UNL - Department of Physics and Astronomy

Preliminary Examination - Day 1
Thursday, May 12, 2016

This test covers the topics of *Quantum Mechanics* (Topic 1) and *Electrodynamics* (Topic 2). Each topic has 4 "A" questions and 4 "B" questions. Work two problems from each group. Thus, you will work on a total of 8 questions today, 4 from each topic.

Note: If you do more than two problems in a group, only the first two (in the order they appear in this handout) will be graded. For instance, if you do problems A1, A3, and A4, only A1 and A3 will be graded.

WRITE YOUR ANSWERS ON ONE SIDE OF THE PAPER ONLY

Quantum Mechanics Group A - Answer only two Group A questions

A1 A proton is confined to an atomic nucleus of diameter 3×10^{-15} m. Use Heisenberg's uncertainty principle to estimate the proton's binding energy. Give the answer in electronvolts (eV).

A2 We consider the Hermitian operator $\hat{A}$ and the non-Hermitian operator $\hat{Q}$.

a. Is $\exp(\hat{A})$ a Hermitian operator?

b. Is $i[\hat{Q},\hat{Q}^{\dagger}]$ a Hermitian operator?

In the following part, $\hat{x}$ and $\hat{p}$ are the position and momentum operators, respectively.

c. Show that $[\hat{x}^n,\hat{p}] = ni\hbar\hat{x}^{n-1}$ for $n = 1, 2, 3, \ldots$

A3 At $t = 0$ the wavefunction of a particle in a one-dimensional, infinitely-deep box with walls at $x = 0$ and $x = a$ is given by

$$\psi(x,0) = C\sin(2\pi x/a)\cos(\pi x/a),$$

where C is a constant.

a. Find $\psi(x,t)$ for $t > 0$.

b. What is the energy expectation value for $t > 0$? Express answers for parts (*a*) and (*b*) in terms of E_n ($n = 1,2,\ldots$), the energies of the stationary states.

c. Find the probability P_n that the measurement of the particle's energy at the time *t* finds the value E_n.

A4 A particle with spin 1 is in the state with the spin projection on the *z* axis $S_z = \hbar$.

a. What are the expectation values of $S_x{}^2$ and $S_y{}^2$ for this state?

b. Suppose S_x is measured. What are the possible outcomes of this measurement and what are the corresponding probabilities?

Quantum Mechanics Group B - Answer only two Group B questions

B1 An electron in the Coulomb field of a proton is in a state described by the wave function

$$\psi(\mathbf{r}) = \tfrac{1}{2}\sqrt{2}\left[\psi_{n00}(\mathbf{r}) - \psi_{210}(\mathbf{r})\right],$$

where $\psi_{n\ell m}(\mathbf{r})$ is the wave function of a stationary state with principal quantum number *n*, angular momentum quantum number ℓ, and its projection *m*.

a. Is this state an eigenstate of the Hamiltonian if $n=1$? If $n=2$? Explain.

b. Find the expectation value of the energy for $n=1$ and for $n=2$. Express your answers in terms of the energy of the ground state, $E_1 = -R$, where *R* is the Rydberg constant.

c. Find the expectation value of $\mathbf{L}^2$.

d. Suppose $n=2$. Find the expectation value of **r** using the following information: $\int z\,\psi_{210}(\mathbf{r})\psi_{200}(\mathbf{r})d\mathbf{r} = -3a_0$ (a_0 is the Bohr radius).

B2 Consider a particle with spin $\frac{1}{2}$ and gyromagnetic ratio γ.

a. Find the eigenvalues and eigenstates of S_x.

b. Suppose the particle is in the state corresponding to the larger eigenvalue. Find the expectation values of S_y, S_z, $S_y^{\ 2}$, and $S_z^{\ 2}$ for this state.

c. Using the results obtained in part (*b*), find $\langle S_x^{\ 2} + S_y^{\ 2} + S_z^{\ 2}\rangle$. Can you obtain the answer in a more straightforward way?

d. Now place the particle in a magnetic field **B** that is directed along the *x* axis. Write the Hamiltonian *H*. Consider the following statement: The solutions found in part (*a*) are eigenstates of the Hamiltonian. Is this statement correct? If yes, find the corresponding energy eigenvalues. If no, find the expectation values $\langle H\rangle$ for these states.

