turning point for the electron motion;
e. Find the probability for the electron to be detected in the classically forbidden region.

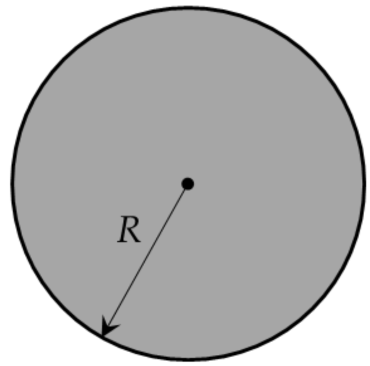
B4 We consider the three operators *A*, *B*, and *C*, given by

$A=p^2-x^2$, $B=xp+px$, and $C=p^2+x^2$.

a. Show that $[A,B]=-4i\hbar C$.
b. Show that $[A,C]=-4i\hbar B$.

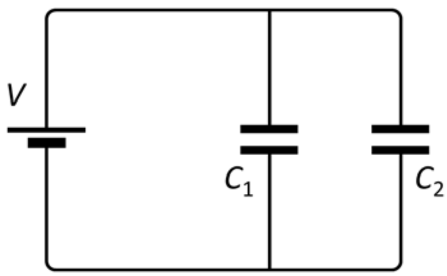
Electrodynamics Group A - Answer only two Group A questions

A1 The figure shows a circular region of radius $R = 3\text{ cm}$ in which a uniform electric flux is directed out of the page. The total electric flux through the region is given by $\Phi_E = (3\text{ mV}\cdot\text{m/s})\,t$, where t is in seconds.



What is the magnitude of the magnetic field that is induced at radial distances

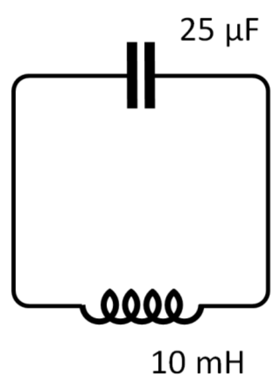
a. 2 cm, and
b. 5 cm?



A2 In the circuit shown, how much charge is stored on the parallel-plate capacitors by the 12 V battery? One is filled with air, and the other is filled with a dielectric for which $\kappa = 3$; both capacitors have a plate area of $5\times10^{-3}\text{ m}^2$ and a plate separation of 2 mm.

A3 A circular current loop of radius 1 m carries a current of 200 A in a counterclockwise direction; a concentric second loop of radius 2 mm lies initially in the plane of the first and carries a current of 3 mA in a clockwise direction. How much work must you do to rotate the inner loop 180° about an axis that intersects the outer loop at two diametrically opposite points?

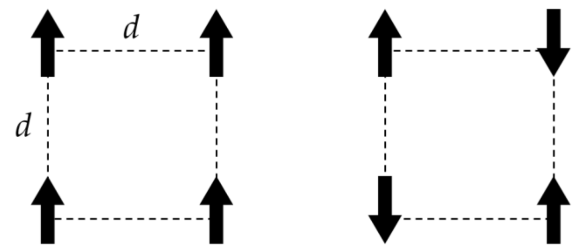
A4 A 300 volts DC power supply is used to charge a 25 μF capacitor. After the capacitor is fully charged, it is disconnected from the power supply, and connected, at time $t = 0$, across a 10 mH inductor.



After how much time are the energies stored in the capacitor and in the inductor equal for the first time?

Electrodynamics Group B - Answer only two Group B questions

B1 The figure shows two different ways to arrange four identical magnetic dipoles (all of magnitude $m = |\mathbf{m}|$) on the corners of a square with sides of length d. In configuration (a) all four dipoles are pointing up. In (b), two dipoles are pointing up, and the other two are pointing down.



(a) (b)

What is the difference in energy between the configurations (a) and (b)?

Note: the magnetic field produced by a magnetic dipole **m** at some point P is

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[3(\mathbf{m}\cdot\mathbf{n})\mathbf{n} - \mathbf{m}\right],$$

where $\mathbf{r}$ is the vector pointing from the dipole to point P, $r = |\mathbf{r}|$, and $\mathbf{n} = \mathbf{r}/r$.

