UNL Department of Physics and Astronomy

Preliminary Examination – Day 1

May, 2012

This test covers the topics of Mechanics (Topic 1) and Thermodynamics and Statistical Mechanics (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work 2 problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Mechanics Group A. Answer only 2 Group A questions.

A1. A one-dimensional potential is given by $V(x) = (1 - e^{-ax^2})$ joules, where $a = 5$ m^{-2}. A point mass of 2 kg executes small-amplitude motion about the bottom of this potential well. Calculate the period of this motion.

A2. Particles of mud are thrown from the rim of a wheel that is rolling without slipping. If the forward speed of the wheel is v and its radius is R, find the maximum height above the ground which the mud reaches. Ignore air drag and fenders.

A3. A plank of weight W and length $\sqrt{3}*R$ lies in a smooth circular trough of radius R. At one end of the plank (attached to it) is a small mass of weight $W/2$. When the plank is in equilibrium, what angle does it make with the horizontal?

A4. Consider the force function

$$\vec{\boldsymbol{F}} = y\hat{\boldsymbol{x}} + x\hat{\boldsymbol{y}} + z^2\hat{\boldsymbol{z}}\,.$$

a) Is it a conservative function?

b) If the answer to a) is "yes,"find the corresponding potential function.

c) Calculate the line integral

$$\int \vec{F} \cdot d\vec{r}$$

for the path $\vec{\boldsymbol{r}} = t\hat{\boldsymbol{x}} + t^2\hat{\boldsymbol{y}} + t\hat{\boldsymbol{z}}$ from (0,0,0) to (3,9,3).

Mechanics Group B. Answer only 2 Group B questions.

B1. The interaction between an atom and an ion at distances greater than contact is given by the potential energy $V(r) = -C\,r^{-4}$.

(a) Sketch the effective potential energy as a function of r, assuming some value of the angular momentum ℓ.

(b) If the total energy of the ion exceeds the maximum value of the effective potential energy V_o, the ion can strike the atom (i.e. reach r = 0). Find V_o in terms of the angular momentum ℓ.

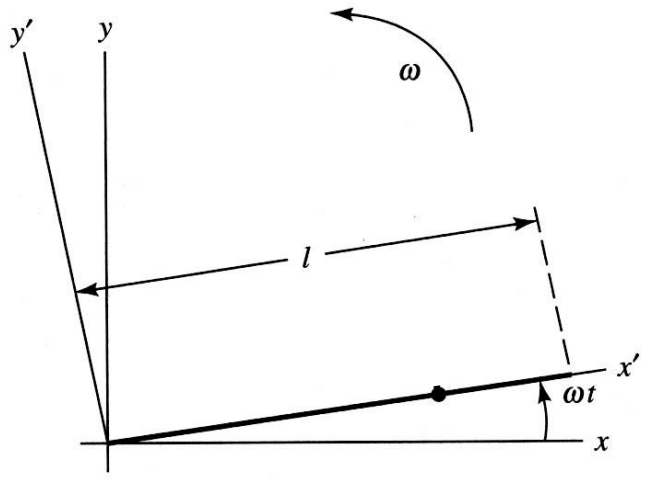
(c) If the ion starts out very far away from the atom with velocity v_o, what is the largest possible value of the impact parameter b such that the ion will strike the atom?

B2. A smooth rod of length l rotates in a plane with a constant angular speed ω about an axis fixed at the end of the rod and perpendicular to the plane of rotation. A bead of mass m is initially released from at rest (relative to the rod) at its midpoint. The bead is constrained to move along the rod without friction. Calculate:

(a) the displacement of the bead along the rod as a function of time.

(b) the time when the bead leaves the end of rod.

(c) the linear speed (relative to the rod) with which the bead leaves the end of rod.