B3 Consider a system whose wavefunction is

$$|\psi\rangle = \frac{1}{\sqrt{7}}|1,-1\rangle + A\,|1,0\rangle + \sqrt{\frac{2}{7}}\,|1,1\rangle ,$$

where the $|\ell, m\rangle$ are angular momentum eigenstates. The quantity A is a real-valued, positive constant.

a. Calculate A so that $|\psi\rangle$ is normalized.
b. Calculate the expectation values of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, and $\hat{L}^2$ for this state $|\psi\rangle$.
c. Find the probability that a measurement of the z component of the angular momentum gives $L_z = \hbar$.
d. Calculate $\langle 1,m|(\hat{L}_+)^2|\psi\rangle$ and $\langle 1,m|(\hat{L}_-)^2|\psi\rangle$.

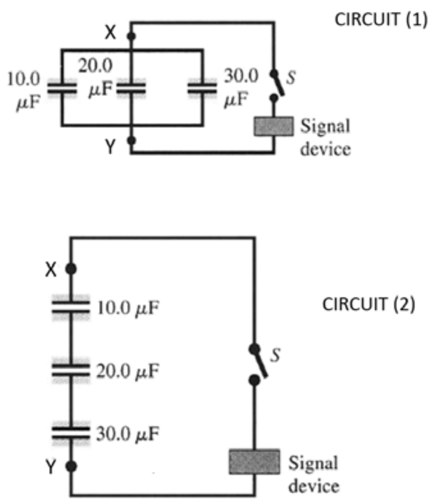
B4 Consider a harmonic oscillator with Hamiltonian $\hat{H} = \dfrac{\hat{p}^2}{2m} + \tfrac{1}{2}m\omega_0{}^2\hat{x}^2 = (\hat{a}^\dagger\hat{a} + \tfrac{1}{2})\hbar\omega_0$, and stationary states given by $\hat{H}|n\rangle = E_n|n\rangle = (n+\tfrac{1}{2})\hbar\omega_0|n\rangle$. The annihilation operator $\hat{a}$ is defined by $\hat{a} = \dfrac{\beta}{\sqrt{2}}\left(\hat{x} + \dfrac{i}{m\omega_0}\hat{p}\right)$ (with $\beta^2 = m\omega_0/\hbar$), and the creation operator $\hat{a}^\dagger$ is its Hermitian adjoint. The annihilation and creation operators act as $\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$, respectively. The oscillator is in a normalized so-called coherent state, $|\psi\rangle = C\sum_{n=0}^{\infty}\dfrac{\alpha^n}{\sqrt{n!}}|n\rangle$, where C and α are constants.

a. Determine the (real-valued) normalization constant C. Note: $e^x = \Sigma_k\, x^k/k!$
b. Show that $|\psi\rangle$ is an eigenstate of the annihilation operator $\hat{a}$ with eigenvalue α.
c. Explain why the creation operator $\hat{a}^\dagger$ cannot have eigenstates.
d. Calculate $\langle x\rangle = \langle\psi|\hat{x}|\psi\rangle$. Hint: write $\hat{x}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$, and use part (*b*).

Electrodynamics Group A - Answer only two Group A questions

A1 A 90 Ω resistor, a 100 μF capacitor, and a 0.45 H inductor are connected in series to a 120 V_{rms} , 60 Hz power source. What is the average power consumption in this circuit?

A2 A microwave oven heats food by using electromagnetic waves that form a standing wave in the oven. When 2.5 GHz waves are used this way in a sample of food, it is found that there are burn marks spaced 5.5 cm apart. What is the index of refraction of the food sample for these electromagnetic waves?

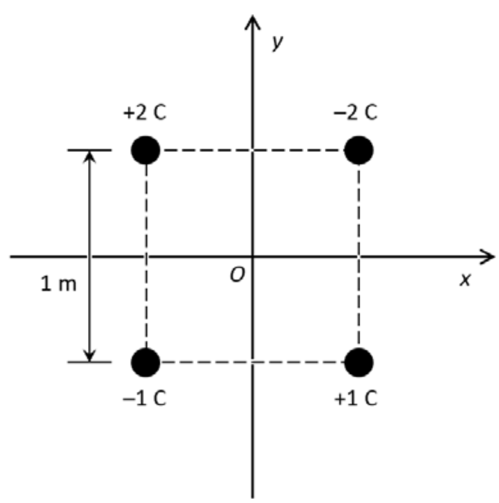


A3 Each combination of capacitors between points X and Y in the figure is first connected across a 120-V battery, charging the combination to 120 V. These combinations are then connected to make the circuits shown. When the switch S is thrown, a surge of charge from the discharging capacitors flows to trigger the signal device. How much charge flows through the signal device in each case?

A4 Four point charges are arranged at the corners of a square, as shown in the figure.

a. Find the electric field **E** at the center of the square.

b. Find the potential V at the center of the square, assuming $V(\infty) = 0$.