B2 A copper wire of length 1 m is stretched tightly between two pegs and is immersed in magnetic field of 30 mT whose direction is perpendicular to the wire. The tension in the wire is 55 N, and its resistance is 0.01 Ω. The wire is vibrating at its fundamental frequency, and the midpoint of the wire has a maximum displacement from its equilibrium position of 1 mm. The resistivity of copper is 1.67 μΩ-cm ; its mass density is 9.0 g/cm^3. What is the voltage amplitude across the wire's ends?

B3 A charge Q is distributed uniformly over a thin rod of length L. The same amount of charge (with the same sign) is distributed uniformly over a second, identical rod infinitely far away from the first. The two rods are now moved in from infinity and both placed on the x-axis, between $x = -L$ and the origin, and between $x = L$ and $x = 2L$, respectively. How much work must be done to assemble this charged-rod combination?

B4 A ferromagnetic rod of diameter 1 cm and length 3 cm has a fixed uniform magnetization $\mathbf{M}_0$ parallel to its cylindrical axis.

a. Sketch the **H** field lines everywhere in the rod and within 3 cm of all its surfaces.
b. Do the same for the **B** field lines (in a separate drawing).

Physical Constants

speed of light.................. $c = 2.998\times10^{8}$ m/s

Planck's constant .... $h = 6.626\times10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055\times10^{-34}$ J·s

Boltzmann constant ...... $k_B = 1.381\times10^{-23}$ J/K

elementary charge .... $e = 1.602\times10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854\times10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257\times10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

Avogrado constant .... $N_A = 6.022\times10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988\times10^{9}$ m/F

electron mass .... $m_{el} = 9.109\times10^{-31}$ kg

electron rest energy...... 511.0 keV

Compton wavelength .. $h / m_{el}c = 2.426$ pm

proton mass .... $m_p = 1.673\times10^{-27}\ \text{kg} = 1836 m_{el}$

1 bohr............................ $a_0 = \hbar^2 / ke^2 m_{el} = 0.5292$ Å

1 hartree (= 2 rydberg) ... $E_h = \hbar^2 / m_{el} a_0^{\,2} = 27.21$ eV

gravitational constant ... $G = 6.674\times10^{-11}$ m^3/ kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

QUANTUM MECHANICS

Energy levels in a one-dimensional, infinitely deep box of width a:

$$E_n = n^2 \frac{\pi^2\hbar^2}{2ma^2}$$

Angular momentum:

$$[L_x, L_y] = i\hbar L_z \quad \text{et cycl.}$$

ELECTROSTATICS

$$\oiint_S \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\ell = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\ell = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2 / 2C$

Multipole expansion:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho(\mathbf{r}')d\tau' + \frac{1}{r^2}\int r'\cos(\theta')\rho(\mathbf{r}')d\tau' + \frac{1}{r^3}\int (r')^2\left\{\tfrac{3}{2}\cos(\theta') - \tfrac{1}{2}\right\}\rho(\mathbf{r}')d\tau' + \ldots\right]$$

where the first term is the monopole term, the second is the dipole term, the third is the quadrupole term … ; $\mathbf{r}$ and $\mathbf{r}'$ are field point and source point and θ' is the angle between them.

$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ $\quad \nabla\cdot\mathbf{D} = \rho_f$ $\quad \rho_b = -\nabla\cdot\mathbf{P}$ $\quad \sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$

The above are true for *all* dielectrics. Confining ourselves to linear, isotropic, and homogeneous (LIH) dielectrics, we also have:

$\mathbf{D} = \varepsilon\mathbf{E}$ $\quad \mathbf{P} = \chi_e\varepsilon_0\mathbf{E}$ $\quad \varepsilon = \varepsilon_0(1+\chi_e)$ $\quad \kappa_e = \varepsilon/\varepsilon_0$ $\quad \chi_e = \kappa_e - 1$

$C(\text{dielectric}) = \kappa_e C(\text{vacuum})$

Boundary condition: $\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$

MAGNETOSTATICS

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$ Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\boldsymbol{\ell}$.

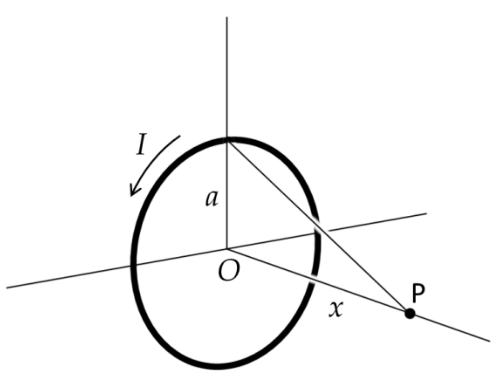
Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{I d\boldsymbol{\ell}\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

For surface currents: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{\mathbf{K}\times\hat{\mathbf{R}}}{R^2}da$

For straight wire segment: $B = \dfrac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]$ where s is the perpendicular distance from wire.