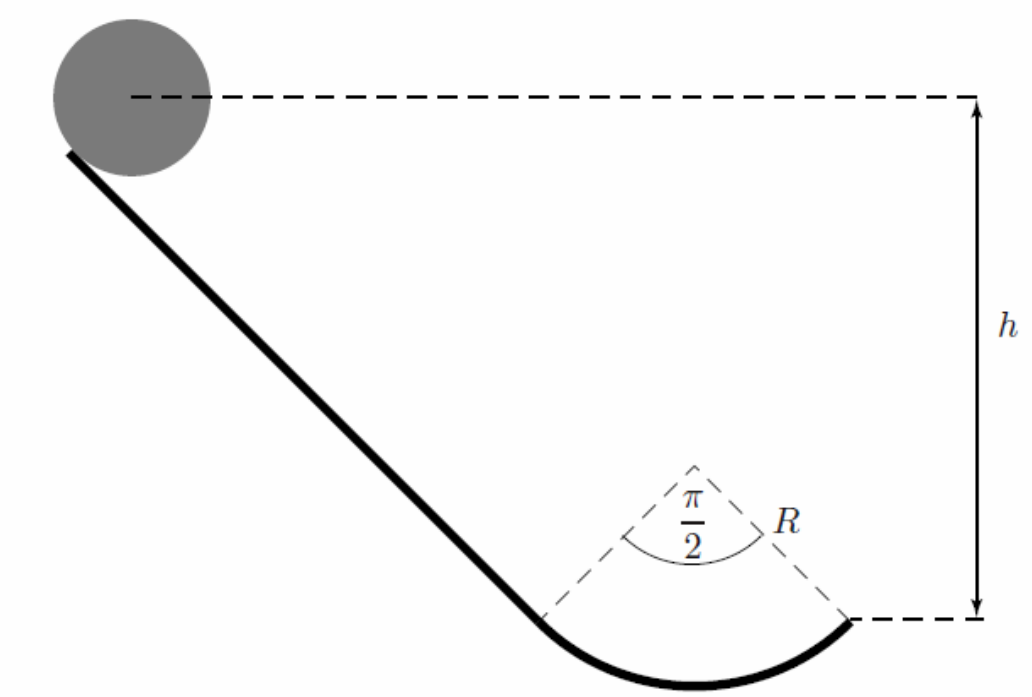


B3. A sphere of radius *r* and mass *m* rolls without slipping down a ramp inclined at 45° to the horizontal. At the bottom of the ramp is a quarter-circle arc of radius R that meets the ramp smoothly (see figure). When the sphere is released, its center is a distance *h* above the right end of the circular track.

(a) What is the velocity of the sphere when it reaches the end of the circular track?

(b) What is the maximum height reached by the sphere after it loses contact with the track?

(c) What is the horizontal distance of the sphere from the end of the track at the time it reaches its maximum height in (b)?

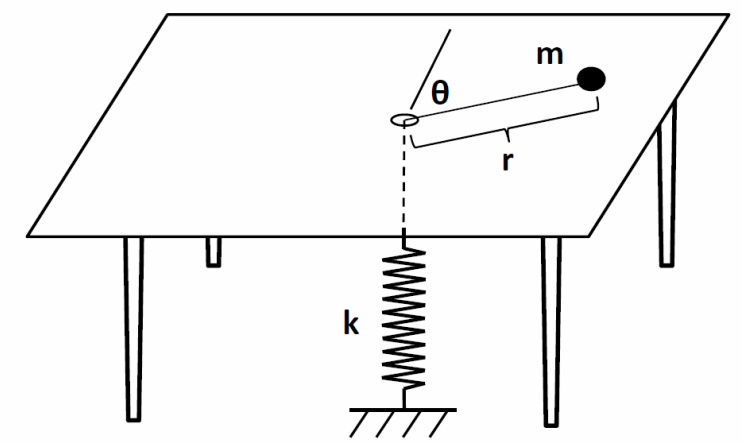


B4. Consider a small mass moving on a frictionless table attached to a vertical massless spring of spring constant *k*, as shown, by a massless string. The spring exerts no force when the mass is directly above it. At t = 0, the mass is given a velocity v_0 perpendicular to the string of which a part of length *a* lies on the table at that time.