Electrodynamics Group B - Answer only two Group B questions

B1 Two semi-infinite line charges with uniform linear charge density $+\lambda$ lie along the x-axis from $-\infty$ to $-a/2$ and from ∞ to $a/2$, respectively. A charge $+q$ is initially placed at rest at the origin. If it receives a small impulse along the x-axis, what is its period of oscillation along this axis?

B2 Two coils (A and B) made out of the same wire are in a uniform magnetic field with the coil axes aligned in the field direction. Coil A has 20 turns of radius 5 cm and coil B has 10 turns with an unknown radius. The field strength is now doubled in 2 s and it is found that the ratio of the emf induced in each coil $\mathrm{EMF_A : EMF_B}$ is 1:2. What is the radius of coil B?

B3 A long, solid metal cylinder with radius a is coaxial with long, thin, and hollow metal tube with radius $b > a$. The positive charge per unit length on the inner cylinder is λ, and there is an equal negative charge per unit length on the outer cylinder. We set $V(r = b) = 0$.

a. Calculate the potential $V(r)$ for $r < a$.
b. Calculate the potential $V(r)$ for $a < r < b$.
c. Calculate the potential $V(r)$ for $r > b$.
d. Calculate the electric field at any point between the cylinders.
e. What would the potential difference between the two cylinders have been if the outer cylinder had had no net charge?

B4 In an *LRC* series a.c. circuit, the voltage amplitude of source is $\mathcal{E} = 120\ \mathrm{V}$, the resistance is $R = 80\ \Omega$, and the reactance of the capacitor is $480\ \Omega$. The voltage amplitude across the capacitor is 135 V.

a. What is the current amplitude in the circuit?
b. What is the impedance?
c. What two values can the reactance of the inductor have?
d. For which of the two values found in part (*c*) is the angular frequency less than the resonance angular frequency?

Physical Constants

speed of light .... $c = 2.998 \times 10^{8}$ m/s

Planck's constant .... $h = 6.626 \times 10^{-34}$ J·s

Planck's constant / 2π.... $\hbar = 1.055 \times 10^{-34}$ J·s

Boltzmann constant ...... $k_B = 1.381 \times 10^{-23}$ J/K

elementary charge .... $e = 1.602 \times 10^{-19}$ C

electric permittivity ...... $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

magnetic permeability ... $\mu_0 = 1.257 \times 10^{-6}$ H/m

molar gas constant.......... $R = 8.314$ J / mol·K

Avogrado constant .... $N_A = 6.022 \times 10^{23}$ mol^{-1}

electrostatic constant ... $k = (4\pi\varepsilon_0)^{-1} = 8.988 \times 10^{9}$ m/F

electron mass .... $m_{el} = 9.109 \times 10^{-31}$ kg

electron rest energy...... 511.0 keV

Compton wavelength .. $\lambda_C = h / m_{el}c = 2.426$ pm

proton mass .... $m_p = 1.673 \times 10^{-27}\,\text{kg} = 1836 m_{el}$

1 bohr............................ $a_0 = \hbar^2 / k e^2 m_{el} = 0.5292$ Å

1 hartree (= 2 rydberg) ... $E_h = \hbar^2 / m_{el} a_0^{\,2} = 27.21$ eV

gravitational constant ... $G = 6.674 \times 10^{-11}$ m^3 / kg s^2

hc .... $hc = 1240$ eV·nm

Equations That May Be Helpful

TRIGONOMETRY

$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

QUANTUM MECHANICS

Particle in one-dimensional, infinitely-deep box with walls at $x = 0$ and $x = a$:

Stationary states $\psi_n = (2/a)^{1/2} \sin(n\pi x / a)$, energy levels $E_n = n^2 \dfrac{\pi^2\hbar^2}{2ma^2}$

Angular momentum: $[L_x, L_y] = i\hbar L_z$ *et cycl.*

Ladder operators:

$$L_+ \,|\,\ell, m\rangle = \hbar\sqrt{(\ell+m+1)(\ell-m)}\,|\,\ell, m+1\rangle$$
$$L_- \,|\,\ell, m\rangle = \hbar\sqrt{(\ell+m)(\ell-m+1)}\,|\,\ell, m-1\rangle$$

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ELECTROSTATICS

$$\oiint_S \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \qquad \mathbf{E} = -\nabla V \qquad \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E}\cdot d\boldsymbol{\ell} = V(\mathbf{r}_1) - V(\mathbf{r}_2) \qquad V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Work done $W = -\int_{\mathbf{a}}^{\mathbf{b}} q\mathbf{E}\cdot d\boldsymbol{\ell} = q\left[V(\mathbf{b}) - V(\mathbf{a})\right]$ Energy stored in elec. field: $W = \frac{1}{2}\varepsilon_0 \int_V E^2 d\tau = Q^2/2C$

MAGNETOSTATICS

Lorentz Force: $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$ Current densities: $I = \int \mathbf{J}\cdot d\mathbf{A}$, $I = \int \mathbf{K}\cdot d\boldsymbol{\ell}$

Biot-Savart Law: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\boldsymbol{\ell}\times\hat{\mathbf{R}}}{R^2}$ ($\mathbf{R}$ is vector from source point to field point $\mathbf{r}$)

For straight wire segment: $B = \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]$ where s is the perpendicular distance from wire.