Infinitely long solenoid: B-field inside is $B = \mu_0 nI$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{enclosed}}$



Field of loop at P, on axis, at distance x from center:

$$|\mathbf{B}_P| = \mu_0\frac{Ia^2}{2(x^2+a^2)^{3/2}}$$

Magnetic vector potential A

$\mathbf{B} = \nabla \times \mathbf{A}$

$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \quad \Rightarrow \quad \mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}}{r - r'} d\tau'$

For line and surface currents $\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{I}{r - r'} d\ell \qquad \mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}}{r - r'} da'$

From Stokes' theorem $\oint \mathbf{A} \cdot d\ell = \int \mathbf{B} \cdot d\mathbf{a}$

For a magnetic dipole $\mathbf{m}$, $\mathbf{A}_{\text{dipole}} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}$

Magnetic dipoles

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I \int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m} \right]$

Material with magnetization M

produces a magnetic field equivalent to that of (bound) volume and surface current densities

$\mathbf{J}_{\text{b}} = \nabla \times \mathbf{M}$ and $\mathbf{K}_{\text{b}} = \mathbf{M} \times \hat{\mathbf{n}}$

$\oint \mathbf{H} \cdot d\ell = I_{\text{free, enclosed}} \qquad \mathbf{H} = \mathbf{B} / \mu_0 - \mathbf{M}$

For linear magnetic material $\mathbf{M} = \chi_{\text{m}} \mathbf{H}$ and $\mathbf{B} = \mu_0 (1 + \chi_{\text{m}}) \mathbf{H}$ or $\mathbf{B} = \mu \mathbf{H}$

Boundary conditions: $\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0 (\mathbf{K} \times \hat{\mathbf{n}})$

Maxwell's Equations in vacuum

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho_{\mathrm{f}}}{\varepsilon}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu \mathbf{J}_{\mathrm{f}} + \varepsilon \mu \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\dfrac{d\Phi}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \frac{1}{2} L I^2 = \frac{1}{2} \oint \mathbf{A} \cdot \mathbf{I}\, d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient : $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl : $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian : $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl :
$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient : $\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence : $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl : $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$

Laplacian : $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2}	$\frac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2}	$\frac{1}{2a}$
$x^2e^{-ax^2}$	$\frac{\sqrt{\pi}}{4a^{3/2}}$
$x^3e^{-ax^2}$	$\frac{1}{2a^2}$
$x^4e^{-ax^2}$	$\frac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5e^{-ax^2}$	$\frac{1}{a^3}$
$x^6e^{-ax^2}$	$\frac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+ay^2}dy = \pi/2a^{1/2}$$

$$\int_0^\infty y^n e^{-ay}dy = \frac{n!}{a^{n+1}}$$

$$\int x^{-2}\ln(x)\,dx = -\frac{\ln(x)}{x} - \frac{1}{x}$$

$$\int x^{-1}\ln(x)\,dx = \tfrac{1}{2}\ln^2(x)$$

$$\int \ln(x)\,dx = x\ln(x) - x$$

$$\int x\ln(x)\,dx = \tfrac{1}{2}x^2\ln(x) - \tfrac{1}{4}x^2$$

$$\int x^2\ln(x)\,dx = \tfrac{1}{3}x^3\ln(x) - \tfrac{1}{9}x^3$$

$$\int \frac{r^3dr}{(x^2+r^2)^{3/2}} = (r^2+x^2)^{1/2} + \frac{x^2}{(r^2+x^2)^{1/2}}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad \left|\tan^{-1}\left(\frac{x}{a}\right)\right| < \frac{\pi}{2}$$

$$\int \frac{xdx}{a^2+x^2} = \frac{1}{2}\ln\left(a^2+x^2\right)$$

$$\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$

$$= -\frac{1}{ab}\coth^{-1}\left(\frac{ax}{b}\right)$$

$$= -\frac{1}{ab}\tanh^{-1}\left(\frac{ax}{b}\right), \quad a^2x^2 < b^2$$