(a) Write down the Lagrangian function for the system in terms of the generalized coordinates *r* = the length of the string on the table, and θ, the angle between the string on the table and some arbitrary radial line painted on the table.

(b) Is either coordinate ignorable? Which, if any?

(c) Write down the Lagrangian equations of motion.



Thermodynamics and Statistics Group A. Answer only 2 Group A questions.

A1. An ideal monatomic gas expands isobarically at 100 kPa from a volume of 1 liter to a volume of 2 liters. What is the amount of heat that the gas absorbs in the process?

A2. An isolated container is split by a wall into two compartments. The first compartment, of volume V_1, contains N molecules of an ideal gas. The other compartment of volume V_2 is evacuated. The wall is removed, the gas fills the whole container, and after some time the system reaches equilibrium. Compute the change in the Helmholtz free energy between the initial and the final equilibrium states.

A3. A 1.5 kg glass brick is heated to 180°C and then plunged into a cold bath containing 10 kg of water at 20°C. Assume that none of the water boils and that there is no heating of the surroundings. What is the final temperature of the water and the glass? The specific heat of glass is approximately 750 J/kg K, and that of water 4184 J/kg K. It takes 334 J to melt 1 gram of ice.

A4. What is the minimum amount of electric energy that a household freezer needs to consume in order to freeze 1 kg of water, assuming that the room temperature is 20°C? The heat capacity of water is 4184 J/kg K.

Thermodynamics and Statistics Group B. Answer only 2 Group B questions.

B1. One mole of an ideal monatomic gas undergoes a process starting from equilibrium at the pressure of 100 kPa and temperature of 100°C. In the final equilibrium state the pressure is 200 kPa and the temperature is 50°C. Find the change in the entropy of the gas in this process assuming that

(a) the process is reversible.

(b) the process is irreversible.

B2. Two dice are rolled and the sum S on their top faces is examined.

(a) List the possible outcomes and their respective probabilities for S.

(b) What is the most probable value of S, and which combination(s) of (die 1, die 2) corresponds to it?

(c) Find the mean value of S and its standard deviation for the case when the double dice throw is made a large number of times.

B3. Show that the heat absorbed by a system in an isobaric process is equal to the change of its enthalpy, under the condition that only mechanical work is involved.

B4. Derive an equation connecting the volume and temperature of a fixed amount of monatomic ideal gas undergoing a reversible adiabatic process.

EQUATIONS THAT MAY BE HELPFUL

General efficiency η of a heat engine producing work $|W|$ while taking in heat $|Q_h|$ is $\eta = \frac{|W|}{|Q_h|}$.

For a Carnot cycle operating as a heat engine between reservoirs at T_h and at T_c the efficiency becomes $\eta_C = \frac{T_h - T_c}{T_h}$.

Clausius' theorem: $\sum_{i=1}^{N} \frac{Q_i}{T_i} \leq 0$ which become $\sum_{i=1}^{N} \frac{Q_i}{T_i} = 0$ for a reversible cyclic process of N steps.

$\frac{dP}{dT} = \lambda/(T\Delta V)$; specific heat of water: 4186 J/(kg*K); Latent heat of ice melting: 334 J/g

$\vec{a'} = \vec{a} - 2\vec{\omega} \times \vec{v'} - \dot{\vec{\omega}} \times \vec{r'} - \vec{\omega} \times (\vec{\omega} \times \vec{r'})$

H = E + PV F = E – TS G = F + PV Ω = F – μN

dE = TdS – PdV + μdN dS = dE/T + PdV/T – μdN/T dH = TdS + VdP + μdN

dF = -SdT – PdV + μdN dG = -SdT + VdP + μdN dΩ = -SdT – PdV - Ndμ