Infinitely long solenoid: B-field inside is $B = \mu_0 n I$ (n is number of turns per unit length)

Ampere's law: $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{enclosed}}$

Magnetic dipole moment of a current distribution is given by $\mathbf{m} = I\int d\mathbf{a}$.

Force on magnetic dipole: $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$

Torque on magnetic dipole: $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$

B-field of magnetic dipole: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}\right]$

The dipole-dipole interaction energy is $U_{\text{DD}} = \frac{\mu_0}{4\pi R^3}\left[(\mathbf{m}_1\cdot\mathbf{m}_2) - 3(\mathbf{m}_1\cdot\hat{\mathbf{R}})(\mathbf{m}_2\cdot\hat{\mathbf{R}})\right]$, where $\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2$.

Maxwell's Equations in vacuum

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ Ampere's Law with Maxwell's correction

Maxwell's Equations in linear, isotropic, and homogeneous (LIH) media

1. $\nabla \cdot \mathbf{D} = \rho_{\mathrm{f}}$ Gauss' Law
2. $\nabla \cdot \mathbf{B} = 0$ no magnetic charge
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ Faraday's Law
4. $\nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{f}} + \dfrac{\partial \mathbf{D}}{\partial t}$ Ampere's Law with Maxwell's correction

Induction

Alternative way of writing Faraday's Law: $\oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\dfrac{d\Phi}{dt}$

Mutual and self inductance: $\Phi_2 = M_{21} I_1$, and $M_{21} = M_{12}$; $\Phi = LI$

Energy stored in magnetic field: $W = \frac{1}{2}\mu_0^{-1} \int_V B^2 d\tau = \frac{1}{2} L I^2 = \frac{1}{2} \oint \mathbf{A} \cdot \mathbf{I}\, d\ell$

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $d\tau = dx\,dy\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient : $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl : $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian : $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $d\tau = s\,ds\,d\phi\,dz$

Gradient : $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence : $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl : $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$$

Laplacian : $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

Second Derivatives

(9) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

(10) $\nabla\times(\nabla f) = 0$

(11) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f)\cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int(\nabla\cdot\mathbf{A})\,d\tau = \oint\mathbf{A}\cdot d\mathbf{a}$

Curl Theorem : $\int(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \oint\mathbf{A}\cdot d\mathbf{l}$

INTEGRALS

$f(x)$	$\int_0^\infty f(x)\,dx$
e^{-ax^2}	$\frac{\sqrt{\pi}}{2\sqrt{a}}$
xe^{-ax^2}	$\frac{1}{2a}$
$x^2e^{-ax^2}$	$\frac{\sqrt{\pi}}{4a^{3/2}}$
$x^3e^{-ax^2}$	$\frac{1}{2a^2}$
$x^4e^{-ax^2}$	$\frac{3\sqrt{\pi}}{8a^{5/2}}$
$x^5e^{-ax^2}$	$\frac{1}{a^3}$
$x^6e^{-ax^2}$	$\frac{15\sqrt{\pi}}{16a^{7/2}}$

$$\int_0^\infty \frac{1}{1+ay^2}dy = \pi / 2a^{1/2}$$

$$\int_0^\infty y^n e^{-ay} dy = \frac{n!}{a^{n+1}}$$

$$\int \frac{r^3 dr}{(x^2+r^2)^{3/2}} = (r^2+x^2)^{1/2} + \frac{x^2}{(r^2+x^2)^{1/2}}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad \left|\tan^{-1}\left(\frac{x}{a}\right)\right| < \frac{\pi}{2}$$

$$\int \frac{xdx}{a^2+x^2} = \frac{1}{2}\ln\left(a^2+x^2\right)$$

$$\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$$

$$\int \frac{dx}{a^2x^2-b^2} = \frac{1}{2ab}\ln\left(\frac{ax-b}{ax+b}\right)$$

$$= -\frac{1}{ab}\coth^{-1}\left(\frac{ax}{b}\right)$$

$$= -\frac{1}{ab}\tanh^{-1}\left(\frac{ax}{b}\right), \quad a^2x^2 < b^2$$